ZCA 110 Kalkulus dan Aljabar
Semester I, Sesi 2005/06
QUIZ 10 (30 Sept 2005)
Techiques of Integration

Nama:
No. Kad Matriks:
Kumpulan Tutorial:
[total (4 + 4+ 2) marks = 10 marks]

(a) Evaluate $\int \ln x \, dx$. You must show your working. [*Hint:* Use integration by parts].

[4 marks]

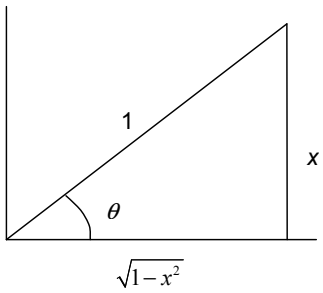
Solution:

$$u \equiv \ln x, \Rightarrow du = \frac{1}{x}dx;$$
$$dv \equiv dx \Rightarrow v = x;$$
$$\int \ln x \, dx \equiv \int u \, dv = uv - \int v \, du$$
$$= x\ln x - \int x \, \frac{1}{x}dx = x\ln x - \int dx$$
$$= x\ln x - x + C$$

(b) Integrate $\int \frac{x}{\sqrt{1-x^2}}dx$. [No hint. Think of a suitable method yourself.]

[4 marks]

Solution: Strategy II, pg. 294.
Let
$x = \sin\theta \Rightarrow dx = \cos\theta d\theta$
$1 - x^2 = \cos^2\theta \Rightarrow \sqrt{1-x^2} = \cos\theta$
$$\Rightarrow \int x\frac{dx}{\sqrt{1-x^2}} = \int \sin\theta \cdot \frac{\cos\theta d\theta}{\cos\theta} = \int \sin\theta d\theta = -\cos\theta + C = -\sqrt{1-x^2} + C$$



(c) To integrate $\int \frac{x-1}{(x-2)(x+2)}dx$ using the method of partial fractions, one breakdowns the integrand $\frac{x-1}{(x-2)(x+2)}$ into its partial fraction sum as per $\frac{x-1}{(x-2)(x+2)} = \frac{A}{(x-2)} + \frac{B}{(x+2)}$. Now consider the integration: $\int \frac{1}{x(x^2+2)}dx$. Express the integrand $\frac{1}{x(x^2+2)}$ in the form of partial fractions. [You do not need to evaluate the integration or the constants of the partial fractions].

[2 marks]

Solution: Example 8, pg. 307.

$$\frac{1}{x(x^2+2)} = \frac{A}{x} + \frac{Cx+D}{x^2+2}$$