ZCA 110 Kalkulus dan Aljabar

Semester I, Sessi 2005/06

QUIZ 2 (29 Julai 2005)

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Kumpulan Tutorial:

Q1 Proof that $f(x) = x$ is everywhere continuous

Solution. Theorem 8.2 (pg. 73).

First, we must prove that $\lim_{x \to a} f(x) = a$, where a is a real number. Moreover $f(a)$ is defined and is given by $f(a)$ = a. Hence, $\lim_{x \to a} f(x) = a = f(a)$. By definition, $f(x) = x$ is thus everywhere continuous.

To prove the existence of $\lim_{x \to a} f(x) = a$, we are required to show that for any arbitrary $\varepsilon > 0$, there exists a $\delta > 0$ such that whenever $|x - a| < \delta$, $|f(x) - a| < \varepsilon$.

To begin with, let us assumes $\delta > 0$, and consider x to be confined within the domain of $(a - \delta, a + \delta)$, but with $x \neq 0$. In other words we consider x that is constrained by the condition $0 < |x - a| < \delta$.

Let $f(x) = x$. Hence, $|f(x) - a| = |x - a|$.
But $|x - a| < \delta$.
$\therefore |f(x) - a| = |x - a| < \delta$.
If we choose $\delta = \varepsilon$, then $|f(x) - a| < \varepsilon$. Thus, $\lim_{x \to a} f(x) = a$ is proven.

Q2

Find the discontinuity of the following functions. Determine whether they are removable or non removable.

(a) $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0$

(b) $f(x) = \begin{cases} 4 & \text{if } x = 2 \\ 0 & \text{if } x \neq 2 \end{cases}$

Solutions

(a) Schaum's series, Solved Pro. 1(d), pg 76

A polynomial has no discontinuity

(b) Schaum's series, Solved Pro. 1(j), pg 76

Has a removable discontinuity at x = 2.