ZCA 110 Kalkulus dan Aljabar

Semester I, Sessi 2005/06

QUIZ 4 (12 August 2005)

Nama:
No. Kad Matriks:
Kumpulan Tutorial:
[7 marks]

Given $y = ax^2 - bx + c$ on the interval [x_1, x_2], where *a*, *b*, *c*, are some real numbers. Assume the function has two roots within [x_1, x_2].

(a) Find a value of x_0 prescribed by the Law of the Mean **[3 marks]**

(b) Find a value of x'_0 prescribed by Rolle's theorem **[2 marks]**

(c) Find the slope (gradient) of the tangent lines at the value of *x* in (b) **[1 mark]**
(d) Find the slope (gradient) of the normal lines at the value of *x* in (b) **[1 mark]**

Solution: modified from Suppl Problem 17(b), pg 114 and Solved problem 1, pg. 111
(a) [3 marks]
Law of the Mean:

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(x_0)$$ 1 mark

LHS, $f'(x_0) = 2ax_0 - b$ 1 mark

$$\therefore 2ax_0 - b = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \Rightarrow x_0 = \frac{f(x_2) - f(x_1)}{2a(x_2 - x_1)} + b$$

$$= \frac{x_2 + x_1}{2}$$ 1 mark

(b) **[2 marks]**
Let the roots be $x = x'_1, x = x'_2 \Rightarrow f(x'_1) = f(x'_2) = 0$
Rolle's Theorem gaurantees that
$f'(x'_0) = 0$ for a point x'_0 between (x'_1, x'_2) 1 mark
LHS, $f'(x'_0) = 2ax'_0 - b$
$\therefore 2ax'_0 - b = 0 \Rightarrow x'_0 = b/2a$ 1 mark

(c) The slope of tangent line at $x'_0 = b/2a$ is zero [1 mark]

(d) The slope of normal line at $x'_0 = b/2a$ is ∞ [1 mark]