

ZCA 110 Kalkulus dan Aljabar

Semester I, Sessi 2005/06

QUIZ 7 (9 Sept 2005)

Nama:

No. Kad Matriks:

Kumpulan Tutorial:

[total (4 + 3) marks = 7 marks]

(a) Estimate $\sec 44^\circ$ without using calculator.

[4 marks]

Leave your answer in terms of π and $\sqrt{2}$ where necessary.

Solution:

Know $\sec 45^\circ = 1/\cos 45^\circ = \sqrt{2}$. Want to find $\sec 44^\circ = \sec(45^\circ - 1^\circ)$.

Let $f(x) = \sec x$

$$\therefore \frac{\sec(x + \Delta x) - \sec x}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \equiv \frac{\Delta y}{\Delta x}$$

$$\sec(x + \Delta x) = \Delta y + \sec x$$

Δy can be taken to be approximated by the differential $df = f'(x)\Delta x$

$$\text{i.e. } \Delta y \approx df = f'(x)\Delta x = \sec x \tan x \Delta x$$

$$\therefore \sec(x + \Delta x) = \sec x \tan x \Delta x + \sec x$$

Here, we want to find $\sec 44^\circ = \sec(45^\circ - 1^\circ)$

$$\Delta x = -1^\circ = -\frac{\pi}{180} \text{ radian}$$

$$\sec 44^\circ = \sec(45^\circ - 1^\circ) = \sec 45^\circ \tan 45^\circ \left(-\frac{\pi}{180}\right) + \sec 45^\circ = -\frac{\sqrt{2}\pi}{180} + \sqrt{2} = \sqrt{2} \left(1 - \frac{\pi}{180}\right)$$

(b) Integrate $\int \sec^2 x \tan x dx$

[3 marks]

Let $u = \sec x \Rightarrow du = \tan x \sec x dx$

$$\therefore \int \sec^2 x \tan x dx = \int \sec x (\sec x \tan x dx) = \int u du = \frac{u^2}{2} + c = \frac{\sec^2 x}{2} + c$$

Or

$$\int \sec^2 x \tan x dx = \int \frac{d}{dx} (\tan x) \tan x dx = \frac{\tan^2 u}{2} + c'$$