ZCA 110B

Calculus and Linear Algebra

Semester I, Sessi 2007/08

QUIZ 9 (27 Sept 2007)

Total = 12 marks

Nama:

No. Kad Matriks:

Q1 The function $f(x) = e^x + x$, being differentiable and one-to-one, has a differentiable inverse, $f^{-1}(x)$. Find the value of df^{-1}/dx at the point $f(\ln 2)$. **[5 marks]**

Q2. (i) Derive $\frac{d}{dx}\tan^{-1}x$. Show your steps clearly. Draw a diagram if it helps in explaining your derivation.

[5 marks]

(ii) Find the derivative $y = \tan^{-1}(\ln x)$. **[2 marks]**

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Solution (Thomas' pg. 549, Chapter 7 Practice exercise Q101).

$$\frac{df}{dx} = e^x + 1$$

$$\Rightarrow \left(\frac{df^{-1}}{dx}\right)_{x=f(\ln 2)} = 1/\left(\frac{df}{dx}\right)_{x=\ln 2}$$

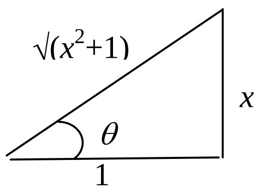
$$\Rightarrow \left(\frac{df^{-1}}{dx}\right)_{x=f(\ln 2)} = \left.\frac{1}{\left(e^x+1\right)}\right|_{x=\ln 2} = \frac{1}{\left(e^{\ln 2}+1\right)} = \frac{1}{(2+1)} = \frac{1}{3}$$

Q2. (i) Derive $\frac{d}{dx}\tan^{-1} x$. Show your steps clearly. Draw a diagram if it helps in explaining your derivation. **[5 marks]**

(ii) Find the derivative $y = \tan^{-1}(\ln x)$. **[2 marks]**

Solution (Thomas' pg. 531, Q62, exercise 7.7)

(i)



$$\frac{d}{dx}\tan^{-1} x = ?$$

Let $\theta = \tan^{-1} x \Rightarrow x = \tan\theta$

$$\rightarrow \frac{d}{dx}x = \frac{d}{dx}\tan\theta$$

$$\rightarrow 1 = \frac{d\theta}{dx}\frac{d}{d\theta}\tan\theta = \frac{d\theta}{dx}\sec^2\theta \rightarrow \frac{d\theta}{dx} = \frac{d}{dx}\tan^{-1} x = \frac{1}{\sec^2\theta} = \cos^2\theta = \frac{1}{x^2+1}$$

(ii)

$$y = \tan^{-1}(\ln x)$$

Let $u = \ln x \Rightarrow y = \tan^{-1}(u)$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1} u = \frac{du}{dx}\frac{d}{dx}\tan^{-1} x = \frac{1}{x}\frac{d}{dx}\tan^{-1} x = \frac{1}{x\left(x^2+1\right)}$$