Chapter 7 The inverse of a matrix

Answer the following designed questions. These questions are designed in accordance to the subsections as sequentially presented in Ayers. Try to identify the questions below with the corresponding subsection from which these questions are based on as it will definitely help while answering these questions.

1. For the following two questions, please refer to the designed questions (12), (13), (14), in Chapter 5. There, $E_{A\to I_3}$ A= I_3. Now, what would you get when if $E_{A\to I_3}$ is operated on I_3 instead of on A? In other words, ask yourself, what is $E_{A\to I_3}$ I_3?

Ans:

Comparing I_3= $E_{A\to I_3}$ A with the definition of the inverse of A, i.e. I_3= $A^{-1}A$, we conclude that $E_{A\to I_3} \equiv A^{-1}$. Hence,

$E_{A\to I_3}$ I_3= $A^{-1}I_3$= A^{-1}.

2. So, by now, have you learnt how to find the inverse of a matrix? Find A^{-1}, where A is as defined in DQ (12), Chapter 5, $A=\begin{pmatrix}1 & 3 & 2\\ 0 & 2 & 0\\ 5 & 0 & 1\end{pmatrix}$.

Ans:

A^{-1}= $E_{A\to I_3}$ I_3=

$$R_2(1/2)\ R_1(-1/9)\ R_3^1(5/9)\ R_1^2(-3/2)\ R_1^3(-2)\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$= R_2(1/2)\ R_1(-1/9)\ R_3^1(5/9)\ R_1^2(-3/2)\begin{pmatrix}1 & 0 & -2\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$= R_2(1/2)\ R_1(-1/9)\ R_3^1(5/9)\begin{pmatrix}1 & -3/2 & -2\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$= R_2(1/2)\ R_1(-1/9)\begin{pmatrix}1 & -3/2 & -2\\ 0 & 1 & 0\\ 5/9 & -5/6 & -1/9\end{pmatrix}$$

$$= R_2(1/2)\begin{pmatrix}-1/9 & 1/6 & 2/9\\ 0 & 1 & 0\\ 5/9 & -5/6 & -1/9\end{pmatrix} =$$

$$\begin{pmatrix}-1/9 & 1/6 & 2/9\\ 0 & 1/2 & 0\\ 5/9 & -5/6 & -1/9\end{pmatrix} = A^{-1}$$

3. Now carry out the $E_{A\to I_3}$ A= and $E_{A\to I_3}$ I_3 operations in a “two-in-one” manner, i.e. if the augmented matrix form. First form the augmented matrix of the form $(A|I)=\left(\begin{array}{ccc|ccc}1 & 3 & 2 & 1 & 0 & 0\\ 0 & 2 & 0 & 0 & 1 & 0\\ 5 & 0 & 1 & 0 & 0 & 1\end{array}\right)$. Carrying out $E_{A\to I_3}$ on both sides to arrive at $(I|A^{-1})$.

4. In the designed questions (12), (13), (14) of Chapter 5, you are asked to find a sequence of elementary transformations that transform a generic matrix A into a unit matrix. Now, if A were our old friend $\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix}$, what would happen if you were to attempt to reduce it into an identity matrix via a sequence of elementary transformations? Explain.

Ans:

Since our old friend is a singular square matrix with no inverse, it is not possible to reduce it into a unit matrix via a finite sequence of elementary transformation. If we were able to achieve that, that means there exists a finite sequence of operation similar to $E_{A\to I_3}$ such that $E_{A\to I_3}$ I_3

gives us the inverse of *A*, which is a contradiction. Hence, it is not possible to obtain a finite sequence of elementary transformation to reduce out old friend to identity matrix.

5. Deduce A^{-1}, where *A* is as defined (2), using $A^{-1}= \text{adj}A/|A|$. Do you get the same answer as in (2) where the inverse is obtained using row reduced echelon form method?

Ans: Of course yes.