

**Chapter 6 The adjoint of a square matrixQ**

Answer the following designed questions. These questions are designed in accordance to the subsections as sequentially presented in Ayers. Try to identify the questions below with the corresponding subsection from which these questions are based on as it will definitely help while answering these questions.

1. Given  $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ . (i) Obtain the cofactor  $\alpha_{ij}$  for all

$i, j$ . (ii) Form the matrix of the cofactors of  $A$ , and call it  $X$ . (iii) How is  $X$  be related to  $\text{adj } A$ ? (iv) What is  $|A|$ ? (You should have been very familiar with  $A$ , see DQ in Chapter 5.) (v) What is  $\text{adj } A \cdot A$ ? Try to relate the answer in (v) to theorem II, page 50, Ayers.

2. How would you convince yourself that indeed  $\text{adj } A \cdot A = A \cdot \text{adj } A = \text{diag}(|A|, |A|, |A|, \dots, |A|)$ ?

3. Say  $A, B$  are two matrices conformable for a product in the order of  $AB$ . What is  $|A||B|$ ?

4. How would you convince yourself that  $|A| |\text{adj } A| = |A|^n$ ? [Hint: use DQ (2).]

5. Consider this statement: If  $X$  a square matrix and singular, then  $|\text{adj } X| = 0$ . Is this statement true?

6. Try to relate this question with what you have learnt in

the secondary school. Given  $G = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ , (i) what is

$\text{Adj } G$ ? You should be able to write down the answer by inspection. (ii) What is  $|G|$ ? (iii) Let  $H = \text{Adj } G / |G|$ , work out what is  $HG$ . (iv) Based on the answer of (iii), what is the product  $GH$ ? (v) So, what can you conclude from the above exercise?