Chapter 2 Some types of matricesQ

Answer the following questions. These questions are designed in accordance to the subsections as sequentially presented in Ayers. Try to identify the questions below with the corresponding subsection from which these questions are based on as it will definitely help while answering these questions.

1. Give an example of a 3×3 (*i*) upper triangular matrix (*ii*) lower triangular matrix. (iii) Give an example of a 3×3 matrix which is both an upper and lower triangular matrix.

2. Give an example of a (*i*) 2×3 upper triangular matrix (*ii*) 2×3 lower triangular matrix.

3. (*i*) Write out explicitly I_4. (*ii*) What is $(I_4)^n$, with n a positive integer.

4. Given a $n \times n$ matrix A with $a_{ii} = \sqrt[n]{n}$, $i = 1, 2, \ldots, n$, where n a positive integer. What is the name for this type of matrix?

5. Refer to (4), express A explicitly if $n=3$.

6. Consider the matrix A as defined in (4). Find A^n.

7. Given B an n-square diagonal matrix, with $b_{ii} = j$, where $j = 1, 2, \ldots, n$. Let $C = AB$. Let c_{ij} denote the (i, j) element of the matrix C. (*i*) What is c_{ij} if $i \neq j$? What the element c_{jj}?

8. Say A is a matrix of order $n \times n$. Give three simplest matrices you can think of that will commute with A?

9. Give an example of a matrix that anti-commute with I_n.

10. Give two simplest example you can think of for an idempotent matrix.

11. Consider the matrix A in (4). What is the period of A?

12. If A is nilpotent of order n, what is A^{n+1}? n positive integer.

13. Given an n-square matrix A is the inverse of matrix B, then we write $A = B^{-1}$. (*i*) Express the inverse of A in term of B. (*ii*) What is AB? (*ii*) What is BA?

14. Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

15. Give an easiest example you can think of where the inverse of a matrix is also the matrix itself.

16. Let Q be a scalar n-square matrix such that $Q = kI_n$, where k an scalar. What is Q^{-1}?

17. Given the order of matrix A is $m \times n$, what is the order of its transpose, A'? Note that sometimes the transpose of matrix A is written as A^{T} instead of A'.

18. Consider a 2 by 2 matrix A. (*i*) what is the sufficient condition that A is also equal to its transpose (i.e. $A'=A$)?

19. Consider a 3 by 3 non-zero diagonal matrix *B*. (*i*) Is *B* symmetric? (*ii*) Could *B* ever be skew-symmetric?

20. Let *A* be an *n*-square matrix. (*i*) What is the symmetry of $A+A^{\mathrm{T}}$ (i.e. is it symmetric or skew-symmetric?). (*i*) What is the symmetry of $A-A^{\mathrm{T}}$?

21. Given the matrix, $C=\begin{pmatrix} w & y \\ x & z \end{pmatrix}$, (*i*) express *C* as a sum of a symmetric matrix and a skew-symmetric matrix.

22. (*i*) Express explicitly $\overline{C}$, the conjugate of the matrix *C* in (21). (*ii*) Express explicitly the conjugate of the matrix *kC*, where *k* a real scalar.

23. (*i*) The transpose of the conjugate of matrix *C* is written as … ? (*ii*) The conjugate of transpose of the matrix *C* is written as …? (*iii*) Are both of these equal each to other?

24. (*i*) What is *A**? (*ii*) If *A* is a matrix containing no complex element, is there any difference between *A** and A^{T}? (*iii*) In general, if *A* is a matrix containing complex element, will *A** equal A^{T} in general?

25. Is a real, symmetric matrix Hermitian?

26. (*i*) By making use of Theorem X in page 13 of Ayers 1982 impression, very quickly give an example of a 2 by 2 Hermitian matrix and an example of a 2 by 2 skew-Hermitian matrix.

27. Let $A=\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, B=\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, C=\begin{pmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{pmatrix}$. (*i*) What is the order of *S* = diag(*A*, *B*, *C*), the direct sum of *A*, *B*, *C*? (*ii*) What is the order of S^2?