Chapter 3 Determinant of a square matrix

Answer the following questions. These questions are designed in accordance to the subsections as sequentially presented in Ayers. Try to identify the questions below with the corresponding subsection from which these questions are based on as it will definitely help while answering these questions.

1. See if you understand what permutation is.

   (*i*) How many inversions are there in the permutation 1234? (*ii*) 4321? (*iii*) state whether (*i*) is odd or even? (*iv*) state whether (*ii*) is odd or even. (*v*) Give an example of an odd permutation.

2. See if you can do this: $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -3 & -3 & -3 \end{vmatrix} =$

3. See if you can do this without expanding the determinant:

   (i) $\begin{vmatrix} 43 & 88 & 22.34 & 113.8 \\ 65 & 43 & 43 & 24.9 \\ 86 & 443 & 3 & 23 \\ 0 & 0 & 0 & 0 \end{vmatrix} =$

   (ii) $\begin{vmatrix} 1 & 0 & 5 \\ 2 & 0 & 6 \\ 4 & 0 & 8 \end{vmatrix} =$

4. Can you define the determinant of a matrix that is not square? How?

5. Given that you know $\begin{vmatrix} 1 & 4 & 1 \\ 2 & 2 & 3 \\ 0 & -3 & -3 \end{vmatrix} = 21$, find

   (Q5 Continue)

   $\begin{vmatrix} 1 & 2 & 0 \\ 4 & 2 & -3 \\ 1 & 3 & -3 \end{vmatrix}$ without expansion.

6. Given that $\begin{vmatrix} 1 & 2 & 1 \\ 8 & 4 & 4 \\ -3 & 2 & -3 \end{vmatrix} = 32$, find

$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ -3 & 2 & -3 \end{vmatrix}$ without expansion.

7. Given that $\begin{vmatrix} g & j & p \\ y & t & l \\ u & s & m \end{vmatrix} = 53.889,$ find

(*i*) $\begin{vmatrix} y & t & l \\ g & j & p \\ u & s & m \end{vmatrix}$, (*ii*) $\begin{vmatrix} u & s & m \\ y & t & l \\ g & j & p \end{vmatrix}$

8. Given $|A| = \begin{vmatrix} l & o & v & e \\ s & u & k & a \\ l & i & k & e \\ a & i & 0 & 0 \end{vmatrix}$ =999, what is

(*i*) $\begin{vmatrix} o & v & l & e \\ u & k & s & a \\ i & k & l & e \\ i & 0 & a & 0 \end{vmatrix}$, (*ii*) $\begin{vmatrix} o & v & e & l \\ u & k & a & s \\ i & k & e & l \\ i & 0 & 0 & a \end{vmatrix}$?

9. How do you convince yourself that

(*i*) $\begin{vmatrix} 0 & 0 & 0 \\ y & t & l \\ g & j & p \end{vmatrix} = \begin{vmatrix} u & s & m \\ 0 & 0 & 0 \\ g & j & p \end{vmatrix} = 0$?

(*ii*) $\begin{vmatrix} o & v & 0 & l \\ u & k & 0 & s \\ i & k & 0 & l \\ i & 0 & 0 & a \end{vmatrix} = \begin{vmatrix} o & v & e & l \\ u & k & a & s \\ i & k & e & l \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$?

10. Given $\begin{vmatrix} 2 & t & l \\ 2 & j & p \\ 4 & s & m \end{vmatrix} = 10, \begin{vmatrix} 3 & t & l \\ 4 & j & p \\ 6 & s & m \end{vmatrix} = 9,$ what is $\begin{vmatrix} 5 & t & l \\ 6 & j & p \\ 10 & s & m \end{vmatrix}$?

11. Say $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = x$. (*i*) Consider the operation "add 2×(second row) to the first row". If this operation is applied to the determinant above, what do you get? Write down the expression explicitly. (*ii*) Without expansion, work out what is the value of the determinant in (*i*).

12. Consider the matrix $X = [x_{ij}] = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

    (i) What is the first minor of x_{11}? Of x_{23}? Conventionally what symbol you will use to represent these quantities?

    (ii) What is the cofactor of x_{11}? Of x_{23}? Conventionally what symbol you will use to represent these quantities?

    (iii) What is the matrix of the minors x_{ij}, $[M_{ij}]$?

    (iv) What is the matrix of the cofactors of x_{ij}, $[\alpha_{ij}]$?

13. Consider the matrix in (12). Calculate

(*i*) $\sum_{j}^{3} x_{1j}\alpha_{1j}$ (*ii*) $\sum_{i}^{3} x_{i3}\alpha_{i3}$

(*ii*) What is the quantity represented by the sum in (*i*) and (*iii*)? Do you get the same value for both (*i*) and (*ii*)? (*iv*) So, what is the determinant of *X*?

14. If you were to manually find the determinant of $\begin{vmatrix} 44.1 & 0 & 52.3 \\ 33.6 & 1.98 & 7.1 \\ 4.2 & 0 & 1.6 \end{vmatrix}$, along which column or row would you like to follow to calculate the determinant?