Chapter 4 Evaluation of determinantsQ

Answer the following questions. These questions are designed in accordance to the subsections as sequentially presented in Ayers. Try to identify the questions below with the corresponding subsection from which these questions are based on as it will definitely help while answering these questions.

Consider the operation "add to row j by ($k \times$ row i)", $j \neq I$, where k is a non-zero scalar. We will conveniently represent the above operation

by $R_j^i(k)$, and we have $A \xrightarrow{R_j^i(k)} A' = R_j^i(k)A$.

Sometimes this operation is symbolized by $\{j\} \rightarrow \{j\} + k\{i\}$, meaning: "replace row j by (row $j + k$ times row i)". In Ayer's notation, this operation is denoted by $H_{ij}(k)$. We will adopt the $R_j^i(k)$ notation

for the rest of the course. There is also a similar operation that acts on the columns, denoted by

$C_j^i(k)$.

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1. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 9 \\ 8 & 10 & 6 \end{pmatrix}$, what is $A' = R_3^2(-2)A$?

2. (*i*) What is the determinant of A in (1)? (*ii*) That of A'? (*iii*) What is your conclusion?

3. If you understand the idea intended to be conveyed in (1), what is your strategy if you were asked to

evaluate $\begin{vmatrix} 1 & 66 & 54 \\ 4 & 11 & 9 \\ 8 & 10 & 43 \end{vmatrix}$? Inspect the values of the elements

then think of the best strategy possible.

4. How about evaluating $\begin{vmatrix} 1 & 4 & 8 \\ 7 & 11 & 10 \\ 7 & 8 & 16 \end{vmatrix}$?

5. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Carry out successive operations

   of $R_j^i(k)$ to transform A into a matrix which has as

   many zero elements as possible in row 3. (*i*) What is
   your resultant matrix? (*ii*) Evaluate |*A*| manually.