ZCA 110 Calculus and Linear Algebra
Semester I, Academic session 2008/09
TEST I   1 August 2008

Name:

Matrix Number:
Group (A or B):

1. Given the 3-square matrix $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 8 \end{pmatrix}$.

(i) Use a sequence of elementary row operation to transform A into its equivalent matrix in reduced row echelon form (RREF). Show your steps clearly.

[4 marks]

(ii) What is the rank of A?

[2 marks]

(iii) Given as set of three 3-vectors, v_1=[1,2,3], v_2=[4,5,6], v_3=[6,7,8]. Based on your answer from (*i*), (*ii*), are these vectors linearly dependent? Explain your answer.

[2 marks]

(iv) Find the determinant of A.

[3 marks]

(v) Does the inverse of A exist? Explain your answer.

[2 marks]

Consider the homogeneous equation system $AX=0$, where $X=[x_1, x_2, x_3]^T$ is the column vectors of 3 unknowns.

(vi) Has the homogeneous equation system any non-trivial solutions? Explain your answer.

[2 marks]

(vii) How many linearly independent solution are there for $AX=0$?

[2 marks]

(viii) Obtain the solution for $AX=0$.

[3 marks]

Solutions:

(i) $A \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ =RREF(A) **4 marks**

(ii) The rank of A = number of leading 1 in RREF(A), $r = 2$. **2 marks**

(iii) Yes. The set of the vectors is linearly dependent. Argument: Since $r = 2 < m = 3$, where m is the number of vectors, according to theorem **V**, Ayers, Page 69, the set of vectors is linearly dependent.

2 marks

(iv) Det[A]=0. **3 marks**

(v) Inverse of A does not exist since $|A|=0$. **2 marks**

(vi) Since the rank $r=2 < n$, where n is the number of unknowns, then, according to theorem **IV**, Ayers, page 78, the HE system has solution other than the trivial one.

Or, alternatively: The HE has non-trivial solution since $|A|=0$, according to theorem **V**, Ayers, page 78.

2 marks

(vii) The number of linearly independent solution is n-r = 3 – 2 = 1.

2 marks

(viii) Use augmented matrix method:

$[A \mid 0] \sim \text{RREF}([A \mid 0]) = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv x_1 - x_3 = 0;\ x_2 + 2x_3 = 0;$

Choose $x_3=a$ to be an independent arbitrary parameter,

$X_{sol} = [x_1, x_2, x_3]^T = [a, -2a, a]^T = a[1, -2, 1]^T$.

3 marks