ZCA 110 Calculus and Linear Algebra
Semester I, Academic session 2008/09
TEST II 19 September 2008

Name:

Matrix Number:
Group (A or B):

Instruction: answer all questions (4 in total) on the question paper. Note that this printed paper comprises of two pages.

1. Graph the function $y = x^{2/3}\left(x-5\right)$. Include the coordinate of any local extreme points and inflection points.

(5 marks)

2. For the function as defined in Q1, find the area bounded by the curve, the *x*-axis, the *y*-axis, and the vertical line $x = 5$.

(5 marks)

3. Obtain the derivative of the function $f(x) = \dfrac{1}{2x+1}$ using the definition, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

(5 marks)

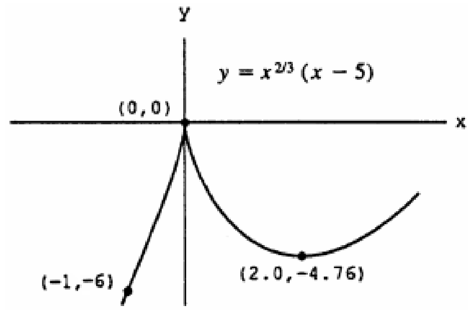
4. Find (*i*) $\dfrac{dy}{dx}$ and (*ii*) $\dfrac{d^2y}{dx^2}$ if $y\cos 2x + \sin^2 x = \pi$.

(2.5 + 2.5 = 5 marks)

Solutions:

1. Exercise 4.4, Q32, pg. 275, Chapter 4.

When $y = x^{2/3}(x-5) = x^{5/3} - 5x^{2/3}$, then $y' = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5}{3}x^{-1/3}(x-2)$ and $y'' = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3} = \frac{10}{9}x^{-4/3}(x+1)$. The curve is rising on $(-\infty, 0)$ and $(2, \infty)$, and falling on $(0, 2)$. There is a local minimum at $x = 2$ and a local maximum at $x = 0$. The curve is concave up on $(-1, 0)$ and $(0, \infty)$, and concave down on $(-\infty, -1)$. There is a point of inflection at $x = -1$ and a cusp at $x = 0$.

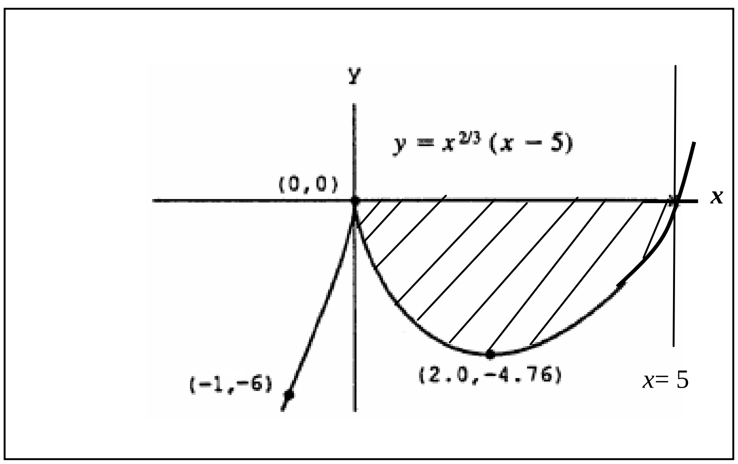


2.

$$\int_0^5 y\,dx = \int_0^5 x^{2/3}(x-5)\,dx = \int_0^5 x^{5/3} - 5x^{2/3}dx = \left[\frac{3}{8}x^{8/3} - 3x^{5/3}\right]_0^5 = x^{5/3}\left[\frac{3}{8}x - 3\right]_0^5 =$$

$$5^{5/3}\left[\frac{15}{8} - 3\right]_0^5 = -\frac{9}{8}(5)^{5/3}$$

Area enclosed = $-\int_0^5 y\,dx = \frac{9}{8}(5)^{5/3}$.



3. Practice exercises 3 Q63, pg. 236, Chapter 3.

$$f(t) = \frac{1}{2t+1} \text{ and } f(t+h) = \frac{1}{2(t+h)+1} \Rightarrow \frac{f(t+h)-f(t)}{h} = \frac{\frac{1}{2(t+h)+1} - \frac{1}{2t+1}}{h} = \frac{2t+1-(2t+2h+1)}{(2t+2h+1)(2t+1)h}$$

$$= \frac{-2h}{(2t+2h+1)(2t+1)h} = \frac{-2}{(2t+2h+1)(2t+1)} \Rightarrow f'(t) = \lim_{h \to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h \to 0} \frac{-2}{(2t+2h+1)(2t+1)}$$

$$= \frac{-2}{(2t+1)^2}$$

4. Practice exercises 3, Q51, pg. 236, Chapter 3.

(*i*)

$r\cos 2s + \sin^2 s = \pi \;\Rightarrow\; r(-\sin 2s)(2) + (\cos 2s)\left(\frac{dr}{ds}\right) + 2\sin s\cos s = 0 \;\Rightarrow\; \frac{dr}{ds}(\cos 2s) = 2r\sin 2s - 2\sin s\cos s$

$\Rightarrow\; \frac{dr}{ds} = \frac{2r\sin 2s - \sin 2s}{\cos 2s} = \frac{(2r-1)(\sin 2s)}{\cos 2s} = (2r-1)(\tan 2s)$

(*ii*)

$$\frac{\mathrm{d}r}{\mathrm{d}s} = (2r-1)\tan 2s$$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}s^2} = \tan 2s\frac{\mathrm{d}}{\mathrm{d}s}(2r-1) + (2r-1)\frac{\mathrm{d}}{\mathrm{d}s}(\tan 2s)$$

$$= 2\tan 2s\frac{\mathrm{d}r}{\mathrm{d}s} + (2r-1)\left(2\sec^2 2s\right) = 2\tan^2 2s(2r-1) + (2r-1)\left(2\sec^2 2s\right)$$

$$= 4s(2r-1)\left(\sec^2 2s + \tan^2 2s\right) = 4s(2r-1)\left(1 + 2\tan^2 2s\right)$$