```
(*tut 2A, Q1*)
A = {{1, 3, 1}, {2, -1, -3}, {1, 1, -2}};
H = {1, 1, 5};
Print["A=", MatrixForm[A]];
```

$$A=\begin{pmatrix}1 & 3 & 1\\ 2 & -1 & -3\\ 1 & 1 & -2\end{pmatrix}$$

```
B = Inverse[A];
Print["Inverse[A]=", MatrixForm[B]];
Print["Check: A*Inverse[A]=", MatrixForm[B.A]];
```

$$\text{Inverse}[A]=\begin{pmatrix}\frac{5}{11} & \frac{7}{11} & -\frac{8}{11}\\ \frac{1}{11} & -\frac{3}{11} & \frac{5}{11}\\ \frac{3}{11} & \frac{2}{11} & -\frac{7}{11}\end{pmatrix}$$

$$\text{Check: } A*\text{Inverse}[A]=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

```
Adjoin = Det[A] * B;
Print["Adjoin=", Adjoin // MatrixForm]
```

$$\text{Adjoin}=\begin{pmatrix}5 & 7 & -8\\ 1 & -3 & 5\\ 3 & 2 & -7\end{pmatrix}$$

```
Print["solution, {x1,x2,x3} = ", LinearSolve[A, H]]
```

$$\text{solution, } \{x1,x2,x3\} = \left\{-\frac{28}{11}, \frac{23}{11}, -\frac{30}{11}\right\}$$

```
In[289]:= (*tut 2A, Q2*)
A = {{1, 3, 1, -2}, {2, -1, -3, 0}, {1, 2, -2, 0}, {0, 1, 0, 3}};
H = {2, 1, 2, 0}; X = {x1, x2, x3, x4};
aug = {{1, 3, 1, -2, 2}, {2, -1, -3, 0, 1}, {1, 2, -2, 0, 2}, {0, 1, 0, 3, 0}};
Print["A=", MatrixForm[A]];
Print["H=", MatrixForm[H]];
Print["Rank[A]=", MatrixRank[A]];
Print["Rank[A|H]=", MatrixRank[aug]];
Print["Since Rank[A]=Rank[A|H], the system is consisten"];
Print["AX=H is ", MatrixForm[A] .MatrixForm[X], "=", MatrixForm[H]];
Print["particular solution, X_p = ", LinearSolve[A, H] // MatrixForm]
Print["A.X_p= ", MatrixForm[A.LinearSolve[A, H]]];
Print["Now solve for the homogeneous solution, X_h, for the non-homogeneous system, AX=0"];
Print["RREF[A]=", RowReduce[A] // MatrixForm];
Print["A.X =", A // MatrixForm , X // MatrixForm, "=", {0, 0, 0, 0} // MatrixForm];
Print["≡ RREF[A].X =", RowReduce[A] // MatrixForm ,
  X // MatrixForm, "=", {0, 0, 0, 0} // MatrixForm];
Print["Hence the solution for the homegenoues equation, AX=0 is X_h=",
  {0, 0, 0, 0} // MatrixForm];
Print["Hence the complete solution for the
    non-homegenoues equation, AX=H is Xc=X_p+X_h=X_p"];
Print["The particular solution for the AX=H, is unique because the number
    of unknonws n is the same as the number of equation, = 4 , "];
```

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 \\ 2 & -1 & -3 & 0 \\ 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 3 \end{pmatrix}$$

$$H = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

Rank [A] = 4

Rank [A | H] = 4

Since Rank [A] = Rank [A | H], the system is consisten

$$AX = H \text{ is } \begin{pmatrix} 1 & 3 & 1 & -2 \\ 2 & -1 & -3 & 0 \\ 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 3 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

$$\text{particular solution, } X\_p = \begin{pmatrix} \frac{8}{25} \\ \frac{27}{50} \\ -\frac{3}{10} \\ -\frac{9}{50} \end{pmatrix}$$

$$A.X\_p = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

Now solve for the homogeneous solution, X_h, for the non-homogeneous system, AX = 0

$$RREF[A] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A.X = \begin{pmatrix} 1 & 3 & 1 & -2 \\ 2 & -1 & -3 & 0 \\ 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\equiv RREF[A].X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Hence the solution for the homegenoues equation, } AX = 0 \text{ is } X\_h = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Hence the complete solution for the non-homegenoues equation, AX = H is Xc = X_p + X_h = X_p

The particular solution for the AX = H, is unique because
the number of unknonws n is the same as the number of equation, = 4 ,

In[681]:=
```
(*tut 2A, Q3*)

A = {{3, 1, 2, 1, 0}, {2, 1, 3, -1, 0}, {1, 0, 1, 2, -1}};
H = {-2, 3, 4}; X = {x1, x2, x3, x4, x5}; n = Length[X];
aug = {{3, 1, 2, 1, 0, -2}, {2, 1, 3, -1, 0, 3}, {1, 0, 1, 2, -1, 4}};
Print["A=", MatrixForm[A]];
Print["H=", MatrixForm[H]];
Print["Rank[A]=", MatrixRank[A]];
Print["Rank[A|H]=", MatrixRank[aug]];
Print["Since Rank[A]=Rank[A|H], the system is consisten"];
Print["AX=H is ", MatrixForm[A] .MatrixForm[X], "=", MatrixForm[H]];
Print["particular solution, X_p = ", LinearSolve[A, H] // MatrixForm]
Print["A.X_p= ", MatrixForm[A.LinearSolve[A, H]]];
Print["Now solve for the homogeneous solution, X_h, for the non-homogeneous system, AX=0"];
Print["n-Rank[A]=", n, "-", MatrixRank[A], "=", n - MatrixRank[A]];
Print["There will be ", n - MatrixRank[A], " linearly independent solution for AX=0"];
Print["A.X =", A // MatrixForm , X // MatrixForm, "=", {0, 0, 0} // MatrixForm];
Print["≡ RREF[A].X =", RowReduce[A] // MatrixForm ,
  X // MatrixForm, "=", RowReduce[A].X // MatrixForm , "=", {0, 0, 0} // MatrixForm];
Print["Let x4=a, x5=b"];
Print["The solution for the homegenoues equation, AX=0 is"];
Print["X_h="
   MatrixForm[X], "=", a * MatrixForm[{-2, 5, 0, 1, 0}] + b * MatrixForm[{1/2, -5/2, 1/2, 0, 1}]
 ];
Print["Hence the complete solution for the non-homegenoues equation, AX=H is Xc=X_p+X_h"];
Print[
  "The particular solution for the AX=H, is not unique because the number of unknonws, n
    = ", n, " is not equal the number of equation, = 3"];

(*
A={{3,1,2,1},{2,1,3,-1},{1,1,2,-1}};
(*H={-2,3,4};*)
H={0,0,0};
Print["A=",MatrixForm[A]];
Print["H=",MatrixForm[H]];
Print["Rank[A]=",MatrixRank[{{3,1,2,1},{2,1,3,-1},{1,1,2,-1}}]]

 Print["Trivial solution, of AX=0 is X={x1,x2,x3,x4} = ",LinearSolve[A,H]]*)
```

$$A=\begin{pmatrix}3&1&2&1&0\\2&1&3&-1&0\\1&0&1&2&-1\end{pmatrix}$$

$$H=\begin{pmatrix}-2\\3\\4\end{pmatrix}$$

Rank[A]=3

Rank[A|H]=3

Since Rank[A]=Rank[A|H], the system is consisten

$AX = H$ is $\begin{pmatrix} 3 & 1 & 2 & 1 & 0 \\ 2 & 1 & 3 & -1 & 0 \\ 1 & 0 & 1 & 2 & -1 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$

particular solution, X_p $= \begin{pmatrix} -\frac{1}{2} \\ -\frac{19}{2} \\ \frac{9}{2} \\ 0 \\ 0 \end{pmatrix}$

$A . X\_p = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$

Now solve for the homogeneous solution, X_h, for the non-homogeneous system, $AX = 0$

$n - \text{Rank}[A] = 5 - 3 = 2$

There will be 2 linearly independent solution for $AX = 0$

$A . X = \begin{pmatrix} 3 & 1 & 2 & 1 & 0 \\ 2 & 1 & 3 & -1 & 0 \\ 1 & 0 & 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\equiv \text{RREF}[A] . X = \begin{pmatrix} 1 & 0 & 0 & 2 & -\frac{1}{2} \\ 0 & 1 & 0 & -5 & \frac{5}{2} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} x1 + 2\,x4 - \frac{x5}{2} \\ x2 - 5\,x4 + \frac{5\,x5}{2} \\ x3 - \frac{x5}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Let $x4 = a$, $x5 = b$

The solution for the homegenoues equation, $AX = 0$ is

$X\_h = \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = a \begin{pmatrix} -2 \\ 5 \\ 0 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} \frac{1}{2} \\ -\frac{5}{2} \\ \frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$

Hence the complete solution for the non-homegenoues equation, $AX = H$ is $Xc = X\_p + X\_h$

The particular solution for the $AX = H$, is not unique because the number of unknonws, $n = 5$ is not equal the number of equation, $= 3$