```
In[176]:= (*Q1, TUT 2B*)
A = {{1, 3, 1}, {2, 1, -3}, {1, -2, 1}};
H = {2, 1, 4}; X = {x1, x2, x3}; n = Length[X];
aug = {{1, 3, 1, 2}, {2, 1, -3, 1}, {1, -2, 1, 4}};
Print["A is ", A // MatrixForm];
Print["H is ", H // MatrixForm];
Print["AX=H is ", A.X // MatrixForm, "=", H // MatrixForm];
Print["Rank[A]=", MatrixRank[A]];
Print["Rank[A|H]=", MatrixRank[aug]];
Print["Hence AX=H is consistant"];
Print["particular solution,X_p = ", LinearSolve[A, H] // MatrixForm];
Print["A.X_p=", MatrixForm[A.LinearSolve[A, H]]];
Print[
  "The complete solution for the AX=H, is unique because the number of unknonws, n = ",
  n, " is equal the number of equation, = 3"];
```

A is $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

H is $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

AX=H is $\begin{pmatrix} x1 + 3\ x2 + x3 \\ 2\ x1 + x2 - 3\ x3 \\ x1 - 2\ x2 + x3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

Rank[A]=3

Rank[A|H]=3

Hence AX=H is consistant

particular solution,X_p = $\begin{pmatrix} \frac{11}{5} \\ -\frac{2}{5} \\ 1 \end{pmatrix}$

A.X_p= $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

The complete solution for the AX=H, is unique because the number of unknonws, n = 3 is equal the number of equation, = 3

```
(*Q2, TUT 2B*)
A = {{1, 1, 1, 1}, {2, 1, 3, -1}, {1, 0, 1, 2}};
H = {-1, 7, 4}; X = {x1, x2, x3, x4};
n = Length[X];
aug = {{1, 1, 1, 1, -1}, {2, 1, 3, -1, 7}, {1, 0, 1, 2, 5}};
Print["A=", MatrixForm[A]];
Print["H=", MatrixForm[H]];
Print["Rank[A]=", MatrixRank[A]];
Print["Rank[A|H]=", MatrixRank[aug]];
Print["Since Rank[A]=Rank[A|H], the system is consisten"];
Print["AX=H is ", MatrixForm[A] .MatrixForm[X], "=", MatrixForm[H]];
Print["particular solution, X_p = ", LinearSolve[A, H] // MatrixForm]
Print["A.X_p= ", MatrixForm[A.LinearSolve[A, H]]];
Print["Now solve for the homogeneous solution, X_h, for the non-homogeneous system, AX=0"];
Print["n-Rank[A]=", n, "-", MatrixRank[A], "=", n - MatrixRank[A]];
Print["There will be ", n - MatrixRank[A], " linearly independent solution for AX=0"];
Print["A.X =", A // MatrixForm , X // MatrixForm, "=", {0, 0, 0} // MatrixForm];
Print["≡ RREF[A].X =", RowReduce[A] // MatrixForm ,
  X // MatrixForm, "=", RowReduce[A].X // MatrixForm , "=", {0, 0, 0} // MatrixForm];
Print["Let x4=a"];
Print["The solution for the homegenoues equation, AX=0 is"];
Print["X_h="
   MatrixForm[X], "=", a * MatrixForm[{-6, 1, 4, 1}]
 ];
Print["Hence the complete solution for the non-homegenoues equation, AX=H is Xc=X_p+X_h"];
Print[
  "The complete solution for the AX=H, is not unique because the number of unknonws, n = ",
  n, " is not equal the number of equation, = 3"];

(*
A={{3,1,2,1},{2,1,3,-1},{1,1,2,-1}};
(*H={-2,3,4};*)
H={0,0,0};
Print["A=",MatrixForm[A]];
Print["H=",MatrixForm[H]];
Print["Rank[A]=",MatrixRank[{{3,1,2,1},{2,1,3,-1},{1,1,2,-1}}]]

 Print["Trivial solution, of AX=0 is X={x1,x2,x3,x4} = ",LinearSolve[A,H]]*)
```

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & -1 \\ 1 & 0 & 1 & 2 \end{pmatrix}$$

$$H = \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix}$$

Rank [A] = 3

Rank [A | H] = 3

Since Rank [A] = Rank [A | H], the system is consisten

Number of equation = 3 ;number of unknown is n = 4

Hence the solution will not be unique

$$AX = H \text{ is } \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & -1 \\ 1 & 0 & 1 & 2 \end{pmatrix} . \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix}$$

$$\text{particular solution, } X\_p = \begin{pmatrix} 0 \\ -5 \\ 4 \\ 0 \end{pmatrix}$$

$$A . X\_p = \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix}$$

Now solve for the homogeneous solution, X_h, for the non-homogeneous system, AX = 0

n - Rank [A] = 4 - 3 = 1

There will be 1 linearly independent solution for AX = 0

$$A . X = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & -1 \\ 1 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\equiv \text{RREF}[A].X = \begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} x1 + 6\,x4 \\ x2 - x4 \\ x3 - 4\,x4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Let x4 = a

The solution for the homegenoues equation, AX = 0 is

$$X\_h = \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = a \begin{pmatrix} -6 \\ 1 \\ 4 \\ 1 \end{pmatrix}$$

Hence the complete solution for the non-homegenoues equation, AX = H is Xc = X_p + X_h

The particular solution for the AX = H, is not unique because the number of unknonws, n = 4 is not equal the number of equation, = 3