In[1]:=

```
(*Q3,TUT 2C *)
A = {{9, 1, 4, 0, 1}, {2, 5, 0, 1, 0},
   {7, 0, 3, 0, -1}};
H = {1, 3, -1};
X = {x1, x2, x3, x4, x5};
aug = {{9, 1, 4, 0, 1, 1}, {2, 5, 0, 1, 0, 3},
   {7, 0, 3, 0, -1, -1}};
Print["A is ", A // MatrixForm ];
Print["H is ", H // MatrixForm ];
Print["AX = H is ", A.X // MatrixForm, "=", H // MatrixForm ];
Print["Rank [A]=", MatrixRank[A]];
Print["Rank [A|H]=", MatrixRank[aug]];
Print["particular solution ,X_p = ", LinearSolve[A, H] // MatrixForm ];
Print["A.X_p=", MatrixForm[A.LinearSolve[A, H ]]];
Print["For homegeneous solutions:"];
Print["n - r =", 5 - MatrixRank[A],
  " hence we exepct two linearly independent solutions for AX =0"];
Print["A .X =", A // MatrixForm, X // MatrixForm, "=", {0, 0, 0} // MatrixForm ];
Print["RREF[A].X =", RowReduce[A ] // MatrixForm, X // MatrixForm,
  "=", RowReduce[A ].X // MatrixForm, "=", MatrixForm[{0, 0, 0}]];
x4 = a; x5 = b;
Print["Let x4= a,x5= b"];
x1 = (1 / 11) (-3 x4 + 35 x5);
x2 = ( 1 / 11) (-1 x4 - 14 x5);
x3 = ( 1 / 11) ( 7 x4 - 78 x5);
Print["Hence , X_h =", {x1, x2, x3, x4, x5} // MatrixForm,
  "= a", MatrixForm[{-3 / 11, -1 / 11, 7 / 11, 1, 0}],
  "+ b", MatrixForm[{35 / 11, -14 / 11, -78 / 11, 0, 1}]];
Print["The complete solution is X_c = X_p + X_h"];
```

A is $\begin{pmatrix} 9 & 1 & 4 & 0 & 1 \\ 2 & 5 & 0 & 1 & 0 \\ 7 & 0 & 3 & 0 & -1 \end{pmatrix}$

H is $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

AX = H is $\begin{pmatrix} 9\,x1 + x2 + 4\,x3 + x5 \\ 2\,x1 + 5\,x2 + x4 \\ 7\,x1 + 3\,x3 - x5 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

Rank [A] = 3

Rank [A | H] = 3

particular solution ,X_p = $\begin{pmatrix} -\frac{26}{11} \\ \frac{17}{11} \\ \frac{57}{11} \\ 0 \\ 0 \end{pmatrix}$

A.X_p = $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

For homegeneous solutions:

n - r = 2 hence we exepct two linearly independent solutions for AX = 0

$$A\,.X = \begin{pmatrix} 9 & 1 & 4 & 0 & 1 \\ 2 & 5 & 0 & 1 & 0 \\ 7 & 0 & 3 & 0 & -1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$RREF[A].X = \begin{pmatrix} 1 & 0 & 0 & \frac{3}{11} & -\frac{35}{11} \\ 0 & 1 & 0 & \frac{1}{11} & \frac{14}{11} \\ 0 & 0 & 1 & -\frac{7}{11} & \frac{78}{11} \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} x1 + \frac{3\,x4}{11} - \frac{35\,x5}{11} \\ x2 + \frac{x4}{11} + \frac{14\,x5}{11} \\ x3 - \frac{7\,x4}{11} + \frac{78\,x5}{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Let x4 = a, x5 = b

$$\text{Hence , X\_h} = \begin{pmatrix} \frac{1}{11}(-3\,a + 35\,b) \\ \frac{1}{11}(-a - 14\,b) \\ \frac{1}{11}(7\,a - 78\,b) \\ a \\ b \end{pmatrix} = a \begin{pmatrix} -\frac{3}{11} \\ -\frac{1}{11} \\ \frac{7}{11} \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} \frac{35}{11} \\ -\frac{14}{11} \\ -\frac{78}{11} \\ 0 \\ 1 \end{pmatrix}$$

The complete solution is X_c = X_p + X_h