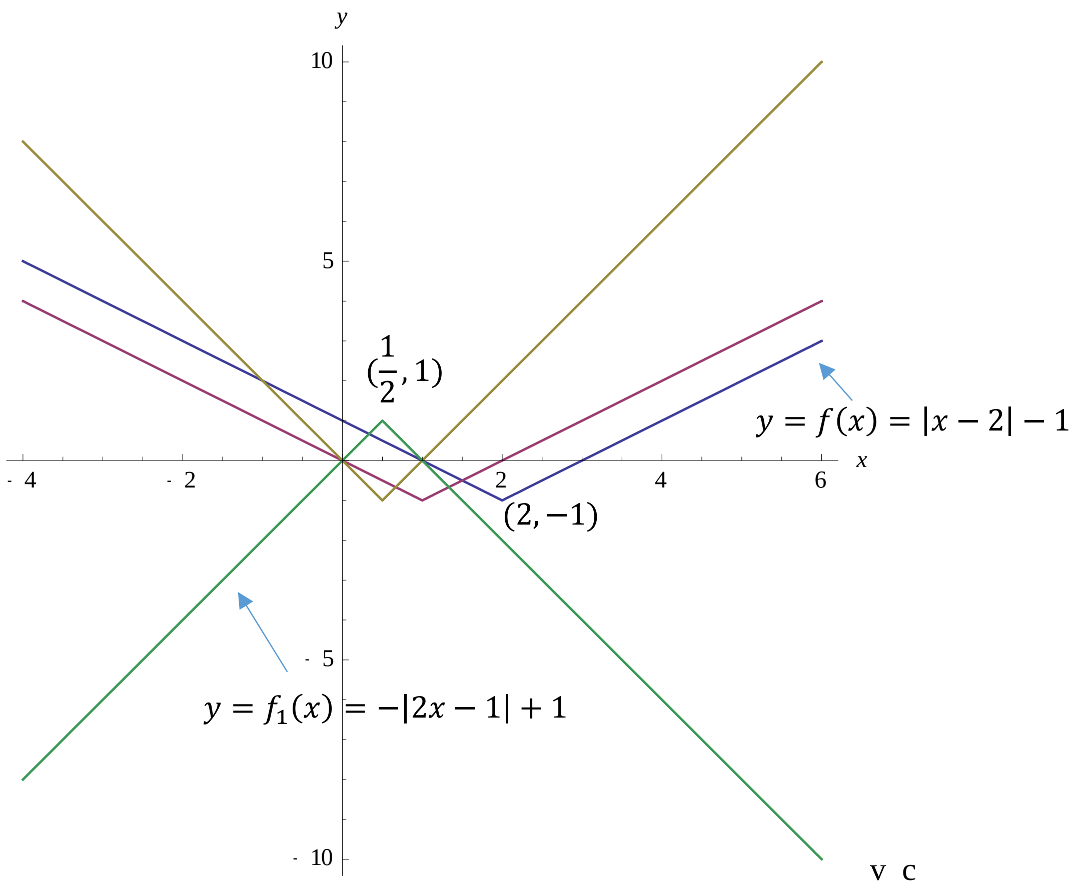
Solutions Q1:

（a)

(I) ***marks: 5 (graph) + 5 (label) = 10***

(II) ***marks: 15 (graph) + 5 (label) = 20***

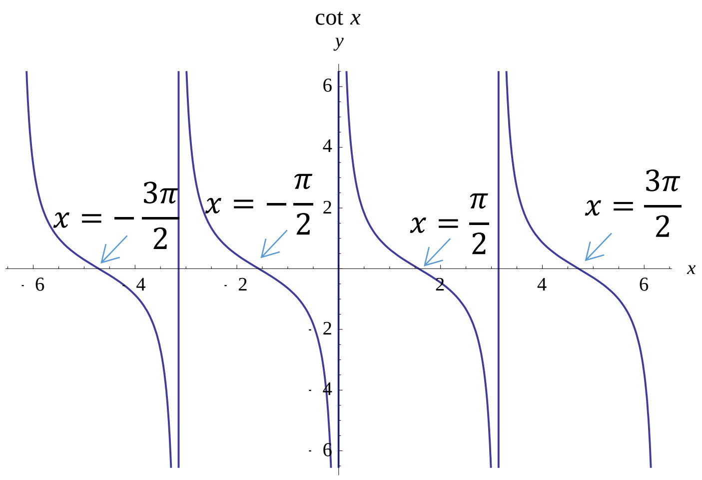


(III) $f_1(x) = -|2x-1| + 1$

marks: 10 marks

(b) (I) [pg 52, Fig. 1.73 (f)]

marks: 10 (graph) + 5 (labelling of the zeros) = 15



(II) ***marks = 5 + 5 + 5 = 15***

Domain = $\{x: -2\pi \leq x \leq 2\pi, x \neq 0, \pm\pi, \pm 2\pi\}$

Range=$(-\infty < x < \infty)$

Period = π

(c) $D_{h\circ g} = (-\infty < x < \infty)$, $R_{h\circ g} = [-1,1]$.

marks: 15 + 15=30

Solutions Q9:

(a) Q35, page 781. Ex 11.4

Marks: 30

35. $\frac{1}{1+2+3+\ldots+n} = \frac{1}{\left(\frac{n(n+1)}{2}\right)} = \frac{2}{n(n+1)}$. The series converges by the Limit Comparison Test (part 1) with $\frac{1}{n^2}$:

$$\lim_{n\to\infty} \frac{\left(\frac{2}{n(n+1)}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{2n^2}{n^2+n} = \lim_{n\to\infty} \frac{4n}{2n+1} = \lim_{n\to\infty} \frac{4}{2} = 2.$$

You may tempted to use ratio test. In ratio test, for $a_n > 0$, $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$ implies S is convergent.

$$\text{Let } a_n = \frac{1}{1+2+3+\cdots+n}; a_n > 0.$$

$$S = \sum_n \frac{1}{1+2+3+\cdots+n} = \sum_{n=1}^{n\to\infty} a_n$$

Note that $a_n = \frac{1}{1+2+3+\cdots+n} = \frac{2}{n(n+1)}$.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2}{(n+1)(n+2)}}{\frac{2}{n(n+1)}} = \lim_{n\to\infty} \frac{n(n+1)}{(n+1)(n+2)} = \lim_{n\to\infty} \frac{n}{(n+2)} = 1.$$

It is not conclusive if one uses ratio test to determine the convergence of S.

However, we observe that

(b) Q2, page 804. Ex. 11.7

Marks: 30

$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1 \Rightarrow \lim_{n \to \infty} \left| \frac{(x+5)^{n+1}}{(x+5)^n} \right| < 1 \Rightarrow |x+5| < 1 \Rightarrow -6 < x < -4$; when $x = -6$ we have $\sum_{n=1}^{\infty} (-1)^n$, a divergent series; when $x = -4$ we have $\sum_{n=1}^{\infty} 1$, a divergent series

the radius is 1; the interval of convergence is $-6 < x < -4$

(c) Marks: 40

- $\tan^{-1} x = \int \frac{1}{1+x^2} dx$ (from integral table)
- Identify $\frac{1}{1+x^2} \equiv \frac{a}{1-r} \Rightarrow a \equiv 1, r \equiv -x^2, |x| < 1.$
- Construct the geometry series:
- $s = ar^0 + ar^1 + ar^2 + ar^3 + ar^4 + \cdots$
  $= 1 \quad - x^2 + x^4 - x^6 + x^8 + \cdots$
- $\Rightarrow \frac{1}{1+x^2} = 1 \quad - x^2 + x^4 - x^6 + x^8 + \cdots$
- $\Rightarrow \int \frac{1}{1+x^2} dx = \int dx(1 - x^2 + x^4 - x^6 + x^8 + \cdots)$
- $\Rightarrow \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} - \frac{x^9}{9} + \cdots$
- Check for the convergence of the series representation at $x = 1, x = -1$: It converges at these values by the virtue of the Leibnitz's theorem.
- Hence,
- $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} - \frac{x^9}{9} + \cdots$

converges for $-1 \leq x \leq 1.$