ZCA 110 Calculus Test 3. Venue: DPD; Date: Friday, 11 am- 12pm

9 Dec 2016

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Matrix No:

Q1.Given that $u(x)$ is a differentiable function, explain how the following integration rule is derived. [Hint: You may have to start from the differentiation of u^n]

$\int u^n\,du = \frac{1}{n+1}u^{n+1} + C;\ n \neq -1.$ $\frac{d}{dx}u^{n+1} = \frac{u^n}{n+1}\frac{du}{dx}$

$\int \frac{d}{dx}u^{n+1}dx = \int \frac{u^n}{n+1}\frac{du}{dx}dx$. By def, differential $dy = \frac{dy}{dx}dx$

$\Rightarrow \int d(u^{n+1}) = \frac{1}{n+1}\int u^n du$

$\Rightarrow \frac{u^{n+1} + c'}{n+1} = \int u^n du \Rightarrow \int u^n du = \frac{1}{n+1}u^{n+1} + c$ ✗

Q2.Given $f(x) = \frac{\sin x + x\cos x}{\text{Exp}(x)+\text{Exp}(-x)}$, evaluate $\int_{-\pi}^{\pi} f(x)\,dx$.

$f(-x) = \frac{\sin(-x) + (-x)\cos(-x)}{Exp(-x) + Exp(x)} = -\frac{\sin x + x\cos x}{Exp(x) + Exp(-x)} = -f(x)$

$\Rightarrow f(x)$ is antisymmetric. $\therefore \int_{-\pi}^{\pi} f(x)dx = 0.$

Q3. Find the length of the curve parametrised by $x = 2t^2+1$, $y = 2t^3$, $0 \le t \le 1$.

$\frac{dx}{dt} = 4t$; $\frac{dy}{dt} = 6t^2$ $L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{16t^2 + 36t^4}\,dt = 4\int_0^1 t\sqrt{1+\frac{9t^2}{4}}$

let $u = \frac{9}{4}t^2 \rightarrow du = \frac{9}{4}\cdot 2t\,dt = \frac{18t\,dt}{4} \Rightarrow t\,dt = \frac{2}{9}du$

$\therefore L = 4\cdot\int_{u(0)}^{u(1)} \sqrt{1+u}\cdot\frac{2}{9}du = \frac{8}{9}\int_0^{9/4}(1+u)^{\frac{1}{2}}du = \frac{8}{9}\left[\frac{2}{3}(1+u)^{3/2}\right]_0^{9/4} = \frac{16}{27}\left[\left(1+\frac{9}{4}\right)^{3/2} - 1\right]$

$= \frac{16}{27}\left[\left(\frac{13}{4}\right)^{3/2} - 1\right]$ ✗

Q4. Explain how the following differential result is derived: $\frac{d}{dx}e^x = e^x$.

By def, $\ln(x) = \int_1^x \frac{1}{t}dt$. By 1st fundamental theorem of calculus,

$\frac{d}{dx}\int_1^x \frac{1}{t}dt = \frac{1}{x} \Rightarrow \frac{d}{dx}\ln x = \frac{1}{x}$. Exp(x) is the inverse of ln(x).

$\therefore$ use $\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(x)}\Big]_{x\to f^{-1}(x)} \Rightarrow \frac{d}{dx}e^x = \frac{1}{\left(\frac{1}{x}\right)_{x\to e^x}} = \frac{1}{\frac{1}{e^x}} = e^x$ ✗

Q5. Prove $\frac{e^{x_2}}{e^{x_1}} = e^{x_2}\cdot e^{-x_1}$, for all real numbers x_1 and x_2.

let $x_2 = \ln y_2 \Rightarrow e^{x_2} = y_2$ —② Also, $-x_2 = -\ln y_2 = \ln y_2^{-1} = \ln\left(\frac{1}{y_2}\right) = \ln\left(\frac{1}{y_2}\right)$

$x_1 = \ln x_1 \Rightarrow e^{x_1} = y_1$ —① Taking exp on both sides $\exp(-x_2) = \exp(\ln(\frac{1}{y_2}))$

$e^{-x_2} = \frac{1}{y_2}$ —③

multiply ① × ③

$e^{x_1}\cdot e^{-x_2} = y_1\cdot\left(\frac{1}{y_2}\right) = \frac{y_1}{y_2} = \frac{e^{x_1}}{e^{x_2}}$ from ②,①

$\therefore e^{x_1}\cdot e^{-x_2} = \frac{e^{x_1}}{e^{x_2}}$ ✗