CALCULUS ASSIGNMENT

QUESTIONS FOR ZCA 101

Tutorial 1 (Chapter 1)
Thomas' Calculus 11th edition

Exercise 1.3

Functions and Their Graphs

find the domain and range of each function.

1. $f(x) = 1 + x^2$

3. $F(t) = \dfrac{1}{\sqrt{t}}$

5. $g(z) = \sqrt{4 - z^2}$

Finding Formulas for Functions

13. Express the edge length of a cube as a function of the cube's diagonal length d. Then express the surface area and volume of the cube as a function of the diagonal length.

Functions and Graphs

Find the domain and graph the functions

16. $f(x) = 1 - 2x - x^2$

18. $g(x) = \sqrt{-x}$

22. Graph the following equations and explain why they are not graphs of functions of *x*.

a. $|x| + |y| = 1$ **b.** $|x + y| = 1$

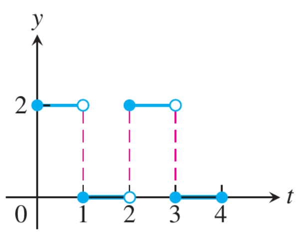
Piecewise-Defined Functions

Graph the function

24. $g(x) = \begin{cases} 1 - x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

27. Find a formula for each function graphed.

b.



Exercise 1.4

Recognizing Functions

In Exercises 3, identify each function as a constant function, linear function, power function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function. Remember that some functions can fall into more than one category.

3. a. $y = \dfrac{3 + 2x}{x - 1}$ **b.** $y = x^{5/2} - 2x + 1$

c. $y = \tan \pi x$ **d.** $y = \log_7 x$

Increasing and Decreasing Functions

Graph the functions. What symmetries, if any, do the graphs have? Specify the intervals over which the function is increasing and the intervals where it is decreasing.

16. $y = (-x)^{3/2}$

18. $y = -x^{2/3}$

Even and Odd Functions

Say whether the function is even, odd, or neither. Give reasons for your answer.

19. $f(x) = 3$

21. $f(x) = x^2 + 1$

23. $g(x) = x^3 + x$

EXERCISES 1.5

Sums, Differences, Products, and Quotients

Find the domains and ranges of f, g, f + g , and $f \cdot g$.

2. $f(x) = \sqrt{x+1}, \quad g(x) = \sqrt{x-1}$

Composites of Functions

6. If $f(x) = x - 1$ and $g(x) = 1/(x + 1)$, find

a. $f(g(½))$

b. $g(f(1/2))$

12. Copy and complete the following table.

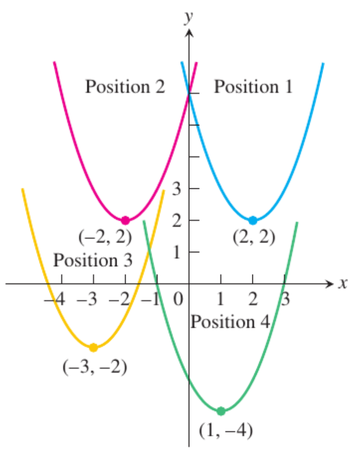
	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$\frac{1}{x-1}$	$\lvert x \rvert$	?
b.	?	$\frac{x-1}{x}$	$\frac{x}{x+1}$
c.	?	$\sqrt{x}$	$\lvert x \rvert$
d.	$\sqrt{x}$	?	$\lvert x \rvert$

Shifting Graphs

17. Match the equations listed in parts (a)–(d) to the graphs in the accompanying figure.

a. $y = (x-1)^2 - 4$ **b.** $y = (x-2)^2 + 2$

c. $y = (x+2)^2 + 2$ **d.** $y = (x+3)^2 - 2$



Graph the functions

39. $y = \sqrt[3]{x-1} - 1$

48. $y = \frac{1}{(x+1)^2}$

Vertical and Horizontal Scaling

Exercises below tell by what factor and direction the graphs of the given functions are to be stretched or compressed. Give an equation for the stretched or compressed graph.

51. $y = x^2 - 1$, stretched vertically by a factor of 3

55. $y = \sqrt{x+1}$, compressed horizontally by a factor of 4

EXERCISES 1.6

Radians, Degrees, and Circular Arcs

4. If you roll a 1-m-diameter wheel forward 30 cm over level ground, through what angle will the wheel turn? Answer in radians (to the nearest tenth) and degrees (to the nearest degree).

Evaluating Trigonometric Functions

5. Copy and complete the following table of function values. If the function is undefined at a given angle, enter "UND." Do not use a calculator or tables.

θ	$-\pi$	$-2\pi/3$	0	$\pi/2$	$3\pi/4$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					
$\cot\theta$					

Find the other two if x lies in the specified interval.

7. $\sin x = \frac{3}{5}, \quad x \in \left[\frac{\pi}{2}, \pi\right]$

Graphing Trigonometric Functions

Graph the functions the *ts*-plane (*t*-axis horizontal, *s*-axis vertical). What is the period of each function? What symmetries do the graphs have?

23. $s = \cot 2t$

25. $s = \sec\left(\frac{\pi t}{2}\right)$

Additional Trigonometric Identities

Use the addition formulas to derive the identity.

31. $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

Using the Addition Formulas

Express the given quantity in terms of sin *x* and cos *x*.

39. $\cos(\pi + x)$

Using the Double-Angle Formulas

Find the function values

47. $\cos^2 \frac{\pi}{8}$

49. $\sin^2 \frac{\pi}{12}$

Tutorial 2 (Chapter 2)
Thomas' Calculus 11th edition

Exercise 2.1

Limits from Graphs

3. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?

a. $\lim_{x\to 0} f(x)$ exists.

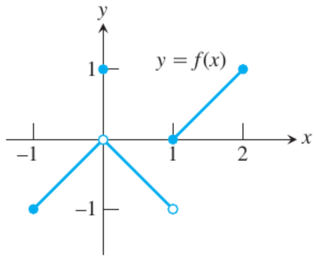
b. $\lim_{x\to 0} f(x) = 0$.

c. $\lim_{x\to 0} f(x) = 1$.

d. $\lim_{x\to 1} f(x) = 1$.

e. $\lim_{x\to 1} f(x) = 0$.

f. $\lim_{x\to x_0} f(x)$ exists at every point x_0 in $(-1, 1)$.



Existence of Limits

9. If $\lim_{x\to 1} f(x) = 5$, must f be defined at $x = 1$? If it is, must $f(1) = 5$? Can we conclude *anything* about the values of f at $x = 1$? Explain.

10. If $f(1) = 5$, must $\lim_{x\to 1} f(x)$ exist? If it does, then must $\lim_{x\to 1} f(x) = 5$? Can we conclude *anything* about $\lim_{x\to 1} f(x)$? Explain.

Limits by Substitution

Find the limits by substitution.

27. $\lim_{x\to \pi/2} x \sin x$

Average Rates of Change

Find the average rate of change of the function over the given interval or intervals.

33. $R(\theta) = \sqrt{4\theta + 1}; \quad [0, 2]$

Exercise 2.2

Limit Calculations

Find the limits.

17. $\lim_{h\to 0} \dfrac{\sqrt{3h+1} - 1}{h}$

19. $\lim_{x\to 5} \dfrac{x - 5}{x^2 - 25}$

27. $\lim_{u\to 1} \dfrac{u^4 - 1}{u^3 - 1}$

Using Limit Rules

39. Suppose $\lim_{x\to c} f(x) = 5$ and $\lim_{x\to c} g(x) = -2$. Find

a. $\lim_{x\to c} f(x)g(x)$

Limits of Average Rates of Change

Because of their connection with secant lines, tangents, and instantaneous rates, limits of the form

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

occur frequently in calculus. Evaluate this limit for the given value of x and function f.

47. $f(x) = \sqrt{x}, \quad x = 7$

Using the Sandwich Theorem

49. If $\sqrt{5 - 2x^2} \le f(x) \le \sqrt{5 - x^2}$ for $-1 \le x \le 1$, find $\lim_{x\to 0} f(x)$.

EXERCISES 2.3

Centering Intervals About a Point

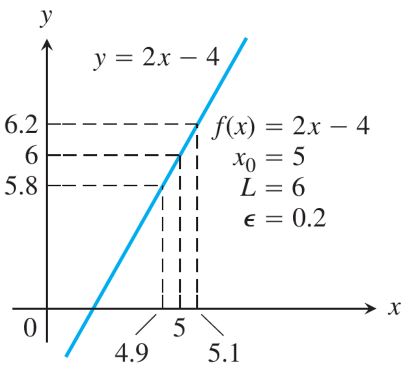
In Exercises 1–6, sketch the interval (a, b) on the x-axis with the point x_0 inside. Then find a value of $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow a < x < b$.

4. $a = -7/2, \quad b = -1/2, \quad x_0 = -3/2$

Finding Deltas Graphically

use the graphs to find a $\delta > 0$ such that for all x

7.



Finding Deltas Algebraically

Each of Exercises 15–30 gives a function $f(x)$ and numbers L, x_0 and $\epsilon > 0$. In each case, find an open interval about x_0 on which the inequality $|f(x) - L| < \epsilon$ holds. Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \epsilon$ holds.

15. $f(x) = x + 1, \quad L = 5, \quad x_0 = 4,$

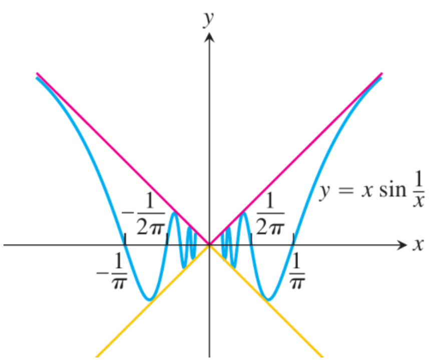
More on Formal Limits

31. $f(x) = 3 - 2x, \quad x_0 = 3, \quad \epsilon = 0.02$

Prove the limit statements

43. $\lim_{x \to 1} \frac{1}{x} = 1$

49. $\lim_{x \to 0} x \sin \frac{1}{x} = 0$



EXERCISES 2.4

Finding Limits Graphically

7. a. Graph $f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1. \end{cases}$

b. Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

c. Does $\lim_{x \to 1} f(x)$ exist? If so, what is it? If not, why not?

Finding One-Sided Limits Algebraically

Find the limit

15. $\lim_{h \to 0^+} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$

Using $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

Find the limits

33. $\lim_{\theta \to 0} \frac{\sin \theta}{\sin 2\theta}$

Calculating Limits as $x \to \pm\infty$

In Exercises 37–42, find the limit of each function **(a)** as $x \to \infty$ and **(b)** as $x \to -\infty$. (You may wish to visualize your answer with a graphing calculator or computer.)

41. $h(x) = \frac{-5 + (7/x)}{3 - (1/x^2)}$

Limits of Rational Functions

In Exercises 47–56, find the limit of each rational function **(a)** $x \to \infty$ and **(b)** as $x \to -\infty$.

49. $f(x) = \frac{x + 1}{x^2 + 3}$

EXERCISES 2.5

Infinite Limits

Find the limits.

13. $\lim_{x \to (\pi/2)^-} \tan x$

Additional Calculations

Find the limits.

25. $\lim \left(\dfrac{1}{x^{2/3}} + \dfrac{2}{(x-1)^{2/3}} \right)$ as

a. $x \to 0^+$

c. $x \to 1^+$

Graphing Rational Functions

Graph the rational functions in Exercises 27–38. Include the graphs and equations of the asymptotes and dominant terms.

36. $y = \dfrac{x^2 - 1}{2x + 4}$

Inventing Functions

Find a function that satisfies the given conditions and sketch its graph. (The answers here are not unique. Any function that satisfies the conditions is acceptable. Feel free to use formulas defined in pieces if that will help.)

43. $\lim_{x \to \pm\infty} f(x) = 0$, $\lim_{x \to 2^-} f(x) = \infty$, and $\lim_{x \to 2^+} f(x) = \infty$

Graphing Terms

The function is given as the sum or difference of two terms. First graph the terms (with the same set of axes). Then, using these graphs as guides, sketch in the graph of the function.

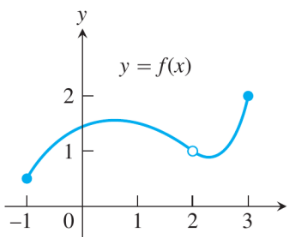
59. $y = \tan x + \dfrac{1}{x^2}, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

EXERCISES 2.6

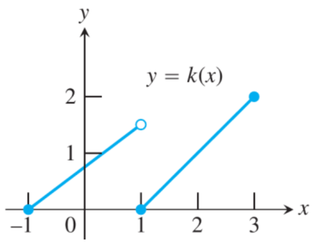
Continuity from Graphs

In the exercises below, say whether the function graphed is continuous on [-1, 3] . If not, where does it fail to be continuous and why?

1.



4.



Applying the Continuity Test

At which points do the functions fail to be continuous? At which points, if any, are the discontinuities removable? Not removable? Give reasons for your

17. $y = |x - 1| + \sin x$

20. $y = \dfrac{x + 2}{\cos x}$

Composite Functions

Find the limits. Are the functions continuous at the point being approached?

29. $\lim_{x \to \pi} \sin (x - \sin x)$

EXERCISES 2.7

Slopes and Tangent Lines

Find an equation for the tangent to the curve at the given point. Then sketch the curve and tangent together.

5. $y = 4 - x^2, \quad (-1, 3)$

Find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

18. $f(x) = \sqrt{x + 1}, \quad (8, 3)$

22. $y = \dfrac{x - 1}{x + 1}, \quad x = 0$

Tangent Lines with Specified Slopes

26. Find an equation of the straight line having slope $1/4$ that is tangent to the curve $y = \sqrt{x}$.

Rates of Change

30. Ball's changing volume What is the rate of change of the volume of a ball ($V = (4/3)\pi r^3$) with respect to the radius when the radius is $r = 2$?

Tutorial 3 (Chapter 3)
Thomas' Calculus 11th edition

EXERCISES 3.1

Finding Derivative Functions and Values

Using the definition, calculate the derivatives of the functions. Then find the values of the derivatives as specified.

6. $r(s) = \sqrt{2s + 1}\,;\quad r'(0), r'(1), r'(1/2)$

Find the indicated derivatives.

10. $\dfrac{dv}{dt}$ if $v = t - \dfrac{1}{t}$

12. $\dfrac{dz}{dw}$ if $z = \dfrac{1}{\sqrt{3w - 2}}$

Slopes and Tangent Lines

Differentiate the functions. Then find an equation of the tangent line at the indicated point on the graph of the function.

18. $w = g(z) = 1 + \sqrt{4 - z},\quad (z, w) = (3, 2)$

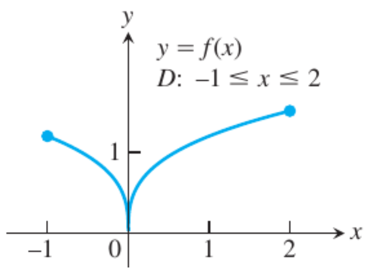
Find the values of the derivative.

21. $\left.\dfrac{dr}{d\theta}\right|_{\theta=0}$ if $r = \dfrac{2}{\sqrt{4 - \theta}}$

Differentiability and Continuity on an Interval

The figure below shows the graph of a function over a closed interval D. At what domain points does the function appear to be
a. differentiable?
b. continuous but not differentiable?
c. neither continuous nor differentiable?

43.



EXERCISES 3.2

Derivative Calculations

Find the derivatives of the functions

24. $u = \dfrac{5x + 1}{2\sqrt{x}}$

28. $y = \dfrac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$

Find the first and second derivatives.

38. $p = \dfrac{q^2 + 3}{(q - 1)^3 + (q + 1)^3}$

Using Numerical Values

39. Suppose u and v are functions of x that are differentiable at $x = 0$ and that

$$u(0) = 5,\quad u'(0) = -3,\quad v(0) = -1,\quad v'(0) = 2.$$

Find the values of the following derivatives at $x = 0$.

a. $\dfrac{d}{dx}(uv)$ **b.** $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$ **c.** $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$ **d.** $\dfrac{d}{dx}(7v - 2u)$

Slopes and Tangents

41. a. Normal to a curve Find an equation for the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point (2, 1).

b. Smallest slope What is the smallest slope on the curve? At what point on the curve does the curve have this slope?

c. Tangents having specified slope Find equations for the tangents to the curve at the points where the slope of the curve is 8.

EXERCISES 3.3

Motion Along a Coordinate Line

7. Particle motion At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ m.

a. Find the body's acceleration each time the velocity is zero.

b. Find the body's speed each time the acceleration is zero.

c. Find the total distance traveled by the body from $t = 0$ to $t = 2$.

Free-Fall Applications

10. Lunar projectile motion A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s = 24t - 0.8t^2$ meters in t sec.

a. Find the rock's velocity and acceleration at time t. (The acceleration in this case is the acceleration of gravity on the moon.)

b. How long does it take the rock to reach its highest point?

c. How high does the rock go?

d. How long does it take the rock to reach half its maximum height?

e. How long is the rock aloft?

Conclusions About Motion from Graphs

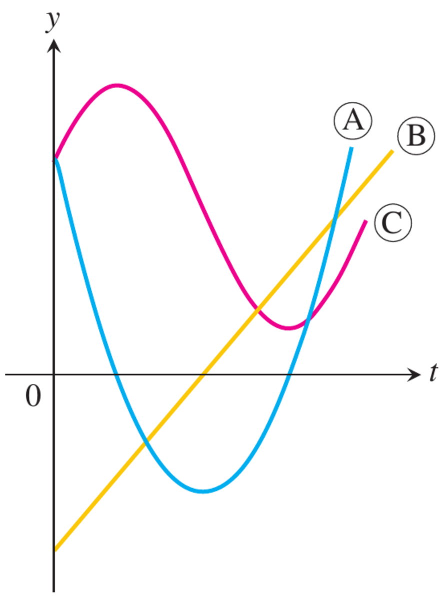


FIGURE 3.21 The graphs for Exercise 21.

35. Find all points on the curve $y = \tan x$, $-\pi/2 < x < \pi/2$, where the tangent line is parallel to the line $y = 2x$. Sketch the curve and tangent(s) together, labeling each with its equation.

27. Draining a tank It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank t hours after the valve is opened is given by the formula

$$y = 6\left(1 - \frac{t}{12}\right)^2 \text{ m}.$$

a. Find the rate dy/dt (m/h) at which the tank is draining at time t.

b. When is the fluid level in the tank falling fastest? Slowest? What are the values of dy/dt at these times?

EXERCISES 3.4

Derivatives

find dy/dx

5. $y = (\sec x + \tan x)(\sec x - \tan x)$

11. $y = x^2 \sin x + 2x \cos x - 2 \sin x$

25. Find y'' if

a. $y = \csc x$.

Tangent Lines

Graph the curves over the given intervals, together with their tangents at the given values of x. Label each curve and tangent with its equation.

29. $y = \sec x, \quad -\pi/2 < x < \pi/2$

$x = -\pi/3, \pi/4$

Trigonometric Limits

Find the limits

39. $\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$

40. $\lim_{x \to -\pi/6} \sqrt{1 + \cos(\pi \csc x)}$

EXERCISES 3.5

Derivative Calculations

In Exercises 9–18, write the function in the form $y = f(u)$ and $u = g(x)$. Then find dy/dx as a function of x.

14. $y = \left(\frac{x}{5} + \frac{1}{5x}\right)^5$

Find the derivatives of the functions

26. $y = \frac{1}{x}\sin^{-5} x - \frac{x}{3}\cos^3 x$

Second Derivatives
Find *y*"

49. $y = \left(1 + \frac{1}{x}\right)^3$

51. $y = \frac{1}{9}\cot(3x - 1)$

Finding Numerical Values of Derivatives

61. Find ds/dt when $\theta = 3\pi/2$ if $s = \cos\theta$ and $d\theta/dt = 5$.

Tangents to Parametrized Curves

94. $x = \sec^2 t - 1, \quad y = \tan t, \quad t = -\pi/4$

EXERCISES 3.6

Derivatives of Rational Powers

Find dy/dx

13. $y = \sin[(2t + 5)^{-2/3}]$

Differentiating Implicitly

Use implicit differentiation to find dy/dx

19. $x^2y + xy^2 = 6$

Second Derivatives

In Exercises 37-42, use implicit differentiation to find dy/dx and then d^2y/dx^2.

41. $2\sqrt{y} = x - y$

Slopes, Tangents, and Normals

Verify that the given point is on the curve and find the lines that are (a) tangent and (b) normal to the curve at the given point.

45. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$

Implicitly Defined Parametrizations

63. $x^2 - 2tx + 2t^2 = 4, \quad 2y^3 - 3t^2 = 4, \quad t = 2$

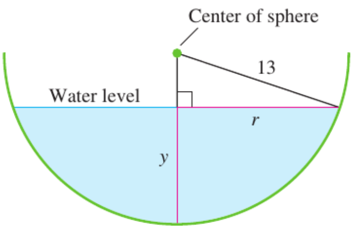
EXERCISES 3.7

12. Changing dimensions in a rectangular box Suppose that the edge lengths x, y, and z of a closed rectangular box are changing at the following rates:

$$\frac{dx}{dt} = 1 \text{ m/sec}, \quad \frac{dy}{dt} = -2 \text{ m/sec}, \quad \frac{dz}{dt} = 1 \text{ m/sec}.$$

20. A growing raindrop Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's

19. A draining hemispherical reservoir Water is flowing at the rate of 6 m^3/min from a reservoir shaped like a hemispherical bowl of radius 13 m, shown here in profile. Answer the following questions, given that the volume of water in a hemispherical bowl of radius R is $V = (\pi/3)y^2(3R - y)$ when the water is y meters deep.



Linearizing Trigonometric Functions

Find the linearization of f at $x = a$.

11. $f(x) = \sin x$ at (a) $x = 0$, (b) $x = \pi$

The Approximation $(1 + x)^k \approx 1 + kx$

17. Faster than a calculator Use the approximation $(1 + x)^k \approx 1 + kx$ to estimate the following.

a. $(1.0002)^{50}$ **b.** $\sqrt[3]{1.009}$

Derivatives in Differential Form

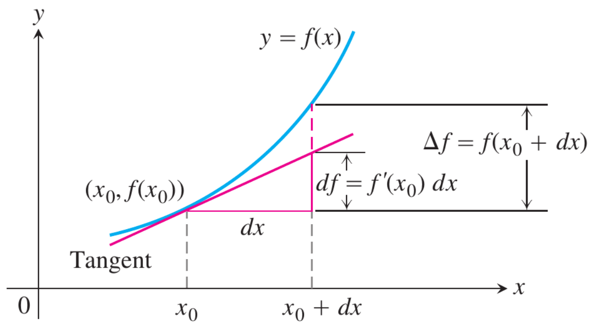
Find dy.

28. $y = \sec(x^2 - 1)$

Approximation Error

The function $f(x)$ changes value when x changes from x_0 to $x_0 + dx$. Find

31. $f(x) = x^2 + 2x, \quad x_0 = 1, \quad dx = 0.1$



EXERCISES 3.8

Finding Linearizations

find the linearization $L(x)$ of $f(x)$ at $x = a$.

4. $f(x) = \sqrt[3]{x}, \quad a = -8$

Linearization for Approximation

You want linearizations that will replace the functions in the following over intervals that include the given points x_0. To make your subsequent work as simple as possible, you want to center each linearization not at x_0 but at a nearby integer $x = a$ at which the given function and its derivative are easy to evaluate. What linearization do you use in each case?

5. $f(x) = x^2 + 2x, \quad x_0 = 0.1$

Tutorial 4 (Chapter 4)
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EXERCISES 4.1

Absolute Extrema on Finite Closed Intervals

Find the absolute maximum and minimum values of the function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

24. $g(x) = -\sqrt{5 - x^2}, \quad -\sqrt{5} \le x \le 0$

Finding Extreme Values

Find the function's absolute maximum and minimum values and say where they are assumed.

44. $y = \dfrac{x + 1}{x^2 + 2x + 2}$

Local Extrema and Critical Points

Find the derivative at each critical point and determine the local extreme values.

48. $y = x^2\sqrt{3 - x}$

54. Let $f(x) = |x^3 - 9x|$.

a. Does $f'(0)$ exist?

b. Does $f'(3)$ exist?

c. Does $f'(-3)$ exist?

d. Determine all extrema of f.

Optimization Applications

Area of an athletic field

62. An athletic field is to be built in the shape of a rectangle *x* units long capped by semicircular regions of radius *r* at the two ends. The field is to be bounded by a 400-m racetrack.

a. Express the area of the rectangular portion of the field as a function of *x* alone or *r* alone (your choice).

b. What values of x and r give the rectangular portion the largest possible area?

EXERCISES 4.2

Finding c in the Mean Value Theorem

Find the value or values of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the functions and intervals.

2. $f(x) = x^{2/3}, \quad [0, 1]$

Checking and Using Hypotheses

10. For what values of a, m and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval [0, 2]?

Roots (Zeros)

Show that the function has exactly one zero in the given interval.

15. $f(x) = x^4 + 3x + 1, \quad [-2, -1]$

Finding Functions from Derivatives

25. Suppose that $f'(x) = 2x$ for all x. Find $f(2)$ if

a. $f(0) = 0$ **b.** $f(1) = 0$ **c.** $f(-2) = 3$.

Finding Position from Acceleration

Exercise 43 give the acceleration $a = d^2s/dt^2$, initial velocity and initial position of a body moving on a coordinate line. Find the body's position at time t.

43. $a = -4\sin 2t, \quad v(0) = 2, \quad s(0) = -3$

EXERCISES 4.3

Analyzing f Given f'

Answer the following questions about the functions whose derivatives are given below:
a. What are the critical points of f?
b. On what intervals is f increasing or decreasing?
c. At what points, if any, does f assume local maximum and minimum values?

7. $f'(x) = x^{-1/3}(x + 2)$

8. $f'(x) = x^{-1/2}(x - 3)$

Extremes of Given Functions

a. Find the intervals on which the function is increasing and decreasing.
b. Then identify the function's local extreme values, if any, saying where they are taken on.
c. Which, if any, of the extreme values are absolute?

24. $f(x) = \dfrac{x^3}{3x^2 + 1}$

Extreme Values on Half-Open Intervals

a. Identify the function's local extreme values in the given domain, and say where they are assumed.
b. Which of the extreme values, if any, are absolute?

32. $g(x) = -x^2 - 6x - 9, \quad -4 \le x < \infty$

36. $k(x) = x^3 + 3x^2 + 3x + 1, \quad -\infty < x \le 0$

Theory and Examples

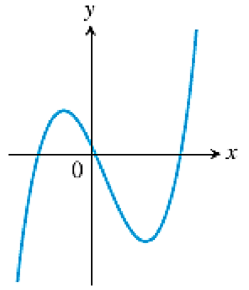
47. As x moves from left to right through the point $c = 2$, is the graph of $f(x) = x^3 - 3x + 2$ rising, or is it falling? Give reasons for your answer.

EXERCISES 4.4

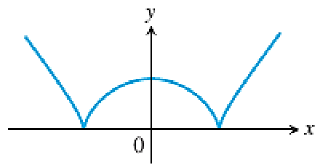
Analyzing Graphed Functions

Identify the inflection points and local maxima and minima of the functions graphed below. Identify the intervals on which the functions are concave up and concave down.

1. $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3}$



3. $y = \frac{3}{4}(x^2 - 1)^{2/3}$



Graph Equations

Use the steps of the graphing procedure to graph the equations below. Include the coordinates of any local extreme points and inflection points.

40. $y = \sqrt{|x - 4|}$

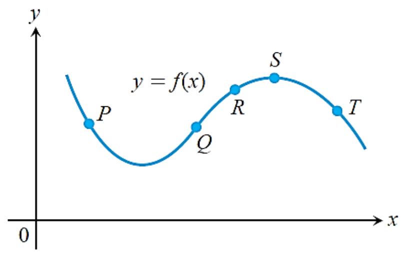
Sketching the General Shape Knowing y'

Each of Exercises below gives the first derivative of a continuous function $= f(x)$. Find y'' and sketch the general shape of the graph of f.

41. $y' = 2 + x - x^2$

Theory and Examples

67. The accompanying figure shows a portion if the graph of a twice-differentiable function $y = f(x)$. At each of the five labelled points, classify y' and y'' as positive, negative, or zero.



75. Suppose the derivative of the function $y = f(x)$ is

$$y' = (x - 1)^2(x - 2)$$

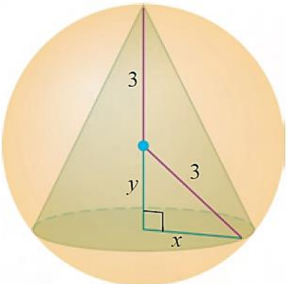
At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection? (Hint: Draw the sign pattern for y'.)

EXERCISES 4.5

Applications in Geometry

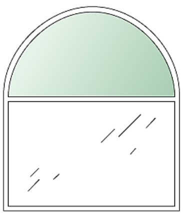
6. You are planning to close off a corner of the first quadrant with a line segment 20 units long running from $(a,0)$ to $(0,\ b)$. Show that the area of the triangle enclosed by the segment is largest when $a = b$.

12. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.



18. A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -\pi$ to $x = \pi$. What are the dimensions of the rectangle with largest area, and what is the largest area?

22. A window is in the form if a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



EXERCISES 4.6

Finding Limits

In Exercises 1 and 5, use l'Hôpital's Rule to evaluate the limit. Then evaluate the limit using a method studied in Chapter 2.

1. $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4}$

Applying l'Hôpital's Rule

Use l'Hôpital's Rule to find the limits in Exercises 22 and 25.

22. $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{\sqrt{x}}\right)$ **25.** $\lim\limits_{x\to\pm\infty} \dfrac{3x-5}{2x^2-x+2}$

Theory and Applications

32. ∞/∞ **Form**
Give an example of two differentiable functions f and g with $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} g(x) = \infty$ that satisfy the following.

a. $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = 3$ **b.** $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = 0$

c. $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \infty$

EXERCISES 4.8

Finding Antiderivatives

In Exercises 8 and 14, find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation.

8. a. $\dfrac{4}{3}\sqrt[3]{x}$ **b.** $\dfrac{1}{3\sqrt[3]{x}}$ **c.** $\sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

14. a. $\csc^2 x$ **b.** $-\dfrac{3}{2}\csc^2 \dfrac{3x}{2}$ **c.** $1 - 8\csc^2 2x$

Finding Indefinite Integrals

In Exercise 31 and 46, find the most general antiderivative or indefinite integral. Check your answer by differentiation.

31. $\displaystyle\int 2x(1 - x^{-3})\,dx$

46. $\displaystyle\int (2\cos 2x - 3\sin 3x)\,dx$

Checking Antiderivative Formulas

Verify the formulas in Exercises 60 by differentiation.

60. $\displaystyle\int \frac{1}{(x+1)^2}\,dx = \frac{x}{x+1} + C$

Theory and Examples

101. Suppose that

$$f(x) = \frac{d}{dx}\left(1 - \sqrt{x}\right) \quad \text{and} \quad g(x) = \frac{d}{dx}(x+2)$$

Find:

a. $\displaystyle\int f(x)\,dx$

b. $\displaystyle\int g(x)\,dx$

c. $\displaystyle\int [-f(x)]\,dx$

d. $\displaystyle\int [-g(x)]\,dx$

e. $\displaystyle\int [f(x) + g(x)]\,dx$

f. $\displaystyle\int [f(x) - g(x)]\,dx$

Tutorial 5 (Chapter 5 and 6)
Thomas' Calculus 11th edition

EXERCISES 5.1

Area

In Exercise 1 use finite approximations to estimate the area under the graph of the function using
a. a lower sum with two rectangles of equal width.
b. a lower sim with four rectangles of equal width.
c. an upper sum with two rectangles of equal width.
d. an upper sum with four rectangles of equal width.

1. $f(x) = x^2$ between $x = 0$ and $x = 1$

Area of a Circle

21. Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of the polygon for the following values of n:

a. 4 (square) **b.** 8 (octagon) **c.** 16
d. Compare the areas in parts (a), (b) and (c) with the area of the circle.

EXERCISES 5.2

Sigma Notation

Write the sums in Exercises 1 without sigma notation. Then evaluate them.

1. $\sum_{k=1}^{2} \frac{6k}{k+1}$

Values of Finite Sums

17. Suppose that $\sum_{k=1}^{n} a_k = -5$ and $\sum_{k=1}^{n} b_k = 6$. Find the values of

c. $\sum_{k=1}^{n} (a_k + b_k)$

Evaluate the sums in Exercise 24.

24. $\sum_{k=1}^{6} (k^2 - 5)$

Limits of Upper Sums

For the functions in Exercise 36, find a formula for the upper sum obtained by dividing the interval $[a,b]$ into n equal subintervals. Then take a limit of this sum as $n \to \infty$ to calculate the area under the curve over $[a,b]$.

36. $f(x) = 2x$ over the interval $[0, 3]$

EXERCISES 5.3

Expressing Limits as Integrals

Express the limits in Exercise 1 as definite integrals.

1. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} c_k^2 \, \Delta x_k$, where P is a partition of $[0, 2]$

Using Properties and Known Values to Find Other Integrals

12. Suppose that $\int_{-3}^{0} g(t)\, dt = \sqrt{2}$. Find

a. $\int_{0}^{-3} g(t)\, dt$ **b.** $\int_{-3}^{0} g(u)\, du$

c. $\int_{-3}^{0} [-g(x)]\, dx$ **d.** $\int_{-3}^{0} \frac{g(r)}{\sqrt{2}}\, dr$

Using Area to Evaluate Definite Integrals

In Exercise 15, graph the integrands and use areas to evaluate the integrals.

15. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

Evaluations

Use the results of Equations (1) and (3) to evaluate the integrals in Exercise 38.

38. $\int_{0}^{3b} x^2\, dx$

Average Value

In Exercise 55, graph the function and find its average value over the given interval.

55. $f(x) = x^2 - 1 \quad \text{on} \quad [0, \sqrt{3}]$

EXERCISES 5.4

Evaluating Integrals

Evaluate the integrals in Exercises 23 and 25.

23. $\displaystyle\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$

25. $\displaystyle\int_{-4}^{4} |x|\, dx$

Derivatives of Integrals

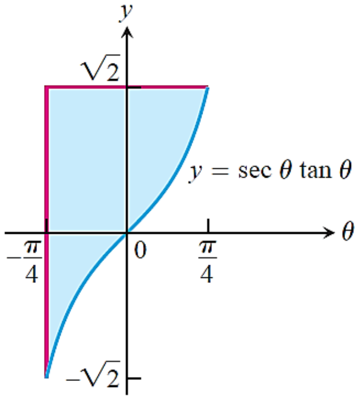
Find dy/dx in Exercise 36.

36. $\displaystyle y = \int_{\tan x}^{0} \frac{dt}{1 + t^2}$

Area

Find the areas of the shaded regions in Exercise 45.

45.



Theory and Examples

62. Find

$$\lim_{x\to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1}\, dt$$

EXERCISES 5.5

Evaluating Integrals

Evaluate the indefinite integrals in Exercise 4 and 11 by using the given substitutions to reduce the integrals to standard form.

4. $\displaystyle\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt, \quad u = 1 - \cos\frac{t}{2}$

11. $\displaystyle\int \csc^2 2\theta \cot 2\theta\, d\theta$

a. Using $u = \cot 2\theta$ **b.** Using $u = \csc 2\theta$

Evaluate the integrals in Exercises 36 and 48.

36. $\displaystyle\int \frac{6\cos t}{(2 + \sin t)^3}\, dt$

48. $\displaystyle\int 3x^5\sqrt{x^3 + 1}\, dx$

Simplifying Integrals Step by Step

Evaluate the integrals in Exercise 51.

51. $\displaystyle\int \frac{(2r - 1)\cos\sqrt{3(2r - 1)^2 + 6}}{\sqrt{3(2r - 1)^2 + 6}}\, dr$

EXERCISES 5.6

Evaluating Definite Integrals

Use the substitution formula in Theorem 6 to evaluate the integrals in Exercises 7 and 14.

7. a. $\displaystyle\int_{-1}^{1} \frac{5r}{(4 + r^2)^2}\, dr$ **b.** $\displaystyle\int_0^1 \frac{5r}{(4 + r^2)^2}\, dr$

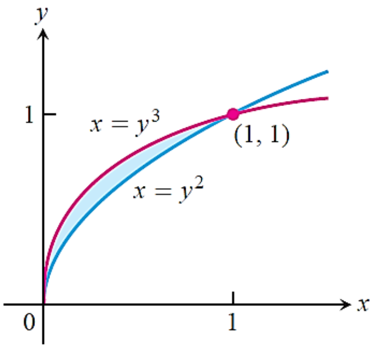
14. a. $\displaystyle\int_{-\pi/2}^{0} \frac{\sin w}{(3 + 2\cos w)^2}\, dw$

b. $\displaystyle\int_0^{\pi/2} \frac{\sin w}{(3 + 2\cos w)^2}\, dw$

Area

Find the total areas of the shaded regions in Exercise 32.

32.



73. Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis.

EXERCISES 6.3

Length of Parametrized Curves

Find the lengths of the curves in Exercise 1.

1. $x = 1 - t, \quad y = 2 + 3t, \quad -2/3 \leq t \leq 1$

Finding Lengths of Curves

Find the lengths of the curves in Exercises 7 and 16. If you have a grapher, you may want to graph these curves to see what they look like.

7. $y = (1/3)(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$

16. $y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt, \quad -2 \leq x \leq -1$

Theory and Applications

27. a. Find a curve through the point (1, 1) whose length integral is

$$L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$$

b. How many such curves are there? Give reasons for your answer.

Tutorial 6 (Chapter 7)
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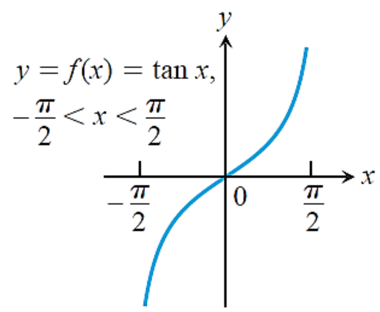
EXERCISES 7.1

Graphing Inverse Functions

Exercise 10 shows the graph of a function $y = f(x)$. Copy the graph and draw in the line $y = x$. Then use symmetry with respect to the line $y = x$ to add the graph of f^{-1} to your sketch. (It is not necessary to find a formula for f^{-1}.) Identify the domain and range of f^{-1}.

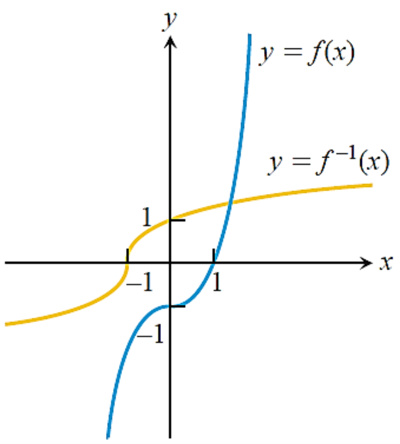
10.



Formulas for Inverse Functions

Exercise 15 gives a formula for a function $y = f(x)$ and shows the graphs of f and f^{-1}. Find a formula for f^{-1} in each case.

15. $f(x) = x^3 - 1$



21. $f(x) = x^3 + 1$

Derivatives of Inverse Functions

In Exercises 25 and 30:
a. Find $f^{-1}(x)$.
b. Graph f and f^{-1} together.
c. Evaluate df/dx at $x = a$ and $\frac{df^{-1}}{dx}$ at $x = f(a)$ to show that at these points $\frac{df^{-1}}{dx} = 1/(\frac{df}{dx})$.

25. $f(x) = 2x + 3, \quad a = -1$

30.
a. Show that $h(x) = x^3/4$ and $k(x) = (4x)^{1/3}$ are inverses of one another.
b. Graph h and k over an x-interval large enough to show the graphs intersecting at (2, 2) and (-2, -2). Be sure the picture shows the required symmetry about the line $y = x$.
c. Find the slopes of the tangents to the graphs at h and k at (2, 2) and (-2, -2).
d. What lines are tangent to the curves at the origin?

EXERCISES 7.2

Using the Properties of Logarithms

1. Express the following logarithms in terms of ln 2 and ln 3.

a. ln 0.75 **b.** ln (4/9) **c.** ln (1/2)
d. ln $\sqrt[3]{9}$ **e.** ln $3\sqrt{2}$ **f.** ln $\sqrt{13.5}$

Derivatives of Logarithms

In Exercise 22, find the derivative of y with respect to x, t, or θ, as appropriate.

22. $y = \frac{x \ln x}{1 + \ln x}$

Integration

Evaluate the integrals in Exercise 39.

39. $\int \frac{2y\,dy}{y^2 - 25}$

Logarithmic Differentiation

In Exercise 64, use logarithmic differentiation to find the derivative of y with respect to the given independent variable.

64. $y = \dfrac{\theta \sin\theta}{\sqrt{\sec\theta}}$

Theory and Applications

69. Locate and identify the absolute extreme values of

a. ln (cos x) on $[-\frac{\pi}{4}, \frac{\pi}{3}]$,

b. cos (ln x) on $[\frac{1}{2}, 2]$.

<u>EXERCISES 7.3</u>

Algebraic Calculations with the Exponential and Logarithm

Find simpler expressions for the quantities in Exercise 2.

2. a. $e^{\ln(x^2+y^2)}$ **b.** $e^{-\ln 0.3}$ **c.** $e^{\ln \pi x - \ln 2}$

Solving Equations with Logarithmic or Exponential Terms

In Exercise 10, solve for y in terms of t or x, as appropriate.

10. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x)$

In Exercise 16, solve for t.

16. $e^{(x^2)}e^{(2x+1)} = e^t$

Derivatives

In Exercises 23 and 36, find the derivative of y with respect to x, t, or θ, as appropriate.

23. $y = (x^2 - 2x + 2)e^x$

36. $y = \displaystyle\int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t\, dt$

Integrals

Evaluate the integrals in Exercises 49 and 56.

49. $\displaystyle\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\, dr$

56. $\displaystyle\int_{\pi/4}^{\pi/2} (1 + e^{\cot\theta}) \csc^2\theta\, d\theta$

Theory and Applications

67. Find the absolute maximum and minimum values of $f(x) = e^x - 2x$ on [0, 1].

<u>EXERCISES 7.4</u>

Algebraic Calculations With a^x and $\log_a x$

Simplify the expressions in Exercise 4.

4. a. $25^{\log_5(3x^2)}$ **b.** $\log_e(e^x)$ **c.** $\log_4(2^{e^x \sin x})$

Derivatives

In Exercises 18 and 29, find the derivative of y with respect to the given independent variable.

18. $y = (\ln\theta)^\pi$

29. $y = \log_3\left(\left(\dfrac{x+1}{x-1}\right)^{\ln 3}\right)$

Logarithmic Differentiation

In Exercises 41 and 46, use logarithmic differentiation to find the derivative of y with respect to the given independent variable.

41. $y = (\sqrt{t})^t$

46. $y = (\ln x)^{\ln x}$

Integration

Evaluate the integrals in Exercise 65.

65. $\displaystyle\int_0^2 \frac{\log_2(x+2)}{x+2}\, dx$

Evaluate the integrals in Exercise 72.

72. $\int_1^{e^x} \frac{1}{t}\,dt$

Theory and Applications

75. Find the area of the region between the curve $y = 2x/(1 + x^2)$ and the interval $-2 \leq x \leq 2$ of the x-axis.

EXERCISES 7.5

6. Voltage in a discharging capacitor
Suppose that electricity is draining from a capacitor at a rate that is proportional to the voltage V across its terminals and that, if t is measured in seconds,

$$\frac{dV}{dt} = -\frac{1}{40}V.$$

Solve this equation for V, using V_0 to denote the value of V when $t = 0$. How long will it take the voltage to drop to 10% of its original value?

8. Growth of bacteria
A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10,000 bacteria. At the end of 5 hours there are 40,000. How many bacteria were present initially?

EXERCISES 7.7

Common Values of Inverse Trigonometric Functions

Use reference triangles to find the angles in Exercise 6.

6. a. $\cos^{-1}\left(\frac{-1}{2}\right)$ **b.** $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

c. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

Trigonometric Function Values

13. Given that $\alpha = sin^{-1}(5/13)$, find cos α, tan α, sec α, csc α, and cot α.

Evaluating Trigonometric and Inverse Trigonometric Terms

Find the values in Exercise 26.

26. $\sec\left(\cot^{-1}\sqrt{3} + \csc^{-1}(-1)\right)$

Finding Derivatives

In Exercise 51, find the derivative of y with respect to the appropriate variable.

51. $y = \sin^{-1}\sqrt{2}\,t$

Evaluating Integrals

Evaluating the integrals in Exercise 72.

72. $\int \frac{dx}{\sqrt{1 - 4x^2}}$

Evaluate the integrals in Exercise 107.

107. $\int \frac{(\sin^{-1} x)^2\,dx}{\sqrt{1 - x^2}}$

Integration Formulas

Verify the integration formulas in Exercise 117.

117. $\int \frac{\tan^{-1} x}{x^2}\,dx = \ln x - \frac{1}{2}\ln(1 + x^2) - \frac{\tan^{-1} x}{x} + C$

EXERCISES 7.8

Hyperbolic Function Values and Identities

Each of Exercise 1 gives a value of sinh x or cosh x. Use the definitions and the identity $\cosh^2 x - \sinh^2 x = 1$ to find the values of the remaining five hyperbolic functions.

1. $\sinh x = -\frac{3}{4}$

Derivatives

In Exercise 16, find the derivative of y with respect to the appropriate variable.

16. $y = t^2 \tanh \frac{1}{t}$

Indefinite Integrals

Evaluate the integrals in Exercise 43.

43. $\int 6 \cosh \left(\frac{x}{2} - \ln 3 \right) dx$

Definite Integrals

Evaluate the integrals in Exercise 60.

60. $\int_0^{\ln 10} 4 \sinh^2 \left(\frac{x}{2} \right) dx$

Evaluating Inverse Hyperbolic Functions and Related Integrals

When hyperbolic function keys are not available on a calculator, it is still possible to evaluate the inverse hyperbolic functions by expressing them as logarithms, as shown here.

$\sinh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right),$	$-\infty < x < \infty$
$\cosh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right),$	$x \geq 1$
$\tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x},$	$\lvert x \rvert < 1$
$\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x} \right),$	$0 < x \leq 1$
$\operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{\lvert x \rvert} \right),$	$x \neq 0$
$\coth^{-1} x = \frac{1}{2} \ln \frac{x + 1}{x - 1},$	$\lvert x \rvert > 1$

Use the formulas in the box here to express the numbers in Exercise 66 in terms of natural logarithms.

66. $\operatorname{csch}^{-1} (-1/\sqrt{3})$

Applications and Theory

83. Arc length

Find the length of the segment of the curve $y = (1/2) \cosh 2x$ from $x = 0$ to $x = \ln \sqrt{5}$.

Tutorial 7 (Chapter 8)
Thomas' Calculus 11th edition

EXERCISES 8.1

Basic Substitutions

Evaluate each integral in Exercise 36 by using a substitution to reduce it to standard form.

36. $\displaystyle\int \frac{\ln x\, dx}{x + 4x \ln^2 x}$

Completing the Square

Evaluate each integral in Exercise 41 by completing the square and using a substitution to reduce it to standard form.

41. $\displaystyle\int \frac{dx}{(x + 1)\sqrt{x^2 + 2x}}$

Improper Fractions

Evaluate each integral in Exercise 50 by reducing the improper fraction and using a substitution (if necessary) to reduce it to standard form.

50. $\displaystyle\int_{-1}^{3} \frac{4x^2 - 7}{2x + 3}\, dx$

Separating Fractions

Evaluate each integral in Exercise 56 by separating the fraction and using a substitution (if necessary) to reduce it to standard form.

56. $\displaystyle\int_{0}^{1/2} \frac{2 - 8x}{1 + 4x^2}\, dx$

Multiplying by a Form of 1

Evaluate each integral in Exercise 59 by multiplying by a form of 1 and using a substitution (if necessary) to reduce it to standard form.

59. $\displaystyle\int \frac{1}{\sec\theta + \tan\theta}\, d\theta$

Eliminating Square Roots

Evaluate each integral in Exercise 68 by eliminating the square root.

68. $\displaystyle\int_{\pi/2}^{\pi} \sqrt{1 - \sin^2\theta}\, d\theta$

Assorted Integrations

Evaluate each integral in Exercise 82 by using any technique you think is appropriate.

82. $\displaystyle\int \frac{dx}{x\sqrt{3 + x^2}}$

Trigonometric Powers

83.
a. Evaluate $\int cos^3\,\theta\; d\theta$. (Hint: $cos^2\,\theta = 1 - sin^2\,\theta$.)
b. Evaluate $\int cos^5\,\theta\; d\theta$.
c. Without actually evaluating the integral, explain how you would evaluate $\int cos^9\,\theta\; d\theta$.

EXERCISES 8.2

Integration by Parts

Evaluate the integrals in Exercise 1, 19 and 24.

1. $\displaystyle\int x \sin\frac{x}{2}\, dx$

19. $\displaystyle\int_{2/\sqrt{3}}^{2} t \sec^{-1} t\, dt$

24. $\displaystyle\int e^{-2x} \sin 2x\, dx$

Substitution and Integration by Parts

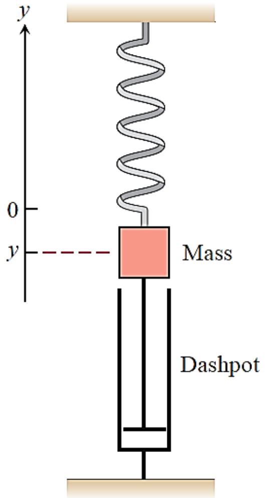
Evaluate the integrals in Exercise 30 by using a substitution prior to integration by parts.

30. $\displaystyle\int z(\ln z)^2\, dz$

37. Average value
A retarding force, symbolized by the dashpot in the figure, slows the motion of the weighted spring so that the mass's position at time t is

$$y = 2e^{-t}\cos t, \quad t \geq 0.$$

Find the average value of y over the interval $0 \leq t \leq 2\pi$.



Reduction Formulas

In Exercise 41, use integration by parts to establish the reduction formula.

41. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx, \quad a \neq 0$

EXERCISES 8.3

Expanding Quotients into Partial Fractions

Expand the quotients in Exercise 6 by partial fractions.

6. $\frac{z}{z^3 - z^2 - 6z}$

Nonrepeated Linear Factors

In Exercise 12, express the integrands as a sum of partial fractions and evaluate the integrals.

12. $\int \frac{2x+1}{x^2 - 7x + 12}\,dx$

Repeated Linear Factors

In Exercise 20, express the integrands as a sum of partial fractions and evaluate the integrals.

20. $\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)}$

Irreducible Quadratic Factors

In Exercise 26, express the integrands as a sum of partial fractions and evaluate the integrals.

26. $\int \frac{s^4 + 81}{s(s^2+9)^2}\,ds$

Improper Fractions

In Exercise 31, perform long division on the intergrand, write the proper fraction as a sum of partial fractions, and then evaluate the integral.

31. $\int \frac{9x^3 - 3x + 1}{x^3 - x^2}\,dx$

Evaluating Integrals

Evaluating the integrals in Exercise 38.

38. $\int \frac{\sin\theta\,d\theta}{\cos^2\theta + \cos\theta - 2}$

EXERCISES 8.4

Products of Powers of Sines and Cosines

Evaluate the integrals in Exercise 6 and 14.

6. $\int_0^{\pi/2} 7\cos^7 t\,dt$

14. $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\,d\theta$

Integrals with Square Roots

Evaluate the integrals in Exercise 22.

22. $\int_{-\pi}^{\pi} (1 - \cos^2 t)^{3/2}\, dt$

Powers of Tan x and Sec x

Evaluate the integrals in Exercise 26.

26. $\int_{0}^{\pi/12} 3 \sec^4 3x\, dx$

Products of Sines and Cosines

Evaluate the integrals in Exercise 38.

38. $\int_{-\pi/2}^{\pi/2} \cos x \cos 7x\, dx$

<u>EXERCISES 8.5</u>

Basic Trigonometric Substitutions

Evaluate the integrals in Exercise 1, 14 and 28.

1. $\int \frac{dy}{\sqrt{9 + y^2}}$

14. $\int \frac{2\, dx}{x^3\sqrt{x^2 - 1}}, \quad x > 1$

28. $\int \frac{(1 - r^2)^{5/2}}{r^8}\, dr$

In Exercise 32, use an appropriate substitution and then a trigonometric substitution to evaluate the integrals.

32. $\int_{1}^{e} \frac{dy}{y\sqrt{1 + (\ln y)^2}}$

Applications

41. Find the area of the region in the first quadrant that is enclosed by the coordinate axes and the curve $y = \sqrt{9 - x^2}/3$.

<u>EXERCISES 8.6</u>

Using Integral Tables

Use the table of integrals to evaluate the integrals in Exercise 8 and 20.

8. $\int \frac{dx}{x^2\sqrt{4x - 9}}$

20. $\int \frac{d\theta}{4 + 5 \sin 2\theta}$

Substitution and Integral Tables

In Exercise 45, use a substitution to change the integral into one you can find in the table. Then evaluate the integral.

45. $\int \cot t \sqrt{1 - \sin^2 t}\, dt, \quad 0 < t < \pi/2$

Using Reduction Formulas

Use reduction formulas to evaluate the integrals in Exercise 60.

60. $\int \csc^2 y \cos^5 y\, dy$

Powers of x Times Exponentials

Evaluate the integrals in Exercise 80 using table Formulas 103-106. These integrals can also be evaluated using integration (Section 8.2).

80. $\int x 2^{\sqrt{2}x}\, dx$

Substitutions with Reduction Formulas

Evaluate the integrals in Exercise 81 by making a substitution (possibly trigonometric) and then applying a reduction formula.

81. $\int e^t \sec^3 (e^t - 1)\, dt$

Hyperbolic Functions

Use the integral tables to evaluate the integrals in Exercise 90.

90. $\int x \sinh 5x \, dx$

EXERCISES 8.8

Evaluating Improper Integrals

Evaluate the integrals in Exercises 1 and 26 without using tables.

1. $\int_0^{\infty} \frac{dx}{x^2 + 1}$

26. $\int_0^1 (-\ln x) \, dx$

Testing for Convergence

In Exercises 35, 50 and 64, use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence. If more than one method applies, use whatever method you prefer.

35. $\int_0^{\pi/2} \tan \theta \, d\theta$

50. $\int_0^{\infty} \frac{d\theta}{1 + e^{\theta}}$

64. $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$

Theory and Examples

65. Find the values of p for which each integral converges.

b. $\int_2^{\infty} \frac{dx}{x(\ln x)^p}$

Tutorial 8 (Chapter 11)
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EXERCISES 11.1

Finding Terms of a Sequence

Exercise 2 gives a formula for the nth term a_n of a sequence $\{a_n\}$. Find the values of a_1, a_2, a_3, and a_4.

2. $a_n = \dfrac{1}{n!}$

Finding a Sequence's Formula

In Exercise 16, find a formula for the nth term of the sequence.

16. The sequence $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$

Reciprocals of squares
of the positive integers,
with alternating signs

Finding Limits

Which of the sequences $\{a_n\}$ in Exercises 25, 49 and 80 converge. and which diverge? Find the limit of each convergent sequence.

25. $a_n = \dfrac{1-2n}{1+2n}$

49. $a_n = \left(1+\dfrac{7}{n}\right)^n$

80. $a_n = \dfrac{(\ln n)^5}{\sqrt{n}}$

EXERCISES 11.2

Finding nth Partial Sums

In Exercise 1, find a formula for the nth partial sum of each series and use it to find the series' sum if the series converges.

1. $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^{n-1}} + \cdots$

Series with Geometric Terms

In Exercise 7, write out the first few terms of each series to show how the series starts. Then find the sum of the series.

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$

Telescoping Series

Find the sum of each series in Exercise 15.

15. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$

Convergence or Divergence

Is Exercise 23 converge or diverge? Give reasons for your answer. If a series converges, find its sum.

23. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n$

Geometric Series

In geometric series in Exercise 41, write out the first few terms of the series to find a and r, and find the sum of the series. Then express the inequality $|r| < 1$ in terms of x and find the values of x for which the inequality holds and the series converges.

41. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$

Repeating Decimals

Express each of the numbers in Exercise 51 as the ratio of two integers.

51. $0.\overline{23} = 0.23\ 23\ 23\ldots$

EXERCISES 11.3

Determining Convergence or Divergence

Which of the series in Exercises 1, 9, 10 and 28 converge, and which diverge? Give reasons for your answers. (When you check an answer, remember that there may be more than one way to determine the series' convergence or divergence.)

1. $\sum_{n=1}^{\infty} \frac{1}{10^n}$

9. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

10. $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$

28. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

EXERCISES 11.4

Determining Convergence and Divergence

Which of the series in 1, 10 and 36 converge, and which diverge? Give reasons for your answers.

1. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$

10. $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$

36. $\sum_{n=1}^{\infty} \frac{1}{1 + 2^2 + 3^2 + \cdots + n^2}$

EXERCISES 11.6

Determining Convergence or Divergence

Is Exercise 1 converge or diverge? Give reasons for your answers.

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$

Absolute Convergence

Which of the series in Exercises 13 and 30 converge absolutely, which converge, and which diverge? Give reasons for your answers.

13. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

30. $\sum_{n=1}^{\infty} (-5)^{-n}$

EXERCISES 11.7

Intervals of Convergence

In Exercise 1, 11 and 22, **(a)** find the series' radius and interval of convergence. For what values of x does the series converge **(b)** absolutely, **(c)** conditionally?

1. $\sum_{n=0}^{\infty} x^n$

11. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

22. $\sum_{n=1}^{\infty} (\ln n) x^n$

In Exercise 36, find the series' interval of convergence and, within this interval, the sum of the series as a function of x.

36. $\sum_{n=0}^{\infty} (\ln x)^n$

EXERCISES 11.8

Finding Taylor Polynomials

In Exercises 1 and 4, find the Taylor polynomials of orders 0, 1, 2, and 3 generated by f at a.

1. $f(x) = \ln x, \quad a = 1$

4. $f(x) = 1/(x + 2), \quad a = 0$

Finding Taylor Series at $x = 0$ (Maclaurin Series)

Find the Maclaurin series for the functions in Exercise 9.

9. e^{-x}

Finding Taylor Series

In Exercises 24 and 28, find the Taylor series generated by f at $x = a$.

24. $f(x) = 3x^5 - x^4 + 2x^3 + x^2 - 2, \quad a = -1$

28. $f(x) = 2^x, \quad a = 1$

EXERCISES 11.9

Taylor Series by Substitution

Use substitution to find the Taylor series at $x = 0$ of the functions in Exercise 1.

1. e^{-5x}

More Taylor Series

Find Taylor series at $x = 0$ for the functions in Exercise 8.

8. $x^2 \sin x$

EXERCISES 11.10

Binomial Series

Find the first four terms of the binomial series for the functions in Exercises 1 and 9.

1. $(1 + x)^{1/2}$

9. $\left(1 + \frac{1}{x}\right)^{1/2}$

Find the binomial series for the functions in Exercise 11.

11. $(1 + x)^4$

EXERCISES 11.11

Finding Fourier Series

In Exercises 1 and 8, find the Fourier series associated with the given functions. Sketch each function.

1. $f(x) = 1 \quad 0 \leq x \leq 2\pi$

8. $f(x) = \begin{cases} 2, & 0 \leq x \leq \pi \\ -x, & \pi < x \leq 2\pi \end{cases}$