Tutorial 2 (Chapter 2)
Thomas' Calculus 11th edition

Exercise 2.1

Limits from Graphs

3. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?

a. $\lim_{x\to 0} f(x)$ exists.

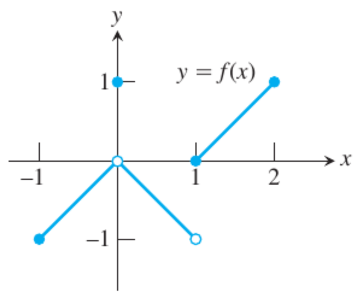
b. $\lim_{x\to 0} f(x) = 0$.

c. $\lim_{x\to 0} f(x) = 1$.

d. $\lim_{x\to 1} f(x) = 1$.

e. $\lim_{x\to 1} f(x) = 0$.

f. $\lim_{x\to x_0} f(x)$ exists at every point x_0 in $(-1, 1)$.



Existence of Limits

9. If $\lim_{x\to 1} f(x) = 5$, must f be defined at $x = 1$? If it is, must $f(1) = 5$? Can we conclude *anything* about the values of f at $x = 1$? Explain.

10. If $f(1) = 5$, must $\lim_{x\to 1} f(x)$ exist? If it does, then must $\lim_{x\to 1} f(x) = 5$? Can we conclude *anything* about $\lim_{x\to 1} f(x)$? Explain.

Limits by Substitution

Find the limits by substitution.

27. $\lim_{x\to \pi/2} x \sin x$

Average Rates of Change

Find the average rate of change of the function over the given interval or intervals.

33. $R(\theta) = \sqrt{4\theta + 1}$; $[0, 2]$

Exercise 2.2

Limit Calculations

Find the limits.

17. $\lim_{h\to 0} \dfrac{\sqrt{3h+1} - 1}{h}$

19. $\lim_{x\to 5} \dfrac{x-5}{x^2 - 25}$

27. $\lim_{u\to 1} \dfrac{u^4 - 1}{u^3 - 1}$

Using Limit Rules

39. Suppose $\lim_{x\to c} f(x) = 5$ and $\lim_{x\to c} g(x) = -2$. Find

a. $\lim_{x\to c} f(x)g(x)$

Limits of Average Rates of Change

Because of their connection with secant lines, tangents, and instantaneous rates, limits of the form

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

occur frequently in calculus. Evaluate this limit for the given value of x and function f.

47. $f(x) = \sqrt{x}$, $x = 7$

Using the Sandwich Theorem

49. If $\sqrt{5 - 2x^2} \le f(x) \le \sqrt{5 - x^2}$ for $-1 \le x \le 1$, find $\lim_{x\to 0} f(x)$.

EXERCISES 2.3

Centering Intervals About a Point

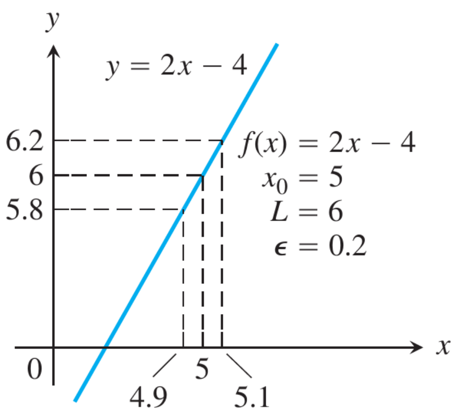
In Exercises 1–6, sketch the interval (a, b) on the x-axis with the point x_0 inside. Then find a value of $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow a < x < b$.

4. $a = -7/2$, $b = -1/2$, $x_0 = -3/2$

Finding Deltas Graphically

use the graphs to find a $\delta > 0$ such that for all x

7.



Finding Deltas Algebraically

Each of Exercises 15–30 gives a function $f(x)$ and numbers L, x_0 and $\epsilon > 0$. In each case, find an open interval about x_0 on which the inequality $|f(x) - L| < \epsilon$ holds. Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \epsilon$ holds.

15. $f(x) = x + 1, \qquad L = 5, \qquad x_0 = 4,$

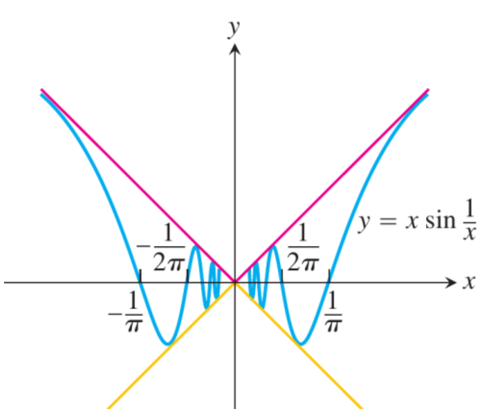
More on Formal Limits

31. $f(x) = 3 - 2x, \qquad x_0 = 3, \qquad \epsilon = 0.02$

Prove the limit statements

43. $\lim_{x \to 1} \frac{1}{x} = 1$

49. $\lim_{x \to 0} x \sin \frac{1}{x} = 0$



EXERCISES 2.4

Finding Limits Graphically

7. a. Graph $f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1. \end{cases}$

b. Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

c. Does $\lim_{x \to 1} f(x)$ exist? If so, what is it? If not, why not?

Finding One-Sided Limits Algebraically

Find the limit

15. $\lim_{h \to 0^+} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$

Using $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

Find the limits

33. $\lim_{\theta \to 0} \frac{\sin \theta}{\sin 2\theta}$

Calculating Limits as $x \to \pm\infty$

In Exercises 37–42, find the limit of each function **(a)** as $x \to \infty$ and **(b)** as $x \to -\infty$. (You may wish to visualize your answer with a graphing calculator or computer.)

41. $h(x) = \frac{-5 + (7/x)}{3 - (1/x^2)}$

Limits of Rational Functions

In Exercises 47–56, find the limit of each rational function **(a)** $x \to \infty$ and **(b)** as $x \to -\infty$.

49. $f(x) = \frac{x + 1}{x^2 + 3}$

EXERCISES 2.5

Infinite Limits

Find the limits.

13. $\lim_{x \to (\pi/2)^-} \tan x$

Additional Calculations

Find the limits.

25. $\lim \left(\dfrac{1}{x^{2/3}} + \dfrac{2}{(x-1)^{2/3}} \right)$ as

a. $x \to 0^+$

c. $x \to 1^+$

Graphing Rational Functions

Graph the rational functions in Exercises 27–38. Include the graphs and equations of the asymptotes and dominant terms.

36. $y = \dfrac{x^2 - 1}{2x + 4}$

Inventing Functions

Find a function that satisfies the given conditions and sketch its graph. (The answers here are not unique. Any function that satisfies the conditions is acceptable. Feel free to use formulas defined in pieces if that will help.)

43. $\lim_{x \to \pm\infty} f(x) = 0$, $\lim_{x \to 2^-} f(x) = \infty$, and $\lim_{x \to 2^+} f(x) = \infty$

Graphing Terms

The function is given as the sum or difference of two terms. First graph the terms (with the same set of axes). Then, using these graphs as guides, sketch in the graph of the function.

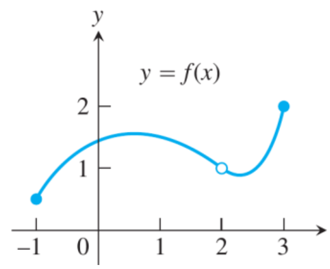
59. $y = \tan x + \dfrac{1}{x^2}, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

EXERCISES 2.6

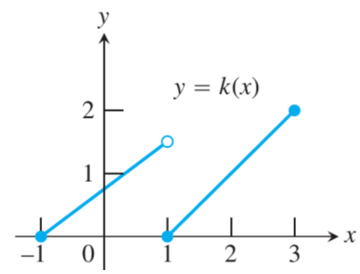
Continuity from Graphs

In the exercises below, say whether the function graphed is continuous on [-1, 3] . If not, where does it fail to be continuous and why?

1.



4.



Applying the Continuity Test

At which points do the functions fail to be continuous? At which points, if any, are the discontinuities removable? Not removable? Give reasons for your

17. $y = |x - 1| + \sin x$

20. $y = \dfrac{x + 2}{\cos x}$

Composite Functions

Find the limits. Are the functions continuous at the point being approached?

29. $\lim_{x \to \pi} \sin(x - \sin x)$

EXERCISES 2.7

Slopes and Tangent Lines

Find an equation for the tangent to the curve at the given point. Then sketch the curve and tangent together.

5. $y = 4 - x^2, \quad (-1, 3)$

Find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

18. $f(x) = \sqrt{x + 1}, \quad (8, 3)$

22. $y = \dfrac{x - 1}{x + 1}, \quad x = 0$

Tangent Lines with Specified Slopes

26. Find an equation of the straight line having slope $1/4$ that is tangent to the curve $y = \sqrt{x}$.

Rates of Change

30. Ball's changing volume What is the rate of change of the volume of a ball ($V = (4/3)\pi r^3$) with respect to the radius when the radius is $r = 2$?