Tutorial 4 (Chapter 4)
Thomas' Calculus 11^{th} edition

EXERCISES 4.1

Absolute Extrema on Finite Closed Intervals

Find the absolute maximum and minimum values of the function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

24. $g(x) = -\sqrt{5 - x^2}, \quad -\sqrt{5} \le x \le 0$

Finding Extreme Values

Find the function's absolute maximum and minimum values and say where they are assumed.

44. $y = \dfrac{x + 1}{x^2 + 2x + 2}$

Local Extrema and Critical Points

Find the derivative at each critical point and determine the local extreme values.

48. $y = x^2\sqrt{3 - x}$

54. Let $f(x) = |x^3 - 9x|$.

a. Does $f'(0)$ exist?

b. Does $f'(3)$ exist?

c. Does $f'(-3)$ exist?

d. Determine all extrema of f.

Optimization Applications

Area of an athletic field
62. An athletic field is to be built in the shape of a rectangle *x* units long capped by semicircular regions of radius *r* at the two ends. The field is to be bounded by a 400-m racetrack.
a. Express the area of the rectangular portion of the field as a function of *x* alone or *r* alone (your choice).
b. What values of x and r give the rectangular portion the largest possible area?

EXERCISES 4.2

Finding c in the Mean Value Theorem

Find the value or values of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the functions and intervals.

2. $f(x) = x^{2/3}, \quad [0, 1]$

Checking and Using Hypotheses

10. For what values of a, m and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval [0, 2]?

Roots (Zeros)

Show that the function has exactly one zero in the given interval.

15. $f(x) = x^4 + 3x + 1, \quad [-2, -1]$

Finding Functions from Derivatives

25. Suppose that $f'(x) = 2x$ for all x. Find $f(2)$ if

a. $f(0) = 0$ **b.** $f(1) = 0$ **c.** $f(-2) = 3$.

Finding Position from Acceleration

Exercise 43 give the acceleration $a = d^2s/dt^2$, initial velocity and initial position of a body moving on a coordinate line. Find the body's position at time t.

43. $a = -4\sin 2t, \quad v(0) = 2, \quad s(0) = -3$

EXERCISES 4.3

Analyzing f Given f'

Answer the following questions about the functions whose derivatives are given below:
a. What are the critical points of f?
b. On what intervals is f increasing or decreasing?
c. At what points, if any, does f assume local maximum and minimum values?

7. $f'(x) = x^{-1/3}(x + 2)$
8. $f'(x) = x^{-1/2}(x - 3)$

Extremes of Given Functions

a. Find the intervals on which the function is increasing and decreasing.
b. Then identify the function's local extreme values, if any, saying where they are taken on.
c. Which, if any, of the extreme values are absolute?

24. $f(x) = \dfrac{x^3}{3x^2 + 1}$

Extreme Values on Half-Open Intervals

a. Identify the function's local extreme values in the given domain, and say where they are assumed.
b. Which of the extreme values, if any, are absolute?

32. $g(x) = -x^2 - 6x - 9, \quad -4 \le x < \infty$

36. $k(x) = x^3 + 3x^2 + 3x + 1, \quad -\infty < x \le 0$

Theory and Examples

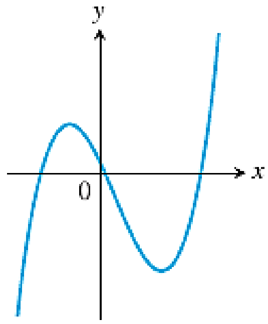
47. As x moves from left to right through the point $c = 2$, is the graph of $f(x) = x^3 - 3x + 2$ rising, or is it falling? Give reasons for your answer.

EXERCISES 4.4

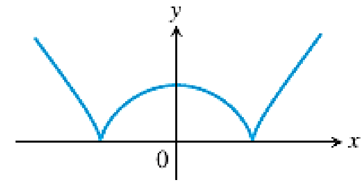
Analyzing Graphed Functions

Identify the inflection points and local maxima and minima of the functions graphed below. Identify the intervals on which the functions are concave up and concave down.

1. $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3}$



3. $y = \frac{3}{4}(x^2 - 1)^{2/3}$



Graph Equations

Use the steps of the graphing procedure to graph the equations below. Include the coordinates of any local extreme points and inflection points.

40. $y = \sqrt{|x - 4|}$

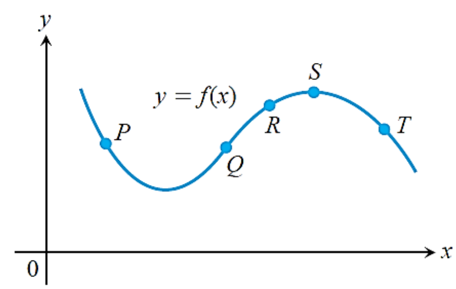
Sketching the General Shape Knowing y'

Each of Exercises below gives the first derivative of a continuous function $= f(x)$. Find y'' and sketch the general shape of the graph of f.

41. $y' = 2 + x - x^2$

Theory and Examples

67. The accompanying figure shows a portion if the graph of a twice-differentiable function $y = f(x)$. At each of the five labelled points, classify y' and y'' as positive, negative, or zero.



75. Suppose the derivative of the function $y = f(x)$ is

$$y' = (x - 1)^2(x - 2)$$

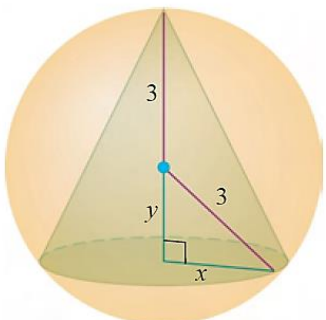
At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection? (Hint: Draw the sign pattern for y'.)

EXERCISES 4.5

Applications in Geometry

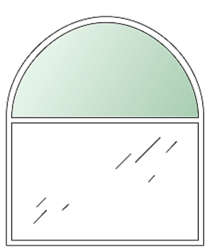
6. You are planning to close off a corner of the first quadrant with a line segment 20 units long running from $(a,0)$ to $(0,\ b)$. Show that the area of the triangle enclosed by the segment is largest when $a = b$.

12. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.



18. A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -\pi$ to $x = \pi$. What are the dimensions of the rectangle with largest area, and what is the largest area?

22. A window is in the form if a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



EXERCISES 4.6

Finding Limits

In Exercises 1 and 5, use l'Hôpital's Rule to evaluate the limit. Then evaluate the limit using a method studied in Chapter 2.

1. $\lim_{x \to 2} \dfrac{x - 2}{x^2 - 4}$

Applying l'Hôpital's Rule

Use l'Hôpital's Rule to find the limits in Exercises 22 and 25.

22. $\lim_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{\sqrt{x}}\right)$ **25.** $\lim_{x \to \pm\infty} \dfrac{3x - 5}{2x^2 - x + 2}$

Theory and Applications

32. ∞/∞ **Form**
Give an example of two differentiable functions f and g with $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$ that satisfy the following.

a. $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = 3$ **b.** $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = 0$

c. $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = \infty$

EXERCISES 4.8

Finding Antiderivatives

In Exercises 8 and 14, find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation.

8. a. $\dfrac{4}{3}\sqrt[3]{x}$ **b.** $\dfrac{1}{3\sqrt[3]{x}}$ **c.** $\sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

14. a. $\csc^2 x$ **b.** $-\dfrac{3}{2}\csc^2 \dfrac{3x}{2}$ **c.** $1 - 8\csc^2 2x$

Finding Indefinite Integrals

In Exercise 31 and 46, find the most general antiderivative or indefinite integral. Check your answer by differentiation.

31. $\displaystyle\int 2x(1 - x^{-3})\, dx$

46. $\displaystyle\int (2\cos 2x - 3\sin 3x)\, dx$

Checking Antiderivative Formulas

Verify the formulas in Exercises 60 by differentiation.

60. $\int \frac{1}{(x+1)^2}\,dx = \frac{x}{x+1} + C$

Theory and Examples

101. Suppose that

$$f(x) = \frac{d}{dx}\left(1 - \sqrt{x}\right) \quad \text{and} \quad g(x) = \frac{d}{dx}(x+2)$$

Find:

a. $\int f(x)\,dx$

b. $\int g(x)\,dx$

c. $\int [-f(x)]\,dx$

d. $\int [-g(x)]\,dx$

e. $\int [f(x) + g(x)]\,dx$

f. $\int [f(x) - g(x)]\,dx$