Tutorial 5 (Chapter 5 and 6)
Thomas' Calculus 11th edition

EXERCISES 5.1

Area

In Exercise 1 use finite approximations to estimate the area under the graph of the function using
a. a lower sum with two rectangles of equal width.
b. a lower sim with four rectangles of equal width.
c. an upper sum with two rectangles of equal width.
d. an upper sum with four rectangles of equal width.

1. $f(x) = x^2$ between $x = 0$ and $x = 1$

Area of a Circle

21. Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of the polygon for the following values of n:

a. 4 (square) **b.** 8 (octagon) **c.** 16
d. Compare the areas in parts (a), (b) and (c) with the area of the circle.

EXERCISES 5.2

Sigma Notation

Write the sums in Exercises 1 without sigma notation. Then evaluate them.

1. $\sum_{k=1}^{2} \frac{6k}{k+1}$

Values of Finite Sums

17. Suppose that $\sum_{k=1}^{n} a_k = -5$ and $\sum_{k=1}^{n} b_k = 6$. Find the values of

c. $\sum_{k=1}^{n} (a_k + b_k)$

Evaluate the sums in Exercise 24.

24. $\sum_{k=1}^{6} (k^2 - 5)$

Limits of Upper Sums

For the functions in Exercise 36, find a formula for the upper sum obtained by dividing the interval $[a,b]$ into n equal subintervals. Then take a limit of this sum as $n \to \infty$ to calculate the area under the curve over $[a,b]$.

36. $f(x) = 2x$ over the interval $[0, 3]$

EXERCISES 5.3

Expressing Limits as Integrals

Express the limits in Exercise 1 as definite integrals.

1. $\lim_{\|P\| \to 0} \sum_{k=1}^{n} c_k^2 \, \Delta x_k$, where P is a partition of $[0, 2]$

Using Properties and Known Values to Find Other Integrals

12. Suppose that $\int_{-3}^{0} g(t)\, dt = \sqrt{2}$. Find

a. $\int_{0}^{-3} g(t)\, dt$ **b.** $\int_{-3}^{0} g(u)\, du$

c. $\int_{-3}^{0} [-g(x)]\, dx$ **d.** $\int_{-3}^{0} \frac{g(r)}{\sqrt{2}}\, dr$

Using Area to Evaluate Definite Integrals

In Exercise 15, graph the integrands and use areas to evaluate the integrals.

15. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

Evaluations

Use the results of Equations (1) and (3) to evaluate the integrals in Exercise 38.

38. $\int_{0}^{3b} x^2\, dx$

Average Value

In Exercise 55, graph the function and find its average value over the given interval.

55. $f(x) = x^2 - 1 \quad \text{on} \quad [0, \sqrt{3}]$

EXERCISES 5.4

Evaluating Integrals

Evaluate the integrals in Exercises 23 and 25.

23. $\displaystyle\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$

25. $\displaystyle\int_{-4}^{4} |x|\, dx$

Derivatives of Integrals

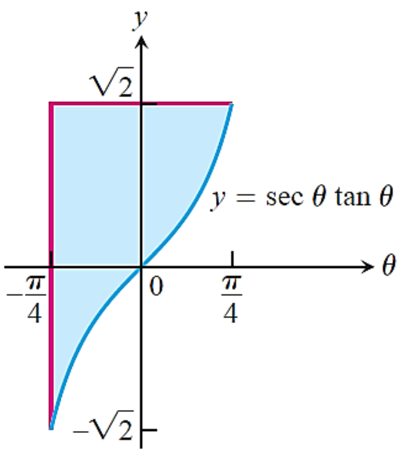
Find dy/dx in Exercise 36.

36. $\displaystyle y = \int_{\tan x}^{0} \frac{dt}{1 + t^2}$

Area

Find the areas of the shaded regions in Exercise 45.

45.



Theory and Examples

62. Find

$$\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1}\, dt$$

EXERCISES 5.5

Evaluating Integrals

Evaluate the indefinite integrals in Exercise 4 and 11 by using the given substitutions to reduce the integrals to standard form.

4. $\displaystyle\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt, \quad u = 1 - \cos\frac{t}{2}$

11. $\displaystyle\int \csc^2 2\theta \cot 2\theta\, d\theta$

a. Using $u = \cot 2\theta$ **b.** Using $u = \csc 2\theta$

Evaluate the integrals in Exercises 36 and 48.

36. $\displaystyle\int \frac{6\cos t}{(2 + \sin t)^3}\, dt$

48. $\displaystyle\int 3x^5 \sqrt{x^3 + 1}\, dx$

Simplifying Integrals Step by Step

Evaluate the integrals in Exercise 51.

51. $\displaystyle\int \frac{(2r - 1)\cos\sqrt{3(2r - 1)^2 + 6}}{\sqrt{3(2r - 1)^2 + 6}}\, dr$

EXERCISES 5.6

Evaluating Definite Integrals

Use the substitution formula in Theorem 6 to evaluate the integrals in Exercises 7 and 14.

7. a. $\displaystyle\int_{-1}^{1} \frac{5r}{(4 + r^2)^2}\, dr$ **b.** $\displaystyle\int_{0}^{1} \frac{5r}{(4 + r^2)^2}\, dr$

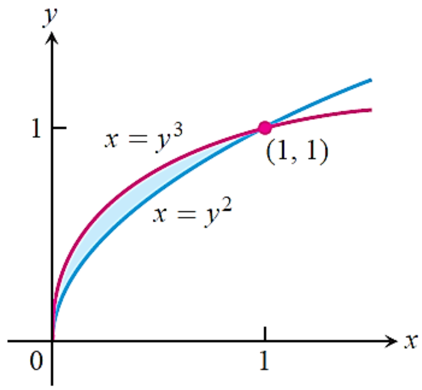
14. a. $\displaystyle\int_{-\pi/2}^{0} \frac{\sin w}{(3 + 2\cos w)^2}\, dw$

b. $\displaystyle\int_{0}^{\pi/2} \frac{\sin w}{(3 + 2\cos w)^2}\, dw$

Area

Find the total areas of the shaded regions in Exercise 32.

32.



73. Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis.

<u>EXERCISES 6.3</u>

Length of Parametrized Curves

Find the lengths of the curves in Exercise 1.

1. $x = 1 - t, \quad y = 2 + 3t, \quad -2/3 \leq t \leq 1$

Finding Lengths of Curves

Find the lengths of the curves in Exercises 7 and 16. If you have a grapher, you may want to graph these curves to see what they look like.

7. $y = (1/3)(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$

16. $y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt, \quad -2 \leq x \leq -1$

Theory and Applications

27. a. Find a curve through the point (1, 1) whose length integral is

$$L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$$

b. How many such curves are there? Give reasons for your answer.