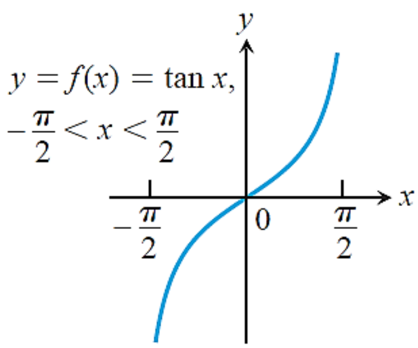
Tutorial 6 (Chapter 7)
Thomas' Calculus 11th edition

EXERCISES 7.1

Graphing Inverse Functions

Exercise 10 shows the graph of a function $y = f(x)$. Copy the graph and draw in the line $y = x$. Then use symmetry with respect to the line $y = x$ to add the graph of f^{-1} to your sketch. (It is not necessary to find a formula for f^{-1}.) Identify the domain and range of f^{-1}.

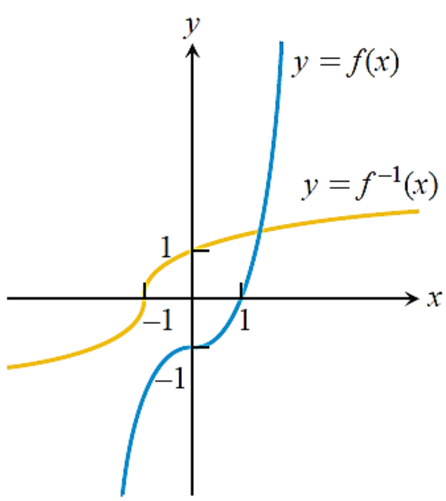
10.



Formulas for Inverse Functions

Exercise 15 gives a formula for a function $y = f(x)$ and shows the graphs of f and f^{-1}. Find a formula for f^{-1} in each case.

15. $f(x) = x^3 - 1$



21. $f(x) = x^3 + 1$

Derivatives of Inverse Functions

In Exercises 25 and 30:
a. Find $f^{-1}(x)$.
b. Graph f and f^{-1} together.
c. Evaluate df/dx at $x = a$ and $\frac{df^{-1}}{dx}$ at $x = f(a)$ to show that at these points $\frac{df^{-1}}{dx} = 1/(\frac{df}{dx})$.

25. $f(x) = 2x + 3, \quad a = -1$

30.
a. Show that $h(x) = x^3/4$ and $k(x) = (4x)^{1/3}$ are inverses of one another.
b. Graph h and k over an x-interval large enough to show the graphs intersecting at (2, 2) and (-2, -2). Be sure the picture shows the required symmetry about the line $y = x$.
c. Find the slopes of the tangents to the graphs at h and k at (2, 2) and (-2, -2).
d. What lines are tangent to the curves at the origin?

EXERCISES 7.2

Using the Properties of Logarithms

1. Express the following logarithms in terms of ln 2 and ln 3.

a. ln 0.75 **b.** ln (4/9) **c.** ln (1/2)
d. ln $\sqrt[3]{9}$ **e.** ln $3\sqrt{2}$ **f.** ln $\sqrt{13.5}$

Derivatives of Logarithms

In Exercise 22, find the derivative of y with respect to x, t, or θ, as appropriate.

22. $y = \dfrac{x \ln x}{1 + \ln x}$

Integration

Evaluate the integrals in Exercise 39.

39. $\displaystyle\int \frac{2y\,dy}{y^2 - 25}$

Logarithmic Differentiation

In Exercise 64, use logarithmic differentiation to find the derivative of y with respect to the given independent variable.

64. $y = \dfrac{\theta \sin\theta}{\sqrt{\sec\theta}}$

Theory and Applications

69. Locate and identify the absolute extreme values of

a. ln (cos x) on $[-\frac{\pi}{4}, \frac{\pi}{3}]$,

b. cos (ln x) on $[\frac{1}{2}, 2]$.

<u>EXERCISES 7.3</u>

Algebraic Calculations with the Exponential and Logarithm

Find simpler expressions for the quantities in Exercise 2.

2. a. $e^{\ln(x^2+y^2)}$ **b.** $e^{-\ln 0.3}$ **c.** $e^{\ln \pi x - \ln 2}$

Solving Equations with Logarithmic or Exponential Terms

In Exercise 10, solve for y in terms of t or x, as appropriate.

10. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x)$

In Exercise 16, solve for t.

16. $e^{(x^2)}e^{(2x+1)} = e^t$

Derivatives

In Exercises 23 and 36, find the derivative of y with respect to x, t, or θ, as appropriate.

23. $y = (x^2 - 2x + 2)e^x$

36. $y = \displaystyle\int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t \, dt$

Integrals

Evaluate the integrals in Exercises 49 and 56.

49. $\displaystyle\int \frac{e^{\sqrt{r}}}{\sqrt{r}} dr$

56. $\displaystyle\int_{\pi/4}^{\pi/2} (1 + e^{\cot\theta}) \csc^2\theta \, d\theta$

Theory and Applications

67. Find the absolute maximum and minimum values of $f(x) = e^x - 2x$ on [0, 1].

<u>EXERCISES 7.4</u>

Algebraic Calculations With a^x and $\log_a x$

Simplify the expressions in Exercise 4.

4. a. $25^{\log_5(3x^2)}$ **b.** $\log_e(e^x)$ **c.** $\log_4(2^{e^x \sin x})$

Derivatives

In Exercises 18 and 29, find the derivative of y with respect to the given independent variable.

18. $y = (\ln\theta)^\pi$

29. $y = \log_3\left(\left(\dfrac{x+1}{x-1}\right)^{\ln 3}\right)$

Logarithmic Differentiation

In Exercises 41 and 46, use logarithmic differentiation to find the derivative of y with respect to the given independent variable.

41. $y = (\sqrt{t})^t$

46. $y = (\ln x)^{\ln x}$

Integration

Evaluate the integrals in Exercise 65.

65. $\displaystyle\int_0^2 \frac{\log_2(x+2)}{x+2} dx$

Evaluate the integrals in Exercise 72.

72. $\int_1^{e^x} \frac{1}{t}\, dt$

Theory and Applications

75. Find the area of the region between the curve $y = 2x/(1+x^2)$ and the interval $-2 \le x \le 2$ of the x-axis.

EXERCISES 7.5

6. Voltage in a discharging capacitor
Suppose that electricity is draining from a capacitor at a rate that is proportional to the voltage V across its terminals and that, if t is measured in seconds,

$$\frac{dV}{dt} = -\frac{1}{40}V.$$

Solve this equation for V, using V_0 to denote the value of V when $t = 0$. How long will it take the voltage to drop to 10% of its original value?

8. Growth of bacteria
A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10,000 bacteria. At the end of 5 hours there are 40,000. How many bacteria were present initially?

EXERCISES 7.7

Common Values of Inverse Trigonometric Functions

Use reference triangles to find the angles in Exercise 6.

6. a. $\cos^{-1}\left(\frac{-1}{2}\right)$ **b.** $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

c. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

Trigonometric Function Values

13. Given that $\alpha = sin^{-1}(5/13)$, find cos α, tan α, sec α, csc α, and cot α.

Evaluating Trigonometric and Inverse Trigonometric Terms

Find the values in Exercise 26.

26. $\sec\left(\cot^{-1}\sqrt{3} + \csc^{-1}(-1)\right)$

Finding Derivatives

In Exercise 51, find the derivative of y with respect to the appropriate variable.

51. $y = \sin^{-1}\sqrt{2}\, t$

Evaluating Integrals

Evaluating the integrals in Exercise 72.

72. $\int \frac{dx}{\sqrt{1-4x^2}}$

Evaluate the integrals in Exercise 107.

107. $\int \frac{(\sin^{-1} x)^2\, dx}{\sqrt{1-x^2}}$

Integration Formulas

Verify the integration formulas in Exercise 117.

117. $\int \frac{\tan^{-1} x}{x^2}\, dx = \ln x - \frac{1}{2}\ln(1+x^2) - \frac{\tan^{-1} x}{x} + C$

EXERCISES 7.8

Hyperbolic Function Values and Identities

Each of Exercise 1 gives a value of sinh x or cosh x. Use the definitions and the identity $\cosh^2 x - \sinh^2 x = 1$ to find the values of the remaining five hyperbolic functions.

1. $\sinh x = -\frac{3}{4}$

Derivatives

In Exercise 16, find the derivative of y with respect to the appropriate variable.

16. $y = t^2 \tanh \frac{1}{t}$

Indefinite Integrals

Evaluate the integrals in Exercise 43.

43. $\int 6 \cosh \left(\frac{x}{2} - \ln 3 \right) dx$

Definite Integrals

Evaluate the integrals in Exercise 60.

60. $\int_0^{\ln 10} 4 \sinh^2 \left(\frac{x}{2} \right) dx$

Evaluating Inverse Hyperbolic Functions and Related Integrals

When hyperbolic function keys are not available on a calculator, it is still possible to evaluate the inverse hyperbolic functions by expressing them as logarithms, as shown here.

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right),$	$-\infty < x < \infty$
$\cosh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right),$	$x \geq 1$
$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x},$	$\lvert x \rvert < 1$
$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right),$	$0 < x \leq 1$
$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{\lvert x \rvert}\right),$	$x \neq 0$
$\coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1},$	$\lvert x \rvert > 1$

Use the formulas in the box here to express the numbers in Exercise 66 in terms of natural logarithms.

66. $\operatorname{csch}^{-1}(-1/\sqrt{3})$

Applications and Theory

83. Arc length

Find the length of the segment of the curve $y = (1/2)\cosh 2x$ from $x = 0$ to $x = \ln\sqrt{5}$.