**Tutorial 7 (Chapter 8)**
**Thomas' Calculus 11th edition**

## EXERCISES 8.1

### Basic Substitutions

Evaluate each integral in Exercise 36 by using a substitution to reduce it to standard form.

**36.** $\int \frac{\ln x \, dx}{x + 4x \ln^2 x}$

### Completing the Square

Evaluate each integral in Exercise 41 by completing the square and using a substitution to reduce it to standard form.

**41.** $\int \frac{dx}{(x + 1)\sqrt{x^2 + 2x}}$

### Improper Fractions

Evaluate each integral in Exercise 50 by reducing the improper fraction and using a substitution (if necessary) to reduce it to standard form.

**50.** $\int_{-1}^{3} \frac{4x^2 - 7}{2x + 3} dx$

### Separating Fractions

Evaluate each integral in Exercise 56 by separating the fraction and using a substitution (if necessary) to reduce it to standard form.

**56.** $\int_{0}^{1/2} \frac{2 - 8x}{1 + 4x^2} dx$

### Multiplying by a Form of 1

Evaluate each integral in Exercise 59 by multiplying by a form of 1 and using a substitution (if necessary) to reduce it to standard form.

**59.** $\int \frac{1}{\sec\theta + \tan\theta} d\theta$

### Eliminating Square Roots

Evaluate each integral in Exercise 68 by eliminating the square root.

**68.** $\int_{\pi/2}^{\pi} \sqrt{1 - \sin^2\theta} \, d\theta$

### Assorted Integrations

Evaluate each integral in Exercise 82 by using any technique you think is appropriate.

**82.** $\int \frac{dx}{x\sqrt{3 + x^2}}$

### Trigonometric Powers

**83.**
**a.** Evaluate $\int cos^3\,\theta\, d\theta$. (Hint: $cos^2\,\theta = 1 - sin^2\,\theta$.)
**b.** Evaluate $\int cos^5\,\theta\, d\theta$.
**c.** Without actually evaluating the integral, explain how you would evaluate $\int cos^9\,\theta\, d\theta$.

## EXERCISES 8.2

### Integration by Parts

Evaluate the integrals in Exercise 1, 19 and 24.

**1.** $\int x \sin\frac{x}{2} dx$

**19.** $\int_{2/\sqrt{3}}^{2} t \sec^{-1} t \, dt$

**24.** $\int e^{-2x} \sin 2x \, dx$

### Substitution and Integration by Parts

Evaluate the integrals in Exercise 30 by using a substitution prior to integration by parts.

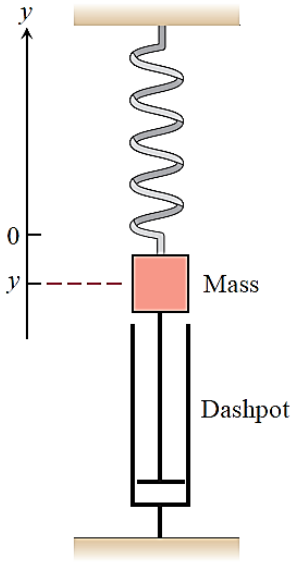
**30.** $\int z(\ln z)^2 \, dz$

**37. Average value**

A retarding force, symbolized by the dashpot in the figure, slows the motion of the weighted spring so that the mass's position at time $t$ is

$$y = 2e^{-t}\cos t, \quad t \geq 0.$$

Find the average value of $y$ over the interval $0 \leq t \leq 2\pi$.




**Reduction Formulas**

In Exercise 41, use integration by parts to establish the reduction formula.

**41.** $$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx, \quad a \neq 0$$

## EXERCISES 8.3

**Expanding Quotients into Partial Fractions**

Expand the quotients in Exercise 6 by partial fractions.

**6.** $$\frac{z}{z^3 - z^2 - 6z}$$

**Nonrepeated Linear Factors**

In Exercise 12, express the integrands as a sum of partial fractions and evaluate the integrals.

**12.** $$\int \frac{2x + 1}{x^2 - 7x + 12}\,dx$$

**Repeated Linear Factors**

In Exercise 20, express the integrands as a sum of partial fractions and evaluate the integrals.

**20.** $$\int \frac{x^2\,dx}{(x - 1)(x^2 + 2x + 1)}$$

**Irreducible Quadratic Factors**

In Exercise 26, express the integrands as a sum of partial fractions and evaluate the integrals.

**26.** $$\int \frac{s^4 + 81}{s(s^2 + 9)^2}\,ds$$

**Improper Fractions**

In Exercise 31, perform long division on the intergrand, write the proper fraction as a sum of partial fractions, and then evaluate the integral.

**31.** $$\int \frac{9x^3 - 3x + 1}{x^3 - x^2}\,dx$$

**Evaluating Integrals**

Evaluating the integrals in Exercise 38.

**38.** $$\int \frac{\sin\theta\,d\theta}{\cos^2\theta + \cos\theta - 2}$$

## EXERCISES 8.4

**Products of Powers of Sines and Cosines**

Evaluate the integrals in Exercise 6 and 14.

**6.** $$\int_0^{\pi/2} 7\cos^7 t\,dt$$

**14.** $$\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\,d\theta$$

**Integrals with Square Roots**

Evaluate the integrals in Exercise 22.

**22.** $\int_{-\pi}^{\pi} (1 - \cos^2 t)^{3/2}\, dt$

**Powers of Tan $x$ and Sec $x$**

Evaluate the integrals in Exercise 26.

**26.** $\int_{0}^{\pi/12} 3 \sec^4 3x\, dx$

**Products of Sines and Cosines**

Evaluate the integrals in Exercise 38.

**38.** $\int_{-\pi/2}^{\pi/2} \cos x \cos 7x\, dx$

## EXERCISES 8.5

**Basic Trigonometric Substitutions**

Evaluate the integrals in Exercise 1, 14 and 28.

**1.** $\int \frac{dy}{\sqrt{9 + y^2}}$

**14.** $\int \frac{2\, dx}{x^3\sqrt{x^2 - 1}}, \quad x > 1$

**28.** $\int \frac{(1 - r^2)^{5/2}}{r^8}\, dr$

In Exercise 32, use an appropriate substitution and then a trigonometric substitution to evaluate the integrals.

**32.** $\int_{1}^{e} \frac{dy}{y\sqrt{1 + (\ln y)^2}}$

**Applications**

**41.** Find the area of the region in the first quadrant that is enclosed by the coordinate axes and the curve $y = \sqrt{9 - x^2}/3$.

## EXERCISES 8.6

**Using Integral Tables**

Use the table of integrals to evaluate the integrals in Exercise 8 and 20.

**8.** $\int \frac{dx}{x^2\sqrt{4x - 9}}$

**20.** $\int \frac{d\theta}{4 + 5 \sin 2\theta}$

**Substitution and Integral Tables**

In Exercise 45, use a substitution to change the integral into one you can find in the table. Then evaluate the integral.

**45.** $\int \cot t\sqrt{1 - \sin^2 t}\, dt, \quad 0 < t < \pi/2$

**Using Reduction Formulas**

Use reduction formulas to evaluate the integrals in Exercise 60.

**60.** $\int \csc^2 y \cos^5 y\, dy$

**Powers of $x$ Times Exponentials**

Evaluate the integrals in Exercise 80 using table Formulas 103-106. These integrals can also be evaluated using integration (Section 8.2).

**80.** $\int x2^{\sqrt{2x}}\, dx$

**Substitutions with Reduction Formulas**

Evaluate the integrals in Exercise 81 by making a substitution (possibly trigonometric) and then applying a reduction formula.

**81.** $\int e^t \sec^3 (e^t - 1)\, dt$

**Hyperbolic Functions**

Use the integral tables to evaluate the integrals in Exercise 90.

**90.** $\int x \sinh 5x \, dx$

## EXERCISES 8.8

**Evaluating Improper Integrals**

Evaluate the integrals in Exercises 1 and 26 without using tables.

**1.** $\int_0^\infty \frac{dx}{x^2 + 1}$

**26.** $\int_0^1 (-\ln x) \, dx$

**Testing for Convergence**

In Exercises 35, 50 and 64, use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence. If more than one method applies, use whatever method you prefer.

**35.** $\int_0^{\pi/2} \tan\theta \, d\theta$

**50.** $\int_0^\infty \frac{d\theta}{1 + e^\theta}$

**64.** $\int_{-\infty}^\infty \frac{dx}{e^x + e^{-x}}$

**Theory and Examples**

**65.** Find the values of $p$ for which each integral converges.

**b.** $\int_2^\infty \frac{dx}{x(\ln x)^p}$