Tutorial 8 (Chapter 11)
Thomas' Calculus 11th edition

EXERCISES 11.1

Finding Terms of a Sequence

Exercise 2 gives a formula for the nth term a_n of a sequence $\{a_n\}$. Find the values of a_1, a_2, a_3, and a_4.

2. $a_n = \dfrac{1}{n!}$

Finding a Sequence's Formula

In Exercise 16, find a formula for the nth term of the sequence.

16. The sequence $1, -\dfrac{1}{4}, \dfrac{1}{9}, -\dfrac{1}{16}, \dfrac{1}{25}, \dots$

Reciprocals of squares
of the positive integers,
with alternating signs

Finding Limits

Which of the sequences $\{a_n\}$ in Exercises 25, 49 and 80 converge. and which diverge? Find the limit of each convergent sequence.

25. $a_n = \dfrac{1-2n}{1+2n}$

49. $a_n = \left(1 + \dfrac{7}{n}\right)^n$

80. $a_n = \dfrac{(\ln n)^5}{\sqrt{n}}$

EXERCISES 11.2

Finding nth Partial Sums

In Exercise 1, find a formula for the nth partial sum of each series and use it to find the series' sum if the series converges.

1. $2 + \dfrac{2}{3} + \dfrac{2}{9} + \dfrac{2}{27} + \cdots + \dfrac{2}{3^{n-1}} + \cdots$

Series with Geometric Terms

In Exercise 7, write out the first few terms of each series to show how the series starts. Then find the sum of the series.

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$

Telescoping Series

Find the sum of each series in Exercise 15.

15. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$

Convergence or Divergence

Is Exercise 23 converge or diverge? Give reasons for your answer. If a series converges, find its sum.

23. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n$

Geometric Series

In geometric series in Exercise 41, write out the first few terms of the series to find a and r, and find the sum of the series. Then express the inequality $|r| < 1$ in terms of x and find the values of x for which the inequality holds and the series converges.

41. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$

Repeating Decimals

Express each of the numbers in Exercise 51 as the ratio of two integers.

51. $0.\overline{23} = 0.23\ 23\ 23\dots$

EXERCISES 11.3

Determining Convergence or Divergence

Which of the series in Exercises 1, 9, 10 and 28 converge, and which diverge? Give reasons for your answers. (When you check an answer, remember that there may be more than one way to determine the series' convergence or divergence.)

1. $\sum_{n=1}^{\infty} \frac{1}{10^n}$

9. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

10. $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$

28. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

EXERCISES 11.4

Determining Convergence and Divergence

Which of the series in 1, 10 and 36 converge, and which diverge? Give reasons for your answers.

1. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$

10. $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$

36. $\sum_{n=1}^{\infty} \frac{1}{1 + 2^2 + 3^2 + \cdots + n^2}$

EXERCISES 11.6

Determining Convergence or Divergence

Is Exercise 1 converge or diverge? Give reasons for your answers.

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$

Absolute Convergence

Which of the series in Exercises 13 and 30 converge absolutely, which converge, and which diverge? Give reasons for your answers.

13. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

30. $\sum_{n=1}^{\infty} (-5)^{-n}$

EXERCISES 11.7

Intervals of Convergence

In Exercise 1, 11 and 22, **(a)** find the series' radius and interval of convergence. For what values of x does the series converge **(b)** absolutely, **(c)** conditionally?

1. $\sum_{n=0}^{\infty} x^n$

11. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

22. $\sum_{n=1}^{\infty} (\ln n) x^n$

In Exercise 36, find the series' interval of convergence and, within this interval, the sum of the series as a function of x.

36. $\sum_{n=0}^{\infty} (\ln x)^n$

EXERCISES 11.8

Finding Taylor Polynomials

In Exercises 1 and 4, find the Taylor polynomials of orders 0, 1, 2, and 3 generated by f at a.

1. $f(x) = \ln x, \quad a = 1$

4. $f(x) = 1/(x + 2), \quad a = 0$

Finding Taylor Series at $x = 0$ (Maclaurin Series)

Find the Maclaurin series for the functions in Exercise 9.

9. e^{-x}

Finding Taylor Series

In Exercises 24 and 28, find the Taylor series generated by f at $x = a$.

24. $f(x) = 3x^5 - x^4 + 2x^3 + x^2 - 2,$
$a = -1$

28. $f(x) = 2^x, \quad a = 1$

EXERCISES 11.9

Taylor Series by Substitution

Use substitution to find the Taylor series at $x = 0$ of the functions in Exercise 1.

1. e^{-5x}

More Taylor Series

Find Taylor series at $x = 0$ for the functions in Exercise 8.

8. $x^2 \sin x$

EXERCISES 11.10

Binomial Series

Find the first four terms of the binomial series for the functions in Exercises 1 and 9.

1. $(1 + x)^{1/2}$

9. $\left(1 + \frac{1}{x}\right)^{1/2}$

Find the binomial series for the functions in Exercise 11.

11. $(1 + x)^4$

EXERCISES 11.11

Finding Fourier Series

In Exercises 1 and 8, find the Fourier series associated with the given functions. Sketch each function.

1. $f(x) = 1 \quad 0 \leq x \leq 2\pi$

8. $f(x) = \begin{cases} 2, & 0 \leq x \leq \pi \\ -x, & \pi < x \leq 2\pi \end{cases}$