Homework assignment 1

1. The velocity of a freely falling object near Earth's surface is described by the equation

$$\frac{dv}{dt} = -g , \qquad (1.8)$$

where v is the velocity and $g = 9.8\ \mathrm{m/s^2}$ is the acceleration due to gravity. Write a program that employs the Euler method to compute the solution to (1.8); that is, calculate v as a function of t. For simplicity, assume that the initial velocity is zero—that is, the object starts from rest—and calculate the solution for times $t = 0$ to $t = 10$ s. Repeat the calculation for several different values of the time step, and compare the results with the exact solution to (1.8). It turns out that for this case the Euler method gives the exact result. Verify this with your numerical results and prove it analytically.

2. The position of an object moving horizontally with a constant velocity, v, is described by the equation

$$\frac{dx}{dt} = v . \qquad (1.9)$$

Assuming that the velocity is a constant, say $v = 40\ \mathrm{m/s}$, use the Euler method to solve (1.9) for x as a function of time. Compare your result with the exact solution.

3. It is often the case that the frictional force on an object will increase as the object moves faster. A fortunate example of this is a parachutist; the role of the parachute is to produce a frictional force due to air drag, which is larger than would normally be the case without the parachute. The physics of air drag will be discussed in more detail in the next chapter. Here we consider a very simple example in which the frictional force depends on the velocity. Assume that the velocity of an object obeys an equation of the form

$$\frac{dv}{dt} = a - b\,v , \qquad (1.10)$$

where a and b are constants. You could think of a as coming from an applied force, such as gravity, while b arises from friction. Note that the frictional force is negative (we assume that $b > 0$), so that it opposes the motion, and that it increases in magnitude as the velocity increases. Use Mathematica to solve (1.10) for v as a function of time. A convenient choice of parameters is $a = 10$ and $b = 1$. You should find that v approaches a constant value at long times; this is called the terminal velocity.