

Homework assignment 3

3.1.0. Derive

$$E_{i+1} = E_i + (1/2)mgl [\omega_i^2 + (g/l)\theta_i^2](\Delta t)^2$$

from

$$E = (1/2)m l^2 [\omega^2 + (g/l)\theta^2]$$

by applying Euler discretisation method [basically, prove Eq. (3.8) using Eq. (3.7), page 51, Giordana and Nakasishi (2nd edition).]

3.1

Investigate the stability of the Euler-Cromer method. Modify our program so that it also calculates the total energy, kinetic plus potential, of the pendulum as a function of time. Show that the energy is conserved over each complete cycle of the motion.

3.2

Repeat the previous problem using the Runge-Kutta method described in Appendix 1. Compare the accuracy of the Runge-Kutta method with that of the Euler-Cromer algorithm using the same time step.

3.7. Numerically investigate the forced (externally driven) pendulum with friction of Eq. (3.14) in page 55, Giordano 2nd edition. Show numerically the existence of the resonance, and confirm the dependence of the resonant amplitude on the driving angular frequency Ω_D . You may either use Euler-Cromer method, or use the command NDSolve in Mathematica. Basically, you will need to plot the amplitude as a function of Ω_D/Ω with fixed value of q , and see whether you get a resonance peak when $\Omega_D/\Omega = 1$.