Homework assignment 4

4.1.0. Show that

$$G\,M_S = 4\,\pi^2\ \mathrm{AU}^3/\mathrm{yr}^2$$

where G gravity constant, M_S the mass of sun. [Basically, prove Eq. (4.6), page 96, Giordano and Nakanishi (2nd edition).]

4.1.1. Given the E.o.M for the planet-Sum system (assumed perfectly circular orbit),

$$\frac{dx}{dt} = v_x \qquad \frac{dv_x}{dt} = -\frac{G\,M_S\,x}{r^3} \qquad \frac{dx}{dt} = v_x \qquad \frac{dv_x}{dt} = -\frac{G\,M_S\,x}{r^3}$$

prove Eq. (4.7), page 96, Giodano and Nakanishi (2nd edition):

$$\begin{aligned}
v_{x,i+1} &= v_{x,i} - \frac{4\,\pi^2\,x_i}{r_i^3}\,\Delta t \\
x_{i+1} &= x_i + v_{x,i+1}\,\Delta t \\
v_{y,i+1} &= v_{y,i} - \frac{4\,\pi^2\,y_i}{r_i^3}\,\Delta t \\
y_{i+1} &= y_i + v_{y,i+1}\,\Delta t\ ,
\end{aligned}$$

4.5 Extend your (Euler-Cromer) orbit program so that it calculates the energy (kinetic, potential, and total) of the planet, and also the angular momentum. Consider the following issues.

(a) Begin with a circular orbit and show that both the kinetic and potential energy are constants. The angular momentum should also be a constant.

(b) Consider an elliptical orbit. An orbit with an initial position 1 AU from the Sun and a velocity of 5 AU/yr is a convenient choice. Show that while the kinetic and potential energies now vary as the planet moves through its orbit, their sum (the total energy) is a constant. Also show that the angular momentum is a constant during the course of the orbit. Prove that

4.6 Consider a planet that begins a distance of 1 AU from the Sun. By trial and error, determine what its initial velocity must be in order for it to escape from the Sun. Compare your estimate with the exact result (which you should also calculate).

4.0.7.
Run the planetary motion program with initial conditions chosen to give orbits that are elliptical. You now can use the *Mathematica* code to "measure" (i) the sizes of the semimajor, (ii) semiminor, hence the positions of the (iii) perihelion (furthest point from the Sun), (iv) aphelion (closest distance to Sun), (vi) positions of the foci, and determine the (vii) planet's period? Write a code that calculates the above mentioned quantities (i) – (vii) for a hypothetical planet. Verify that the planet satisfies Kepler's third law by calculating the value of T^2/a^3.