**Homework assignment 6**

0. Derive the difference equation

$$y(i, n+1) = 2[1 - r^2]\, y(i, n) - y(i, n-1) + r^2 \,[y(i+1, n) + y(i-1, n)] \,, \qquad (6.3)$$

by discretising the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

1. Write a program to simulate wave motion on a string with free ends. Do this by using boundary conditions that always give the ends of the string the same displacement as the points that are one spatial unit in from the ends. Study how the waves are reflected from the ends of the string and compare the results with the behavior with fixed ends. You should find that the reflected wave packets are not inverted.

4. Study the propagation of wavepackets with other shapes. For example, a guitar string that is plucked has an initial profile like that shown in Figure 6.3. Calculate how this excitation splits and moves with time.

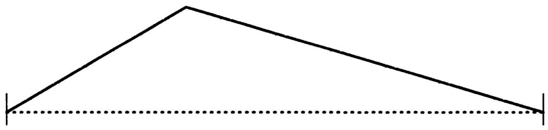

**Figure 6.3:** Realistic excitation profile for a guitar string that is plucked. The initial string displacement consists of two straight lines, which join with different slopes at the excitation (plucking) point.

'3. Set up a wavepacket that doesn't split up into two pieces (as we observed with our gaussian packet in Figure 6.1), but moves uniformly in one direction. Hint: In the simulations discussed (so far) in this section we have assumed that the string is at rest prior to $t = 0$. In order to construct a single wavepacket that does not immediately split you will have to properly specify both the initial displacement and velocity of the string.

6. An important feature of a linear equation is that the sum of two solutions is also a solution. One consequence of this is that two wavepackets will travel independently of each other. An especially clear way to demonstrate this is to set up a string with an initial profile such that there are two gaussian wavepackets, located at different places on the string. These wavepackets (or components of them) may then propagate toward each other and collide. Show that the wavepackets are unaffected by these collisions. That is, show that two such wavepackets pass through each other without changing shape or speed.