ZCE 111
Assignment 10

Q1: Stochastic integration

Write a stochasitc integraton code so that it can integtate a function with both potisive and negative signs in the range of integration. Test it on the following integral. Let the integration limits be from x_0=-2.5 to x_1=+5.0.

$$f(x) = \frac{x}{(z^2 + x^2)^{3/2}}$$

$$\int_{x_0}^{x_1} f(x)\,dx = ?$$

Application of first order differential equation (DE) in physics

The lecture notes by Prof. Tai-Ran Hsu, San José State University, California, provides many useful application of first order differential equation in real life physics. We will attempt to solve some of these problems numerically using Mathematica's NDSolve and Nsolve.

For each of the first oder DE (with boundary condition provided) listed:

- Obtain the Analytical Solution using Dsolve.
- Obtain the Numerical Solution using Ndsolve.
- Overlay both solutions you have obtained on the same graph.
- At the mean time, try read up the lecture note to understand how these equations are derived and interpreted.

Q2

Solve the following differential equation

$$\frac{du(x)}{dx} - (Sin\ x)u(x) = 0$$

with condition u(0) = 2

Ans: The analytical solution is $u(x) = 5.4366\ e^{-Cos\ x}$

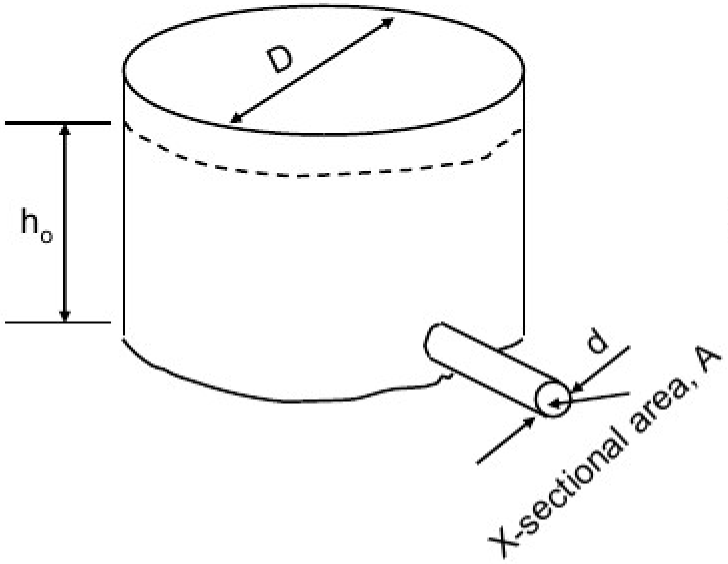
Q3

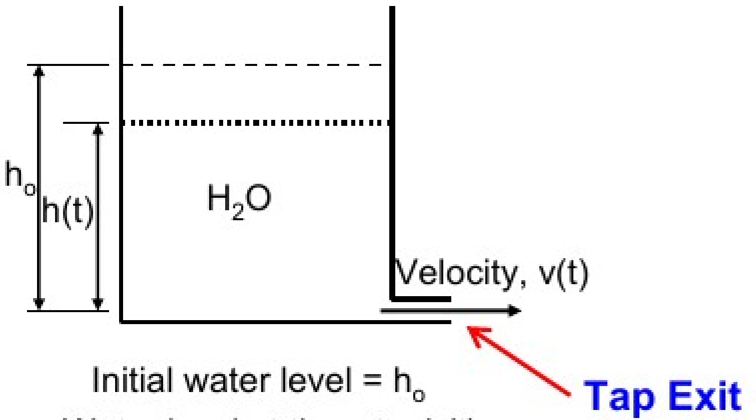
$$\frac{du(x)}{dx} + 2u(x) = 2$$

with condition u(0) = 2

Ans: The analytical solution is $u(x) = 1 + \frac{1}{e^{2x}} = 1 + e^{-2x}$

Q4 Application of 1st Order DE in Drainage of a Water Tank





Initial water level = h_o
Water level at time, t = h(t)

$$\frac{dh(t)}{dt} = -\sqrt{2g}\left(\frac{d^2}{D^2}\right)\sqrt{h(t)}$$

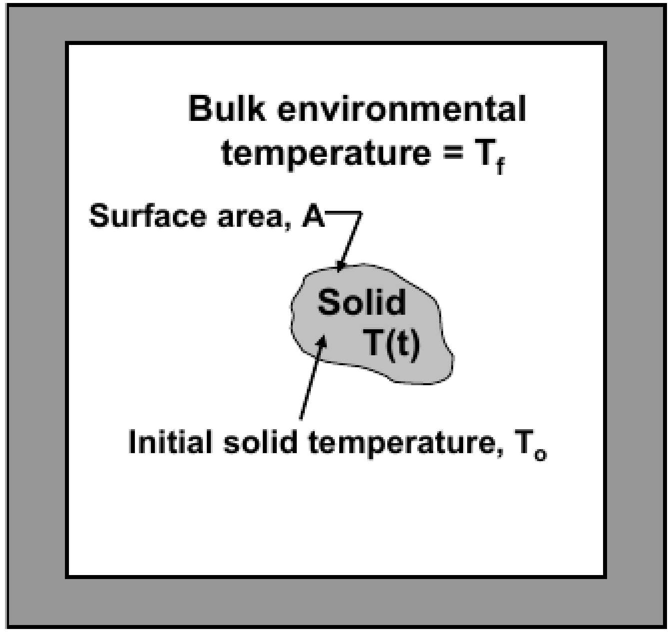
Tank diameter, D = 12" = 1 ft.
Drain pipe diameter, d = 1" = 1/12 ft.
Initial water level in the tank, ho = 12" = 1 ft.
Gravitational acceleration, g = 32.2 ft/sec.

The time required to drain the tank is the time t_e.
Find out, numerically, t_e

Ans: The analytical solution is $h(t) = \left[-\sqrt{\frac{g}{2}}\left(\frac{d^2}{D^2}\right)t + \sqrt{h_o}\right]^2$

$t_e = 35.89$ seconds

Q5 Heat Transfer in Solids Submerged in Fluids



$$\frac{dT(t)}{dt} = -\alpha A\left[T(t) - T_f\right]$$

$$T(t)\big|_{t=0} = T(o) = T_0$$

T_o = 80°C, T_f = 5°C, α = 0.002/m²-s and A = 0.2 m²

Determine the time required to cool down a solid object initially at 80°C to 8°C.

Ans: The analytical solution is $T(t) = 5 + 75e^{-0.0004t}$

t_e = 8047 s