ZCE 111
Assignment 12

Q1: SK2 code for forced pendulum

Develop a code to implement SK2 for the case of a forced pendulum experiencing no drag force but a driving force $F_D \sin(\Omega_D t)$, $\Omega = \sqrt{g/l}$, $l = 1.0$ m, m=1kg; $F_D =$ 1N; Ω_D=0.99 Ω;

Boundary conditions: $\theta(0) = 0.0; \omega(t = 0) = 0$;

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt} + \frac{F_D \sin(\Omega_D t)}{ml}$$

Q2: Stability of the total energy a SHO in RK2.

$\omega = \frac{d\theta}{dt}$, angular velocity. *m*=1kg; *l*=1m.

The total energy of the SHO in can be calculated as

$$E_{i+1} = K_{i+1} + U_{i+1} = \frac{1}{2} m \left(l \omega_{i+1} \right)^2 + mgl \left(1 - \cos \theta_{i+1} \right)$$

$$\approx \frac{1}{2} m l^2 \omega_{i+1}{}^2 + mgl \left[1 - \left(1 - \frac{\theta_{i+1}^2}{2} \right) \right]$$

$$= \frac{1}{2} m l^2 \omega_{i+1}{}^2 + \frac{1}{2} mgl \theta_{i+1}^2$$

User your RK2 code to track the total energy for *t* running from *t=0* till t=25*T*; *T*=$\sqrt{g/l}$. Boundary conditions: $\omega(0) = \sqrt{\frac{g}{l}}; \theta(0) = 0$. Check that indeed E_i should remain constant throughout all t_i.

Q3: Planetary motion in polar coordinates

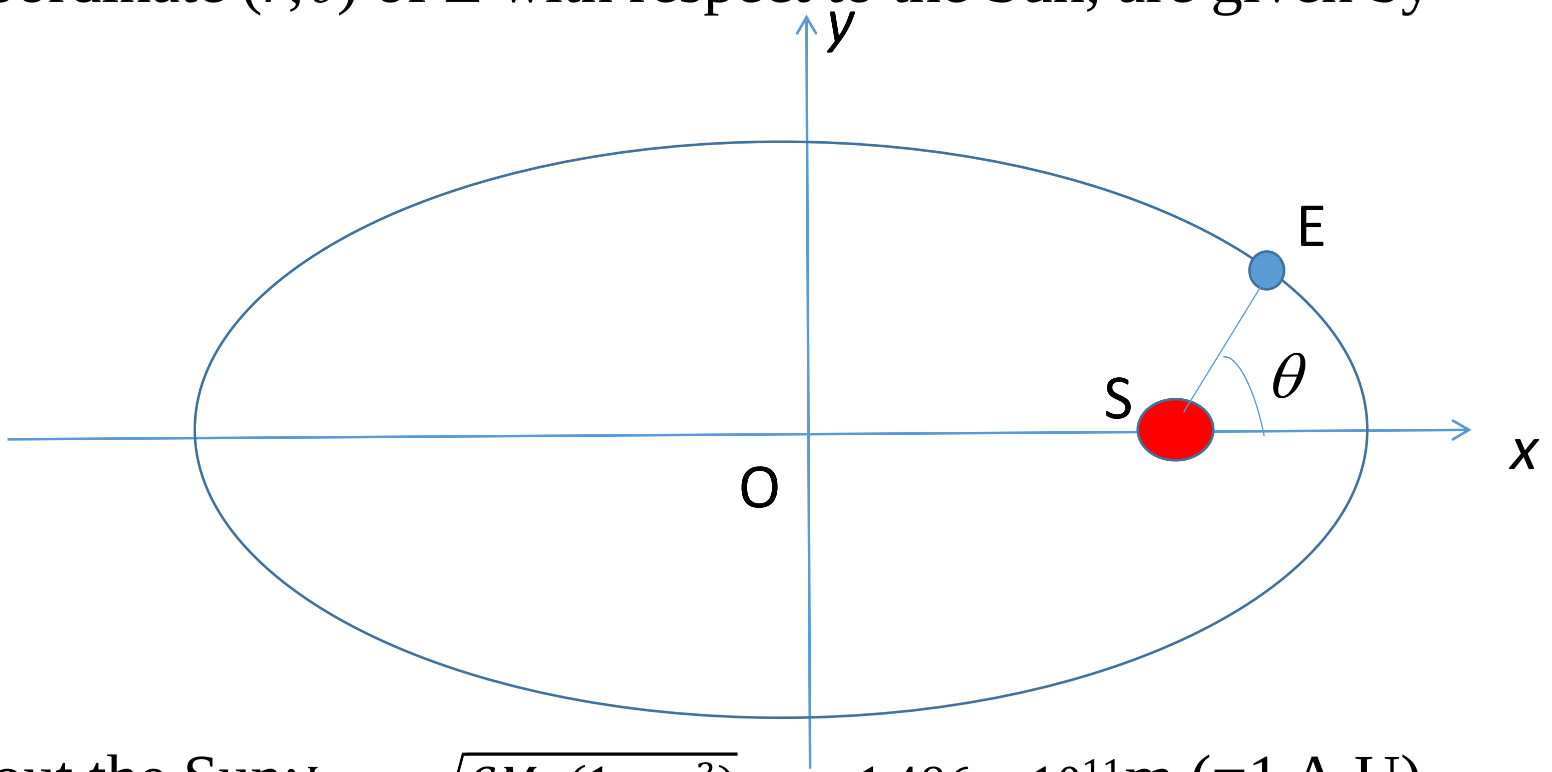
The Earth (E) moves in an elliptical orbit around the Sun (S). In polar coordinate, the equation of motions for the radial and angular coordinate (r,θ) of E with respect to the Sun, are given by

$$m\ddot{r} - mr\left(\frac{L}{mr^2}\right)^2 = -\frac{GMm}{r^2}$$

$$mr^2\dot{\theta} = L$$

where

$$\ddot{r} = \frac{d^2r}{dt^2};\ \dot{\theta} = \frac{d\theta}{dt}$$



L is the angular moment of Earth about the Sun: $L = m\sqrt{GMa(1-\epsilon^2)}$; $a = 1.496 \times 10^{11}$m (=1 A.U) (semimajor); m= 5.97×10^{24}kg, mass of Earth; $M = 1.987 \times 10^{30}$kg, Mass of the Sun; $G = 6.673 \times 10^{-11}$Nm^2/kg^2; $\epsilon = 0.017$ (eccentricity). The period is $T = 2\pi\sqrt{\frac{a^3}{GM}}$

Q3: Planetary motion in polar coordinates (cont.)

(i) Solve the radial equation, $m\ddot{r} - mr\left(\frac{L}{mr^2}\right)^2 = -\frac{GMm}{r^2}$, numerically using your RK2 code, so that you can plot the graph of *r* as a function of time for *t* from 0 to the period of the orbit, *T* (= 1 year). Boundary conditions: $r(0) = \frac{L^2}{GMm^2}\frac{1}{(1+\epsilon)}, \dot{r}(0) = 0$.

(ii) Hence, plot the angular velocity function $\dot{\theta}(t)$ as the time varies throughout the year.

(iii) Plot the normalised angular function $\frac{\theta(t)}{2\pi}$ as the time varies throughout the whole period.
(*Think carefully how you are going to do this using the prior knowledge you have acquired. Think Euler.)

(iv) Visualise the evolution of the Earth's location, $(x(t),y(t))$ on the orbit on a *x*-*y* plot using the Manipulate function. Your visualization should contain the origin, *x*- and *y*-axes.

Q4: Planetary motion in polar coordinates, for Halle's comet

Repeat Q2 for Halle's comet. The parameters of Halle's comet are:

$a = 17.8$ A.U; m= 2.2×10^{14}kg; $\epsilon = 0.967$.

Check that the period is indeed $\approx$ 75 years.

Q5: Coulomb charge scattering in 3D

Develop a code based on velocity Verlet algorithm (see the sample code verlet_algorithm_2D_coulomb_scatterings.nb as a reference) to simulate the scattering process of a fixed charge $Q = +1$ unit, located at $(r_{20}\sin\theta_2\sin\phi_2,\ r_{20}\cos\phi_2,\ r_{20}\sin\theta_2\cos\phi_2$, by a moving charge $q = +1$ unit (with mass $m = 1$ unit), initially located at (0,0,0), and is shooting towards Q with an initial speed $v_0 = 1$ unit.

In your simulation, set $k = \frac{1}{4\pi\epsilon_0} = 1$, $r_{20} = 1$ unit, $\theta_2 = \phi_2 = 45^\circ$.

Q6: Charge moving in a magnetic field

A charge (mass m and charge q) moving with velocity $\mathbf{v} = (v_x, v_y, v_z)$ in a magnetic field $\mathbf{B} = (B_x, B_y, B_z)$ experiences a velocity-dependent Lorentz force $\mathbf{F} = (F_x, F_y, F_z) = q\, \mathbf{v} \times \mathbf{B}$. Develop a code based on the Störmer-Verlet integration algorithm to simulate the dynamical path of the charge particle moving through the magnetic field. Assume: $q = +1$ unit, mass $m = 1$ unit, initially located at (0,0,0), initial velocity (v_{0x}, v_{0y}, v_{0z}), $v_{0x} = v_{0y} = 0.1$ unit, $v_{0z} = 0.05$ unit, $\mathbf{B} = (0, 0, B_z)$, $B_z = 0.1$ unit. You should see a helical trajectory circulating about the z-direction.

(my code: verlet_algorithm_3D_coulomb_helix.nb)