ZCE 111
Assignment 9

Q1 Trapezoid rule for numerical integration

Write a code to evalate the following integral using both Trapezoid rule. z is a constant set to 1. Let the integration limits be from x_0=-1.5 to x_1=+5.0.

$$f(x) = \frac{x}{(z^2 + x^2)^{3/2}}$$

$$\int_{x_0}^{x_1} f(x)\,dx = ?$$

Q2 Simpson's rule for numerical integration

Write a code to evalate the following integral using both Simpson's rule. z is a constant set to 1. Let the integration limits be from x_0=-1.5 to x_1=+5.0.

$$f(x) = \frac{x}{\left(z^2 + x^2\right)^{3/2}}$$

$$\int_{x_0}^{x_1} f(x)\,dx = ?$$

Q3 Generate gamma function using Simpson's rule

Gamma function is formally defined as

$$\Gamma(z)=\int_0^\infty f(t)\,dt\,; f(t)=t^{z-1}e^{-t}; \Re(z)>0.$$

http://functions.wolfram.com/GammaBetaErf/Gamma/02/

(*i*) Use Mathematica command **Gamma[z]** to plot the gamma function for the interval 1 < *z* < 5.

(*ii*) For a few selected values of *z* in the intervals [1,5], plot *f*(*t*) for *t* from close to zero up to a value at which *f*(*t*) appears to be flatten and negligible.

(*iii*) Hence, design a Simpson integration code that could approximate the Gamma function at a fixed *z*.

(iv) Then, use your Simpson code in (*iii*) to List Plot Gamma[*z*] for *z* in the interval [1,5], for *z* = 1.0,1.1,1.2,1.3,...,5.0.

(v) Overlap the ListPlot of (iv) on the graph plotted in (*i*). Both code must agree.

Q4 Generate logarithmic integral function using Simpson's rule

Logarithmic integral function is formally defined as

$$li(x) = \int_0^x f(t)\,dt\;; f(t) = \frac{1}{\ln t}.$$

http://functions.wolfram.com/GammaBetaErf/LogIntegral/02/

(*i*) Use Mathematica command **LogIntegral[x]** to plot the function for the interval $0 < x < 1$ (note: the end points are not included).

(*ii*) Design a Simpson integration code that could approximate the logarithmic integral function at a fixed *x*.

(*iii*) Then, use your Simpson code in (*ii*) to List Plot LogIntegral[*x*] for *x* in the interval (0,1) for some choice of Δx.

(*v*) Overlap the ListPlot of (*iii*) on the graph plotted in (*i*). Both code must agree.

Q4 Generate Bessel functions numerically

Bessel function of the first kind, (see http://en.wikipedia.org/wiki/Bessel_function), for positive values of real α ,is defined as

$$J_{\alpha}(z) = \frac{1}{\pi}\int_0^{\pi}\cos(\alpha\tau - z\sin\tau)\,d\tau - \frac{\sin(\alpha\pi)}{\pi}\int_0^{\infty} e^{-x\sinh(t)-\alpha t}\,dt$$

(*i*) Use Mathematica command **BesselJ[n, z]** to plot the function for the interval 0 <= *z* <= 10.

(*ii*) Design a Mathematica code using **Nintegrate** to generate Bessel functions $J_{\alpha}(z)$ for α = 0,1.5,2,2.5. Plot

$\{J_0(z), J_{1.5}(z), J_2(z), J_{2.5}(z)\}$ on the same graphs for *z* running from 0 to 10.

(*iii*) You should also compare your Bessel curves against those generated using the command **BesselJ[n, z]**.

Q5 Associated Legendre polynomials

Associated Legendre polynomials, (see http://en.wikipedia.org/wiki/Associated_Legendre_polynomials), for integer values such that $0 <= |m| <= l$.

$$P_l^m(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{m/2}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l$$

(*i*) Use Mathematica command **LegendreP[n,m,x]** to plot the function for the interval -10 <= *x* <= 10 for (*m*,*l*)=(-1,1),(0,1),(1,1).

(ii) Generate the functions $P_l^m(x)$ for arbitrary values of (*m*,*l*) (but obey the constrain $0 <= |m| <= l$) using Mathematica's command **D[f[x],x]**.

(*iii*) Plot the functions as generated in (*ii*) with same set of (*m*,*l*) and range of *x* as in (*i*).

Q5 Associated Legendre polynomials (cont.)

(*iv*) Use Mathematica command **Nintegrate** to show that the three Legredre polynomials generated in (*i*) are orthornormal, i.e.,

$$\int_{-1}^{1} \frac{P_\ell^m P_\ell^n}{1-x^2} dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{(\ell+m)!}{m(\ell-m)!} & \text{if } m = n \neq 0 \\ \infty & \text{if } m = n = 0 \end{cases}$$