2D projectile revisited

$x(t) = x_0 + v_0 t \cos\theta; y(t) = y_0 + v_0 t \sin\theta + \frac{g}{2} t^2.$

T :time of flight; T =-2$(y_0 + v_0 \sin\theta)/g.$

$g = -9.81;$

2D projectile using list

Revisit 2D projectile motion: Using **ListPlot** instead.

Generate a list containing the coordinates of a 2D projectile for a time from 0 to T=-$2(y_0 + v_0 \sin\theta)/g$ at an time interval of Detalt=0.01T

See sample code 1 and 2 in listprojectile.nb.

list=Table[{x[t,theta],y[t,theta]},{t,0,T,Deltat}] comprises of a list of the coordinates of the projectile at discrete values of t.

Abstract information of this list: see sample code 3 in listprojectile.nb

Syntax: Length[list]; list[[2]]; list[[2,1]]; list[[2,2]; list[[-1]]

Manipulating list (case study)

See listprojectile.nb as a case study on how we manipulate the list of a 2D projectile to find the maximum values of y and the corresponding value of x at which ymax occurs.

Syntax: **Max[];Min[];Sort;Ordering;**

Exercise: Use your code to tell you when does ymax occurs.

Measuring speed in a simulation: case study 1

As a case study, recycle your code for simulating a single pendulum.

By creating a list for the locations of the pendulum at different time step,

(*i*) "measure" the speed of the pendulum in each time step.

(*ii*) Plot the speed of the pendulum as a function of time.

(*ii*) Plot the square of speed of the pendulum as a function of displacement from the equilibrium position. You should obtain a quadratic relation between these two dynamical variables.

See measure_pendulum_speed.nb

Measuring speed in a simulation: case study 2

Recycle your code for simulating an ellipse. Imagine a planet is orbiting along the ellipse.

By creating a list for the locus of the planet in the ellipse at different time step,

(*i*) "measure" the speed of the planet moving in the ellipse in each time step.

(*ii*) Plot the speed of the planet as a function of time. Does the planet move at the same or different speed in the ellipse? Which are the locations in the orbit where the speed is largest? Smallest?

Directory hopping, **Export**, **Import**

Try to generate the coordinates of a semicircle (in the upper half of the x-y plane) of radius *R* that is centered at (0,0)

y[t_]:=R*Cos[t]; y[t_]:=R*Sin[t];

datasemicircle=Table[{x[t],y[t]},{t,0,Pi,0.05Pi}];

Export[“datasemicircle.dat”,datasemicircle];

Import[“datasemicircle.dat”];

See datasemicircle.nb.

Syntax: **SetDirectory[];NotebookDirectory[];Import[];Export[];AspectRatio->1; FileExistsQ[]**

Simple curve fitting

Download the data "datasemicircle.dat" online. It is supposed to have been generated by your friend who decline to disclose what value of *R* she used when generating the data, except notifying that the center of the circle was located at (0,0).

Now, can you write a code to decipher what value of *R* she uses to generate the data?

To this end, you need to quantify the error of the trial values of R with respect to the "true" R value of datasemicircle.dat.

See decipher_R_semicircle.nb

Merit function”

A circle centered at (*h*,*k*) with radius *R* is described by the equation

$$(x-h)^2+(y-k)^2=R^2$$

Say you are given a set of coordinates of a circle with known center (*h*,*k*) but unknown radius, $\{(x_1, y_1), (x_2, y_2), (x_3, y_3),\ldots, (x_i, y_i), \ldots,(x_N, x_N)\}$.

How do we find out what value of R is exactly, using numerical method?

Consider an equation based on the circle equation, defined as

$$\Delta^2=(R^2-[(x-h)^2+(y-k)^2])^2$$

Since we do not know what the true value of *R* is, let’s make a guess, say, *r*=1. Slotting the guessed value *r* the values of any pair of (*x*,*y*) values from the data set into the equation Δ^2, we have

$$\Delta^2(r,x_i,y_i)=\left(r^2-[(x_i-h)^2+(y_i-k)^2]\right)^2$$

The function $\Delta^2(r,x_i,y_i)$ dictates the discrepancy between the guessed value *r* and the true value of *R* as contributed by the data point (x_i, y_i).

With the guessed value *r*, $\Delta^2(r,x_i,y_i)$ generally does not equal zero for all *i.*

"Merit function", cont.

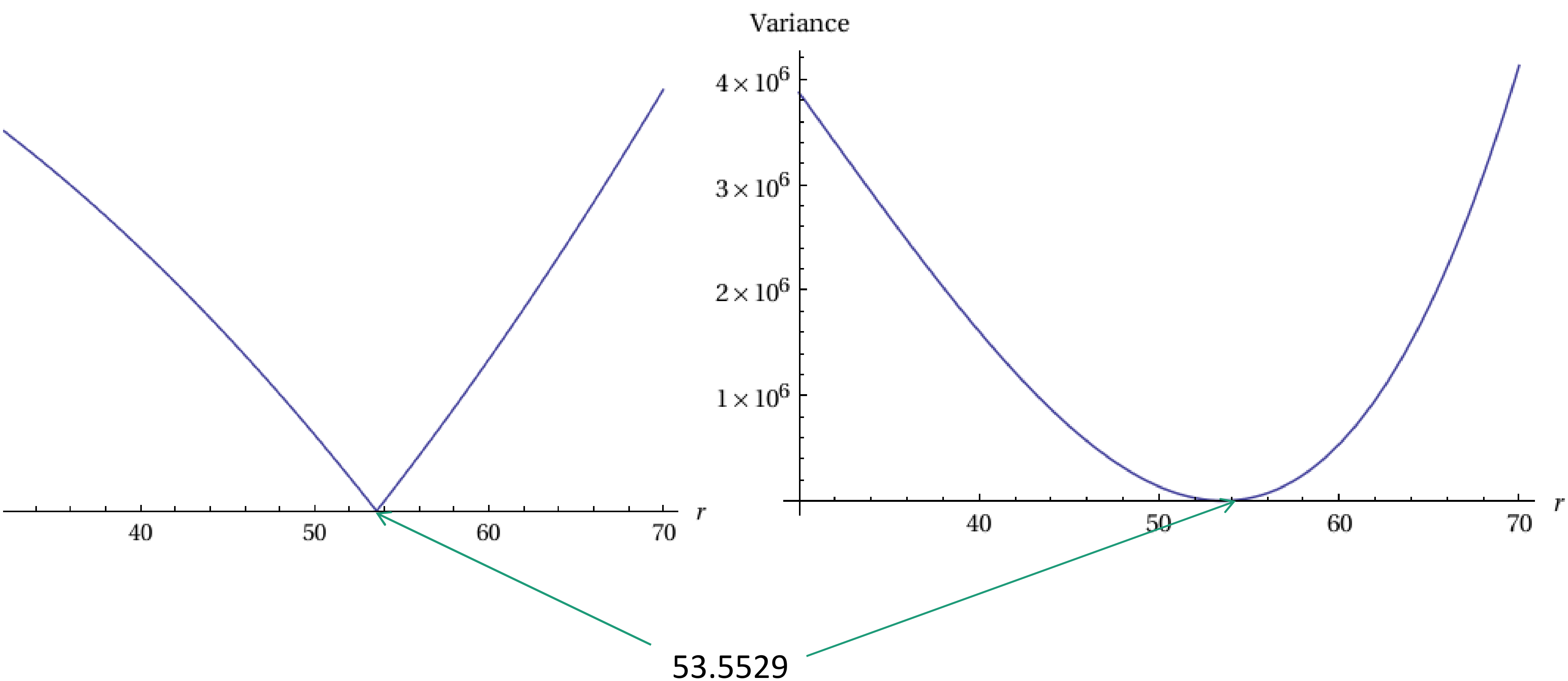
Since every data point $\{x_i, y_i\}$ contribute differently to $\Delta^2(r, x_i, y_i)$, we should sum up these contribution. To these end, we define the variance and standard deviation,

$$\sigma^2(r) = \frac{\sum_{i=1}^{i=N} \Delta^2(r, x_i, y_i)}{N} = \frac{\sum_{i=1}^{i=N} (r^2 - [(x_i - h)^2 + (y_i - k)^2])^2}{N} \quad \text{(variance)}$$

$$\sigma(r) = \sqrt{\sigma^2(r)}, \quad \text{(standard deviation)}$$

If the guessed value *r* happens to hit upon the "true" value *R*, then both the variance or standard deviation will become zero. Standard error and variance are both examples of "merit functions", functions that when minimized will tell what the true value of *R* is.

Standard deviation and variance of a semicircle data with R= 53.5529



Minimise you merit function

Merit function is system-dependent.

To find out the values for the unknown parameter describing a set of data points, you must form a suitable merit function.

Upon minimizing the merit function numerically, these unknown values could be obtained.

Exercise: Decipher the radius of a full circle

Download the data "datacircle.dat" online. It is supposed to have been generated by your friend who decline to disclose what value of *R* she used when generating the data, except notifying that the cente of the circle was located at (0,0).

Now, modify the code decipher_R_semicircle.nb to decipher what value of *R* she uses to generate the data.

Note: Be aware!! You got to redesign the function used to quantify the error between the trial value y and the true y.

Solution: see decipher_R_circle.nb.

Ξxample of a two-variables curve fitting

In previous exercises, you was fitting a data set with an equation with only a single unknown.

Say if you were given a data set of a circle with a known radius *R* but unknown center (*h*,*k*), can you still able to write a code to figure out what values of (*h*,*k*) are?

Download the data file, data_circle_unknown_center.dat. It contains data of a 2-D circle with radius *R*=0.75. The center of the circle (*h*,*k*) is not known. It was generated using the formula:

$$(x-h)^2+(y-k)^2=R^2$$

)r equivalently, in parametrized form,

$$x=Rcos\,\theta+h, y=Rsin\,\theta+k, \theta\in[0,2\pi].$$

Vrite a code to find out the (*h*,*k*). See decipher_R_circle_2D.nb

The merit function for two unknown parameters, {*h*, *k*}.

$$(x-h)^2+(y-k)^2=R^2$$

$$\Delta^2(h,k,x_i,y_i)=\left(R^2-\left[(x_i-h)^2+(y_i-k)^2\right]\right)^2$$

$$\sigma^2(h,k)=\frac{1}{N}\sum_i^N \Delta^2(h,k,x_i,y_i)$$

$$\sigma_s(h,k)=\sqrt{\sigma^2(h,k)}$$

In this case, you have to scan the two-dimensional parameter space spanned by *h* and *k* to search for which values of {*h*,*k*} are such that σ^2 (or σ_s) is minimised.

Minimised standard deviation in (h,k) parameter space