Related topics

Oscillation period, harmonic oscillation, mathematical pendulum, physical pendulum, decomposition of force, moment of inertia.

Principle and task

Investigate the oscillation behaviour of a pendulum (rod pendulum) by varying the magnitude of the components of the acceleration of gravity which are decisive for the oscillation period. The pendulum that is to be used is constructed in such a manner that its oscillation plane can be progressively rotated from a vertical orientation to a horizontal one. The angle Φ, by which the oscillation plane deviates from its normal vertical position, can be read from a scale.

Equipment

Variable g pendulum	02817.00	1
Holder for light barrier	02817.10	1
Light barrier, compact	11207.20	1
Timer 4-4	13605.99	1
Tripod base -PASS-	02002.55	1
Connecting cord, 500 mm, red	07361.01	1
Connecting cord, 500 mm, yellow	07361.02	1
Connecting cord, 500 mm, blue	07361.04	1

Fig. 1: Experimental set-up: Variable "g" pendulum.

Problems

1. Measurement of the oscillation period of the pendulum as a function of the angle of inclination Φ of the oscillation plane for two different pendulum lengths.

2. Graphical analysis of the measured correlations and a comparison with the theoretical curves, which have been standardised with the measured value at $\Phi = 0$.

3. Calculation of the effective pendulum length l for the acceleration of gravity, which is assumed to be known. Comparison of this value with the distance between the pivot point of the pendulum and the centre of gravity of the mobile pendulum weight.

4. On the moon's surface the "lunar acceleration of gravity" g_m is only 16.6 % of the earth's acceleration of gravity g. Calculate the angle Φ and set it on the device such that the pendulum in the laboratory oscillates with the same oscillation period with which it would oscillate on the moon in a perpendicular position. Compare the measured oscillation period with the calculated one.

Set-up and procedure

Clamp the pendulum into the tripod support stand. Move it down as far as possible on its support rod to achieve the greatest possible set-up stability. The pendulum rod should extend approximately 2 cm through the bottom of the weight. Measure the geometric pendulum length L (distance between the pivot point and the centre of the weight) and record it.

The pendulum must be carefully adjusted with the aid of the levelling rings on the tripod support stand in order to ensure that the value for Φ read from the angular scale coincides with the slope of the oscillation plane to the vertical. The following procedure has proved effective:

– One leg of the tripod support stand should be oriented in the direction of the pendulum's resting position, and the foot located there should be adjusted to its shortest possible length with the corresponding levelling ring. The two other feet should be located approximately in the middle of their adjusting range.

– Set the pendulum rod horizontally ($\Phi = 90°$)

– With the aid of the two levelling rings on the tripod base which point away from the pendulum, adjust the resting position of the pendulum so that it lies in the middle of its oscillation range.

– Slowly extend the other feet by unscrewing them so that the pendulum oscillates ever more slowly. If the location of the resting position shifts, this change should be compensated for by adjusting the other two feet.

– The procedure is completed as soon as the pendulum remains at rest in nearly any position.

After making these adjustments, the support stand on the table should not be moved.

Fig. 1 shows the complete experimental set-up for the measurement of T (for an intermediate value of the angle Φ) The light barrier is connected to input port #1 of the time measuring device and is also supplied with power by the device. Select the "Oscillation period" operation mode on the time

measuring device. The timer begins counting the first time the light beam of the barrier is interrupted; the second interruption has no effect; the third interruption ends the time measurement. In this manner the complete oscillation period T is directly measured.

Alter the angle Φ successively, beginning at 85° until the pendulum is again in a vertical position, and measure the oscillation period T for small oscillation amplitude in each case. Always make several measurements in order to increase the exactness by forming means if necessary (in particular for large values of Φ) . Subsequently, perform a second measuring series with approximately half of the pendulum length L.

Theory and evaluation

As a good approximation, the pendulum used here can be treated as a mathematical (simple) pendulum having a length l. However, depending on the position of the pendulum weight, the length l deviates more or less from the geometric pendulum length L, which is measured between the pivot point and the centre of the moveable weight (compare Problem 3). A retracting force acts on the pendulum mass m at a deflection equal to the angle φ

$$F (\varphi) = - mg \cdot \sin\varphi \approx - mg\varphi \quad \text{for small angle } \varphi$$

If one ensures that the amplitudes remain sufficiently small while experimenting, the movement can be described by the following differential equation:

$$l \cdot \frac{d^2\varphi}{dt^2} = - g\varphi$$

The following is obtained as the solution:

$$\varphi = \varphi_0 \cdot \sin\left(\sqrt{\frac{g}{l}} \cdot t\right)$$

This is a harmonic oscillation having the amplitude φ_0 and the oscillation period T.

$$T = 2\pi \sqrt{\frac{l}{g}}.$$

If one rotates the oscillation plane around the angle Φ with respect to the vertical plane, the components of the acceleration of gravity $g(\Phi)$ which are effective in its oscillation plane are reduced to $g(\Phi) = g \cdot \cos\Phi$ and the following is obtained for the oscillation period:

$$T(\Phi) = 2\pi \sqrt{\frac{l}{g\cdot\cos\Phi}} \qquad (1)$$

A sample measurement for the geometric pendulum lengths L = 270 mm and L = 141 mm is shown in Fig. 2. The theoretical curves, which are standardised to the measured value for Φ = 0, are drawn as solid lines for comparison. In accordance with formula (1) , the increase in the oscillation period, which is proportional to the square root of $1/\cos\Phi$, is confirmed by Fig. 2 with a high degree of accuracy.

In our sample measurement the values for the oscillation period given in the table were obtained for the two measured pendulum lengths L for a vertical orientation (Φ = 0). Using these

values, the effective ("reduced") pendulum lengths were calculated under the assumption that in Göttingen g = 9.806 ms^{-2}.

$$l = \frac{gT^2}{4\pi^2}$$

L/mm	T/s	l/mm
270	1.033	265
141	0.787	154

One observes that for a "normal" pendulum form (movable weight near the lower end of the pendulum rod) the geometric pendulum length L deviates only slightly from the length l of a mathematical pendulum having the same oscillation period. As expected, the effective pendulum length l is slightly less (approximately 2 %) than L due to the mass of the pendulum rod, which cannot be neglected. The relationships are reversed when the weight is approximately in the middle of the pendulum rod as in the second case. Due to its large contribution to the pendulum's moment of inertia, the part of the pendulum rod which extends beyond the pendulum weight causes the effective pendulum length to be clearly larger (approximately 8 %) than the geometric length.
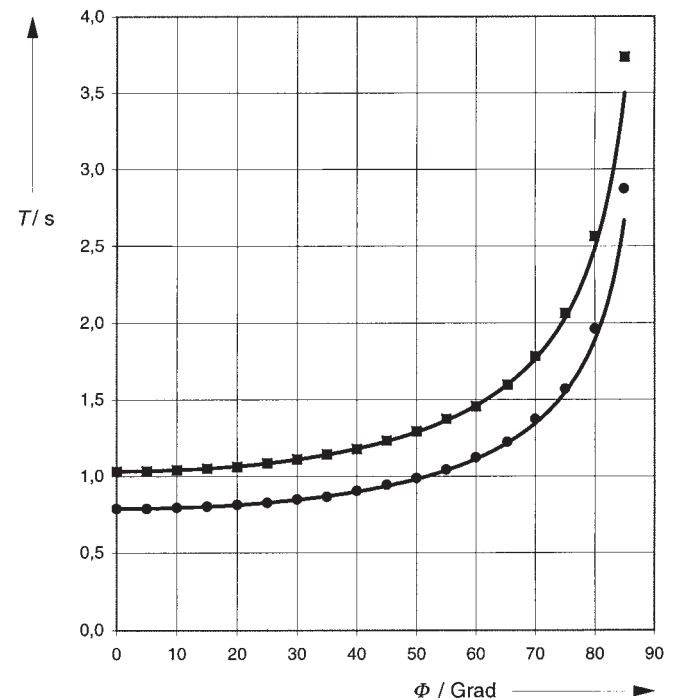
Fig. 2. Oscillation period of the pendulum as a function of the slope Φ. of the oscillation plane. The measured points are plotted above the corresponding theoretical curve (solid line). Upper curve: L = 270 mm; lower curve: L = 141 mm.