

ZCE 111
Assignment 10

Application of first order differential equation (DE) in physics

The lecture notes by Prof. Tai-Ran Hsu, San José State University, California, provides many useful application of first order differential equation in real life physics (Ch. 3 - First Order DEs, Ch. 4 - Second Order DEs). We will attempt to solve some of these problems numerically using Mathematica's NDSolve and Nsolve.

For each of the first order DE (with boundary condition provided) listed in Q2 and Q3:

Obtain the Analytical Solution using Dsolve.

Obtain the Numerical Solution using Ndsolve.

Overlay both solutions you have obtained on the same graph.

At the mean time, try read up the lecture note to understand how these equations are derived and interpreted.

Q2

Solve the following differential equation

$$\frac{du(x)}{dx} - (\sin x)u(x) = 0 \quad \text{with condition } u(0) = 2$$

Ans: The analytical solution is $u(x) = 5.4366 e^{-\cos x}$

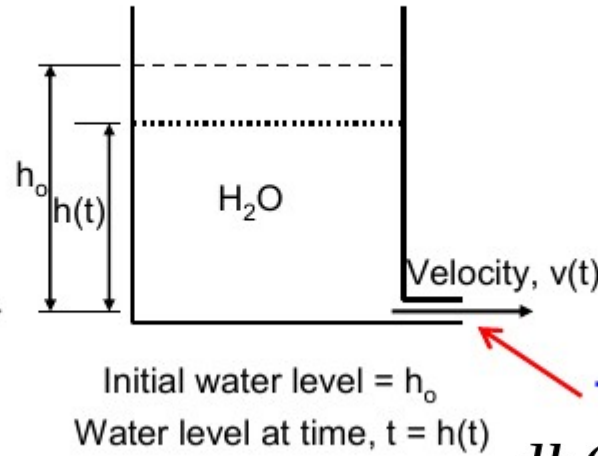
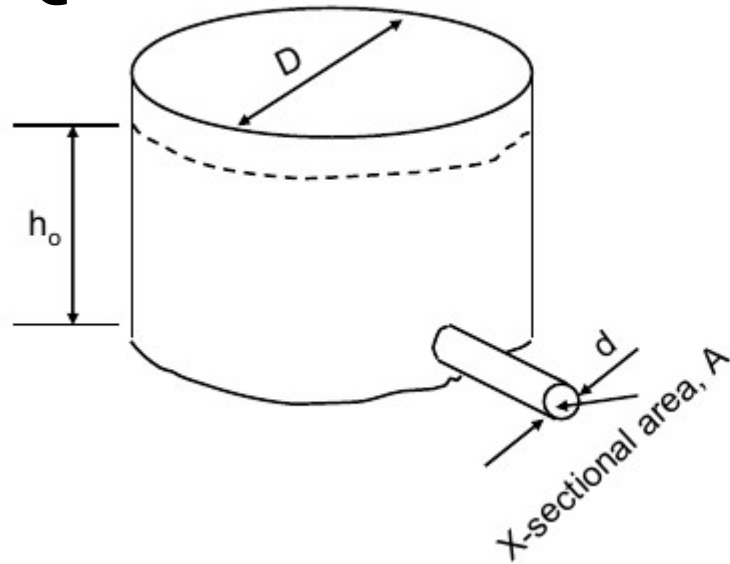
Q3

$$\frac{du(x)}{dx} + 2u(x) = 2 \quad \text{with condition } u(0) = 2$$

Ans: The analytical solution is $u(x) = 1 + \frac{1}{e^{2x}} = 1 + e^{-2x}$

Q4

Application of 1st Order DE in Drainage of a Water Tank



$$\frac{dh(t)}{dt} = -\sqrt{2g} \left(\frac{d^2}{D^2} \right) \sqrt{h(t)}$$

- Tank diameter, $D = 12'' = 1 \text{ ft.}$
- Drain pipe diameter, $d = 1'' = 1/12 \text{ ft.}$
- Initial water level in the tank, $h_0 = 12'' = 1 \text{ ft.}$
- Gravitational acceleration, $g = 32.2 \text{ ft/sec.}$

The time required to drain the tank is the time t_e .
Find out, numerically, t_e

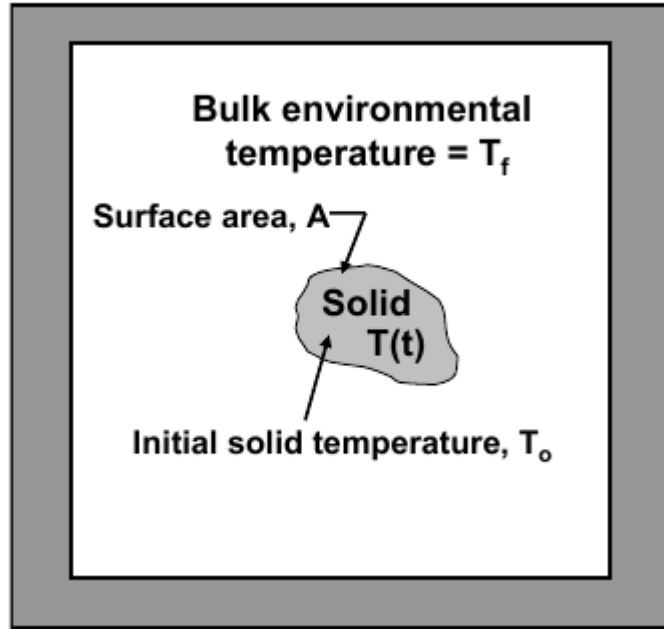
Ans: The analytical solution is

$$t_e = 35.89 \quad \text{seconds}$$

$$h(t) = \left[-\sqrt{\frac{g}{2}} \left(\frac{d^2}{D^2} \right) t + \sqrt{h_0} \right]^2$$

Q5

Heat Transfer in Solids Submerged in Fluids



$$\frac{dT(t)}{dt} = -\alpha A [T(t) - T_f]$$

$$T(t)|_{t=0} = T(0) = T_0$$

$$T_0 = 80^\circ\text{C}, T_f = 5^\circ\text{C}, \alpha = 0.002/\text{m}^2\text{-s} \text{ and } A = 0.2 \text{ m}^2$$

Determine the time required to cool down a solid object initially at 80°C to 8°C .

Ans: The analytical solution is $T(t) = 5 + 75e^{-0.0004t}$

$$t_e = 8047 \text{ s}$$