

ZCE 111  
Assignment 11

# Q1: Euler's method for 1<sup>st</sup> order DE

## Free fall with drag force

A freely falling object through a fluid medium can alternatively be modeled such that the drag force is proportional to the square of its speed. The first order differential equation for such an object is given by

$$\frac{dv}{dt} = -g + kv^2$$

Let  $k=0.01$ , and the boundary condition is  $v(0) = -20$  m/s.

Develop a code that implements Euler's method to numerically solve the equation for  $t_{\text{final}}=20.0$ s.

Use **Dsolve[]** to obtain the analytical solution for  $v(t)$ .

Overlap your numerical solution on top of the analytically obtained plot. Both should agree to each other.

## Q2: Forced pendulum

Use **Dsolve[]** to solve the forced oscillator. Plot on the same graph the analytical solutions of  $\theta(t)$  for  $t$  from 0 to 10 T, where  $T = 2\pi / \Omega$ ,  $\Omega = \sqrt{g/l}$ , for  $\Omega_D = 0.01\Omega, 0.5\Omega, 1.0\Omega, 1.5\Omega, 4\Omega$ .

Assume the boundary conditions  $\theta(t=0)=0$ ;  $d\theta/dt(t=0)=0$ ;  $m=l=F_D=1$ ;  $q=0$ .

$$\frac{d^2 \theta}{dt^2} = -\frac{g}{l} \theta - q \frac{d\theta}{dt} + \frac{F_D \sin(\Omega_D t)}{m l}$$

## Q3: Forced pendulum, again

Use **Dsolve[]** to solve the forced oscillator. Plot on the same graph the analytical solutions of  $\theta(t)$  for  $t$  from 0 to  $10 T$ , where  $T = 2\pi/\Omega$ ,  $\Omega = \sqrt{g/l}$ , for  $\Omega_D = 1.0\Omega$ ,  $q = 0.01, 0.1, 1.0$ . Assume the boundary conditions  $\theta(t=0) = 0$ ;  $d\theta/dt(t=0) = 0$ ;  $m = l = F_D = 1$ ;

$$\frac{d^2 \theta}{dt^2} = -\frac{g}{l} \theta - q \frac{d\theta}{dt} + \frac{F_D \sin(\Omega_D t)}{m l}$$