

Assignment 3

Given the following arbitrary function $f(x)$ below,

(I) $f(x) = \tan^{-1}(x), a = 1.$

(II) $f(x) = \sinh^{-1}(x), a = 0.$

(III) $f(x) = 1/\sqrt{1-x^2}, a = 0.$

(a). Obtain the analytical expression of the i -th coefficient a_i in the Taylor series for $f(x)$ expanded at the center $x=a$ using Mathematica command **D[]**. Display the first 10 coefficients using **Table[]**.

(b) Check the correctness of your answer by comparing the coefficients in (a) against those obtained via the command **Series[]**.

(c) Form the explicit expression of the Taylor series representation for $f(x)$ at $x=a$ up to the n -th order, $P_n(x)$ using the command **Sum[]**. Note that n in theory should be chosen to be infinity, but in numerical practice it just needs to be set to a large positive number. You should choose an appropriate value for n .

(d) Plot $P_n(x)$ along with $f(x)$ on the same graph. The range of x should include $x=a$. To this end you need to fine-tune the range of x for which the Taylor series plots are convergent.