Assignment 4

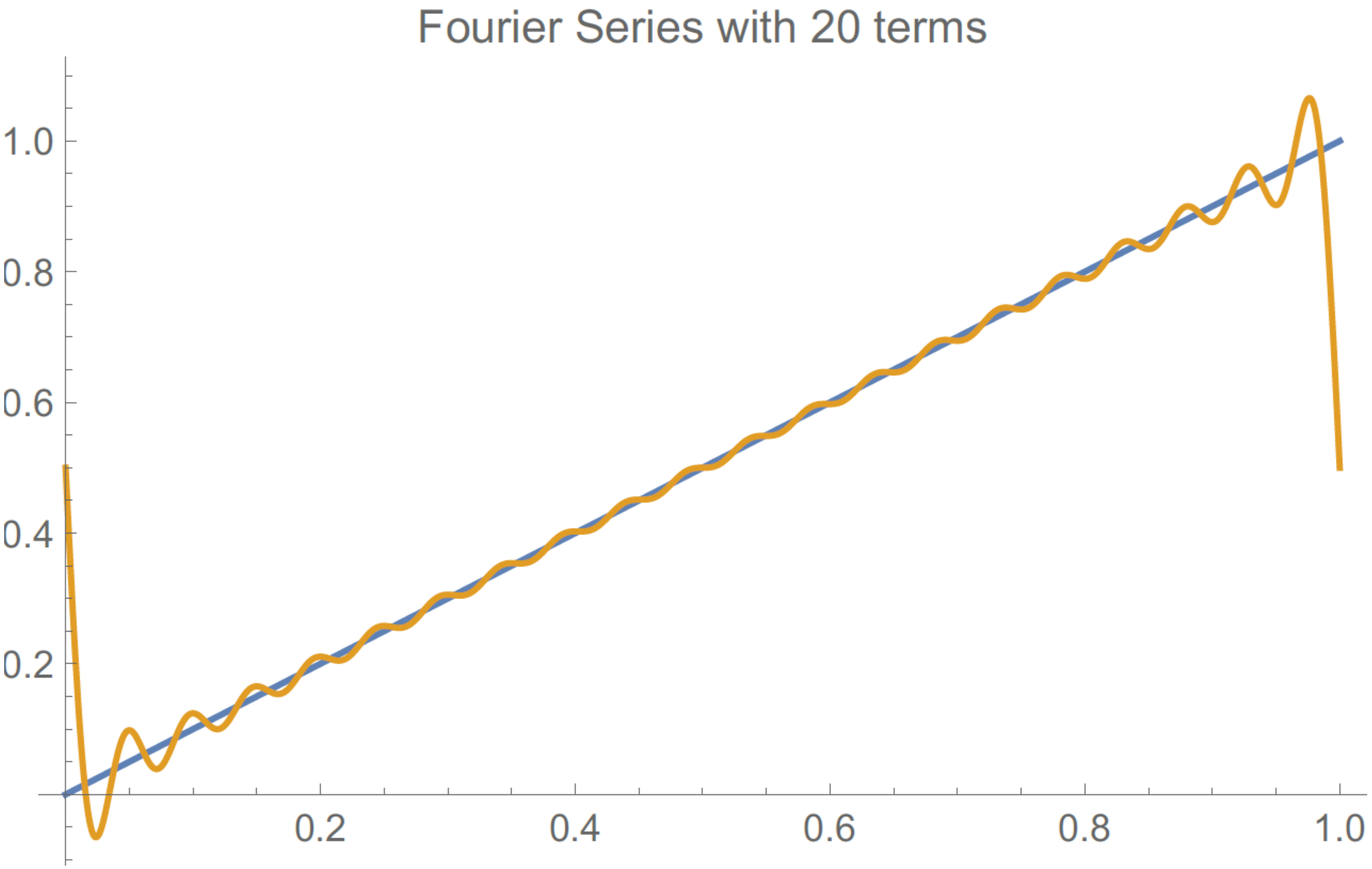
Q1

Given a function $f(x)$ defined in an interval $x \in \{x_1, x_2\}, 0 < L = x_2 - x_1$, its Fourier series representation is given by

$$a_0 + \sum_{k=1}^{n} a_k \cos\frac{2\pi kx}{L} + b_k \sin\frac{2\pi kx}{L} x$$

- Consider the function $f(x) = mx$, where m is the slope of the function, defined for $x \in \{0, L\}, L > 0$. The Fourier coefficients are $\{a_k, b_k\}$.
- Obtain the analytical expression of the k-th coefficient a_k and b_k using Mathematica command **Integrate[ ]**. Display the first 10 coefficients using **Table[ ]**.
- Form the explicit expression of the Fourier series representation for $f(x)$ at up to the n-th order using the command **Sum[ ]**. Note that n in theory should be chosen to be infinity, but in numerical practice it just needs to be set to a large positive number. You should choose an appropriate value for n.
- Plot the Fourier series with $n = 5$ terms, and overlap it with the function $f(x)$ on the same plot. The output should looks like Fig. 1 in the following page. Assume $L = m = 1$.

Fig. 1



Q2.

- Repeat Q1 with $f(x)$ defined as the following triangular function.

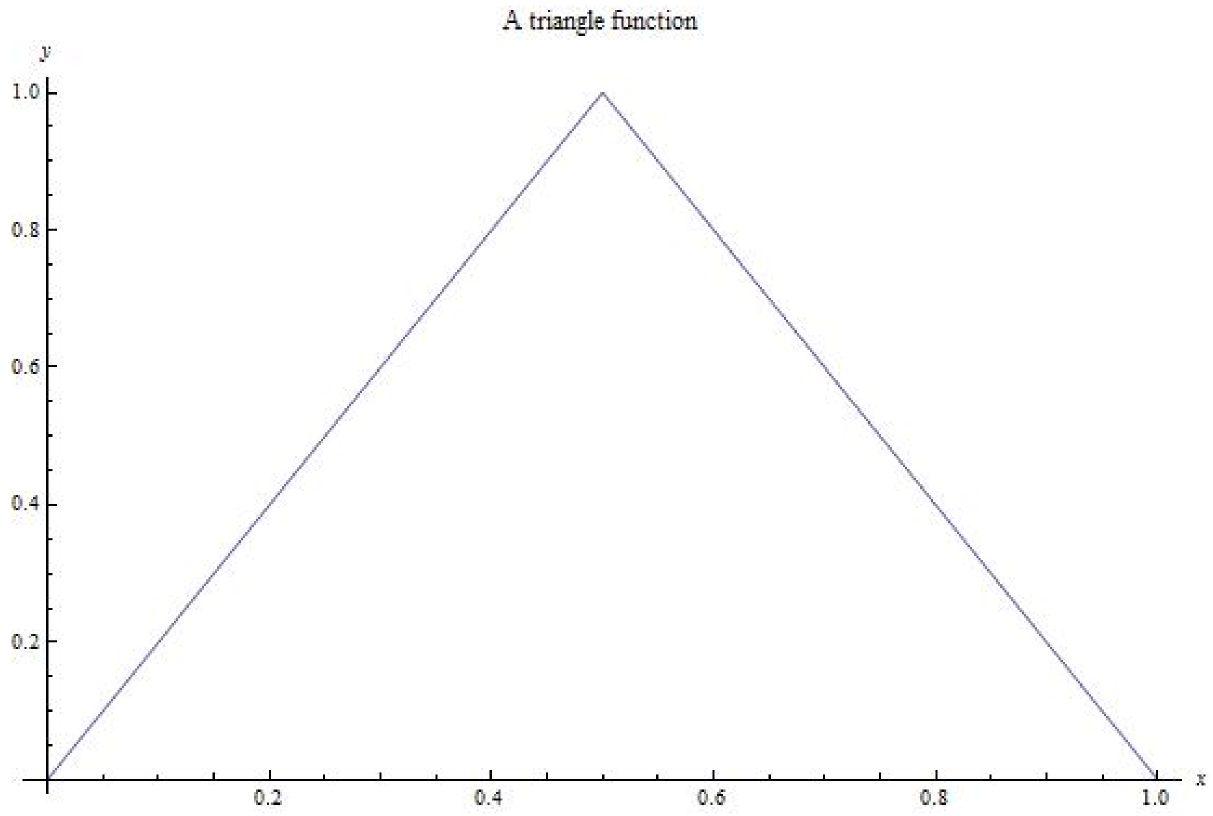
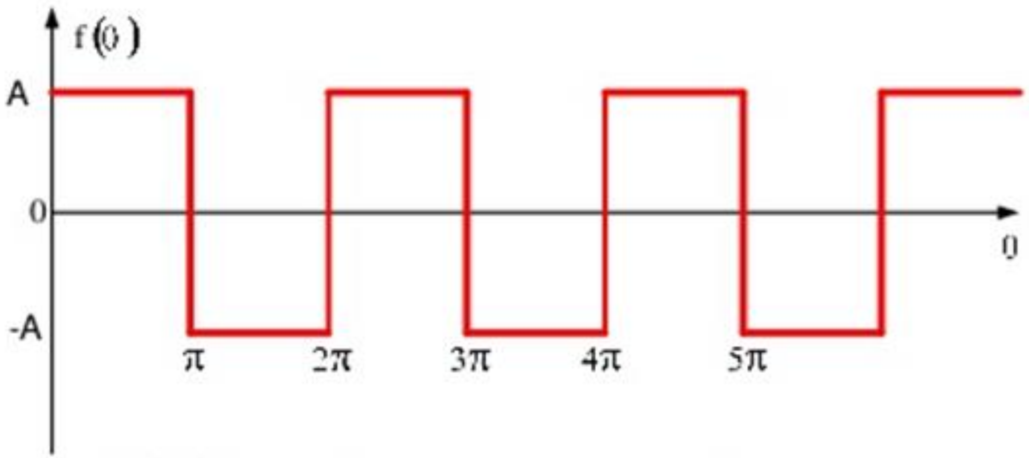


Fig.2

Q3. Repeat Q1 with $f(x)$ defined as $f(x) = e^x, 0 \leq\ x \leq 2\pi$

Q4, Repeat Q1 with $f(x)$ defined as $f(x) = \begin{cases} e^x, 0 < x < \pi \\ 0, \pi < x < 2\pi \end{cases}$

Q5 (Challenging Question)



$$f(\theta) = A \quad \textit{when} \quad 0 < \theta < \pi$$

$$= -A \quad \textit{when} \quad \pi < \theta < 2\pi$$

$$f(\theta + 2\pi) = f(\theta)$$

- Use Mathematica to generate a plot that display a rectangular period function with arbitrary period θ for x in any arbitrary range, $x \in \{x_1, x_2\}, (x_2 - x_2) = n\theta$ for any arbitrary positive integer n.