Lecture 4
Visualization and Animation

Use Mathematica to visualize the motion of physical systems

Parametric equations

- Given a set of parametric equation describing the motion of a particle in space as a function of time,

$$x = f(t), y = g(t)$$

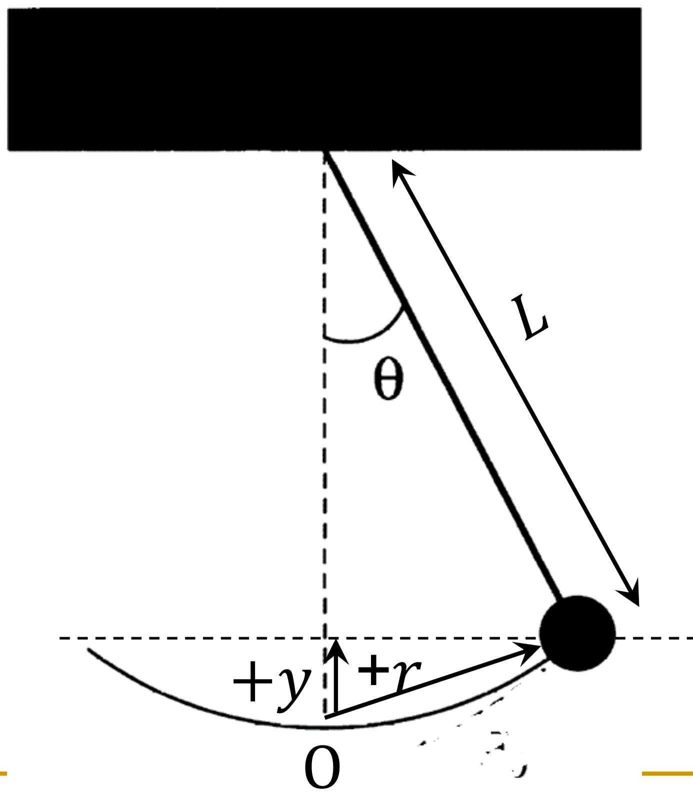
- one can easily visualize the motion using the command **Manipulate**[ ]
- To this end you may have to also invoke a For loop for generating the time-dependent coordinate variables before visualizing them.
- **ParametricPlot[]** is another useful command for this purpose.

Examples of parametric equations

- SHO

$$\theta(t) = \theta_0 \sin(\omega t + \phi)\,;$$

$$x = L\sin(\theta(t))\ ,\ y = L - L\cos(\theta(t))$$



$$\omega = \sqrt{\frac{g}{l}}$$

Derivation of the equation of motion for SHO

Force on the pendulum $F_\theta = -m\,g\,\sin\theta$

for small oscillation, $\sin\theta \approx \theta$.

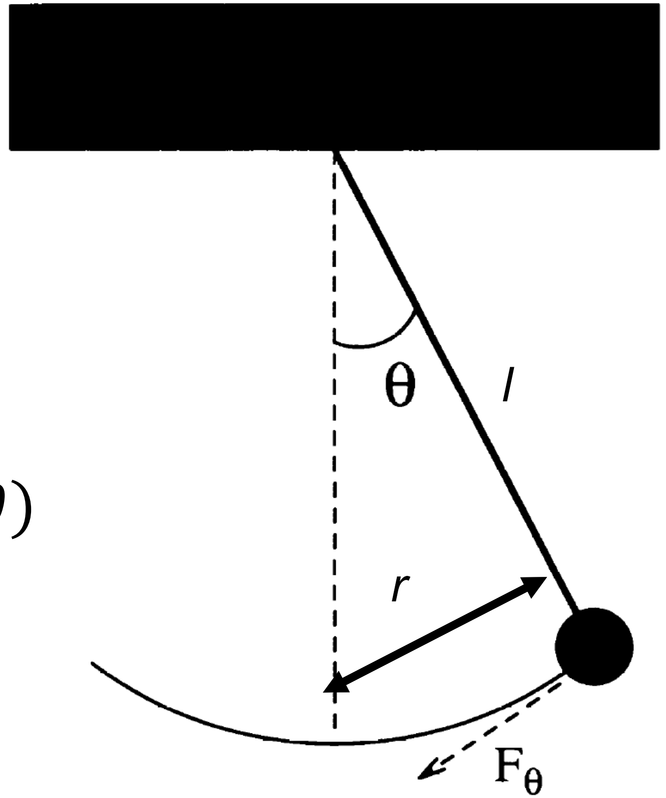
Equation of motion (EoM)

$$F_\theta = ma_\theta$$

$$-mg\sin\theta = m\frac{dv_\theta}{dt} = m\frac{d}{dt}\left(\frac{dr}{dt}\right) \approx m\frac{d^2}{dt^2}(l\theta)$$

$$\frac{d^2\theta}{dt^2} \approx -\frac{g\theta}{l} = -\omega^2\theta$$

$$\theta(t) = \theta_0\sin(\omega t + \phi) \qquad \omega = \sqrt{\frac{g}{l}};T = \frac{2\pi}{\omega}$$



Examples of parametric equations: 2D projectile motion

- The trajectory of a 2D projectile with initial location (x_0, y_0), speed v_0 and launching angle θ are given by the equations:

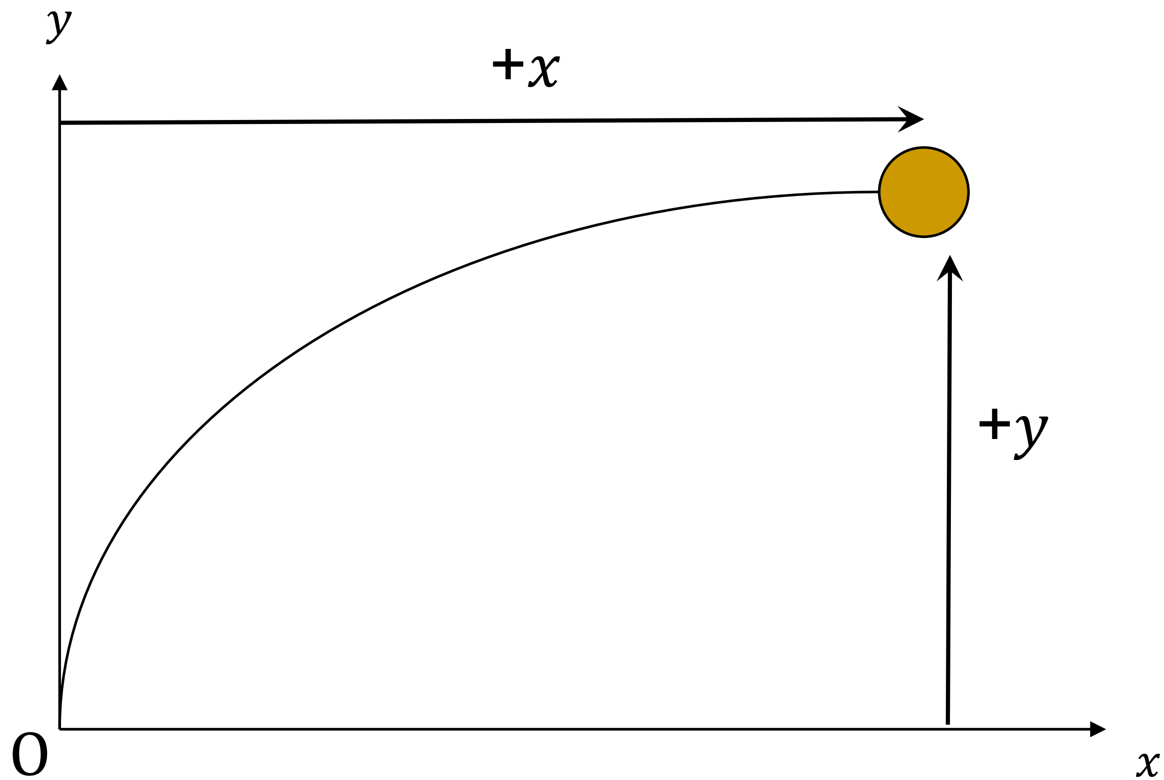
$$x(t) = x_0 + v_0 t \cos\theta;$$

$$y(t) = y_0 + v_0 t \sin\theta + \frac{g}{2} t^2$$

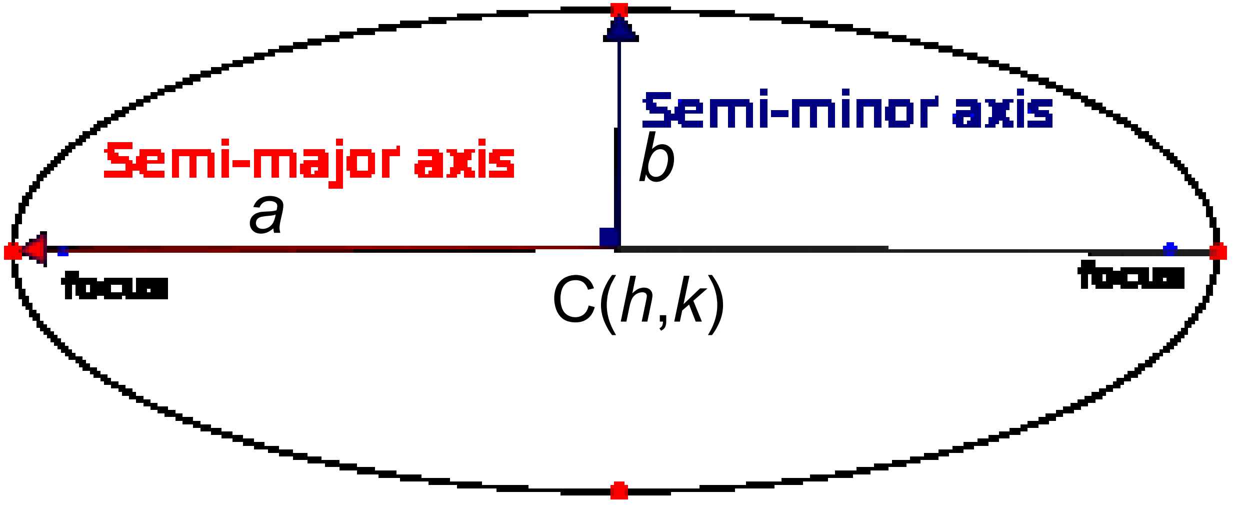
for *t* from 0 till *T*, defined as the time of flight,

$$T = -2(y_0 + v_0 \sin\theta)/g.$$

- $g = -9.81$;


y
+x
+y
O
x

Examples of parametric equations: 2-body Planetary motion



- Consider a planet orbiting the Sun which is located at one of the foci of the ellipse.
- The coordinates of the planet at time *t* can be expressed in parametrised form:

$$x(t) = h + a\cos(\omega_0 t),\ y(t) = k + b\sin(\omega_0 t)$$

Simple examples

Animate the following:

- A freely moving particle bounded to a finite line: 1D motion
- A freely moving particle bounded to a square box: 2D motion
- A freely moving particle bounded to a square rectangular box: 3D motion
- N freely moving particle bounded to a square rectangular box: 3D motion

Assignments

By using the corresponding parametric equations for the (x, y) coordinates,

1. Animate a SHO (w/o drag force and driving force)
2. Animate 2D projectile motion (w/o drag force and driving force)
3. Animate 2-body planetary motion

More complicated examples

- Damped, forced SHO
- 2D projectile motion with drag force from the air
- Three-body planetary motion