Wave Descriptions in Classical Physics

Particles

- Common sense: an ideal particle can be localized completely, has mass and electric charge that can be determined with high precision (just as what is implicitly assumed in Newtonian mechanics). The energy carried by a particle is given by
  $E^2 = m_0^2c^4 + p^2c^2$
- Particles are discrete, or in another words, corpuscular, in nature

- A particle can be modelled as point mass (where all the mass of the particle is thought to concentrate in its centre of mass, internal structure and finite size are ignored) in scenario such as molecules in kinetic theory and stars in galaxies

- In short, an object is effectively a particle whenever its dimensions are very small relative to the dimensions of the system of which it is a part, and when its internal structure is unimportant to the problem under consideration

Wave

- The simplest type of wave is strictly sinusoidal and is characterised by its frequency ν, wavelength λ and its travelling speed c

- E.g. Electric field E of a perfectly monochromatic electromagnetic wave travelling in the $+x$ direction

$$c = \lambda\nu$$

$$E = E_0 \sin(\omega t - kx + \phi) \qquad \text{(Eq. 0)}$$

where $\omega = 2\pi\nu$ (*angular frequency*), $k = 2\pi/\lambda$ (wave number), ϕ phase constant

◆ The **electric field** *E* is sinusoidal variation in time for any fixed point *x* in space extends over all possible values of *x*

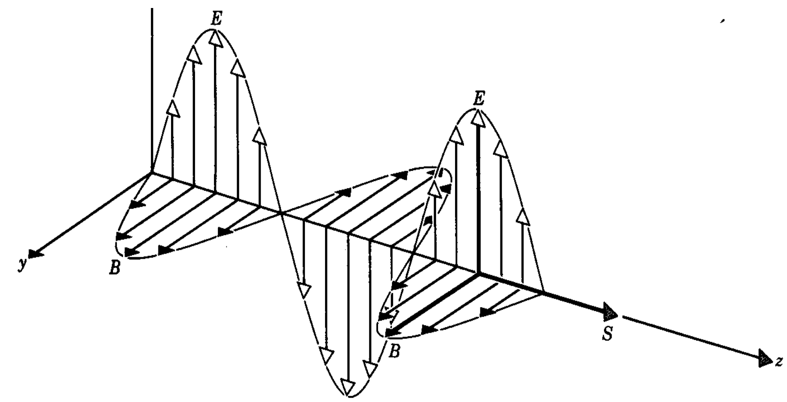


FIGURE 3.1 An electromagnetic wave traveling in the *z* direction. The electric field **E** lies in the *xz* plane and the magnetic field **B** lies in the *yz* plane.

◆ If we know the wavelength and frequency of a pure wave (plain wave of the form as in (Eq. 0) with infinite precision, this would mean the wave cannot be confined to any restricted region of space but must have an infinite extension along the direction in which it is propagated (i.e. the wave exists *everywhere* in space)

◆ In other words, if the frequency of a wave is known (via some measurement) without uncertainty, i.e. $\Delta\nu \longrightarrow 0$ (or equivalently $\Delta\lambda \longrightarrow 0$), the wave must be infinite (i.e. the space in which it is present extent to infinity, $\Delta x \longrightarrow \infty$)

◆ We shall prove that for a pure wave,

$$\Delta x \, \Delta\lambda \geq \lambda^2$$

◆ This is in contrast to the concept of a particle which is localised in space (the extent of space in which the particle is present, Δx, is, obviously, restricted to within the size of the particle itself, Δx = the size of the particle)

Read it yourself (adopted from pg.7 - pg.8, Weidner and Sells):

Use a standard pure wave with known frequency ν_1 to measure the unknown frequency ν_2 of another wave by combining them into beat pattern. We want to determine what is the uncertainty in the location of the measured wave, Δx, in terms of the uncertainty in its wavelength (or equivalently, its frequency)

In beat phenomena, the number of beats per unit time (its beat frequency) is given by $\Delta\nu = |\nu_2 - \nu_1|$. The beat takes a period of $T = 1/\Delta\nu$ to complete one cycle. By measuring the beat in some finite time interval, Δt, we could then tell what ν_1 is.

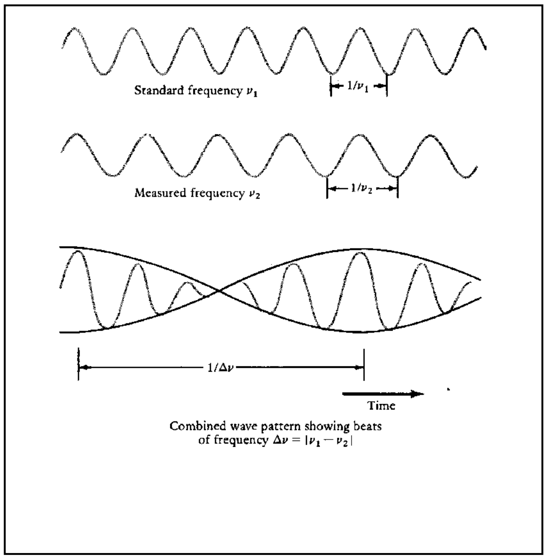


FIGURE 1-3
Beat pattern resulting from the superposition of waves of frequencies ν_1 and ν_2

As far as measurement is concerned, we have to wait for at least $\Delta t \geq T = \dfrac{1}{\Delta\nu}$ in order to see the beat to complete one cycle, hence

$$\Delta t \geq \frac{1}{\Delta\nu} \tag{1-2}$$

Suppose the wave is observed within the finite time interval Δt, then during this time the wave will have travelled a distance $\Delta x = c\,\Delta t$

We then say that the wave has been observed within the distance Δx,

$$\Delta x = c\,\Delta t \tag{1-3}$$

[Note that the wave is not located within a point (such as a particle does) but extent over a finite space, Δx, within which we could not tell exactly where the wave is

located. Thus Δx also represents the uncertainty in the ``position'' of the wave in space.]

Plug Equation (1-3) into Equation (1-2),

$$\Delta x\ \Delta\nu \geqslant \lambda\nu \qquad (1\text{-}3)$$

We then express $\Delta\nu$ in Equation (1-2) in terms of λ and ν via

$$c = \lambda\nu$$

$$\Rightarrow \Delta\nu/\nu = \Delta\lambda/\lambda, \qquad (1\text{-}4)$$

(only magnitudes are concerned)

With Equation (1-4), Equation (1-3) then becomes

$$\Delta x\ \Delta\lambda \geqslant \lambda^2 \qquad (1\text{-}5)$$

Equation (1-4) says that if we could locate the wave to infinite precision, i.e. $\Delta x = 0$, we would require $\Delta\lambda = \infty$.

On the other hand, if the frequency or the wavelength of the wave is known to infinite precision, i.e. $\Delta\lambda = 0$, or $\Delta\nu = 0$, we would require $\Delta x \to \infty$, i.e the space in which the wave is present has to extent to infinity

- $\Delta x\ \Delta\lambda \geqslant \lambda^2$ (or, equivalently, $\Delta t \Delta\nu \geq 1$) are the uncertainty relationships for classical waves: the position and wavelength (the measurement time and frequency) are mutually ``uncertain'' to the degree given by $\Delta x\ \Delta\lambda \geqslant \lambda^2$ ($\Delta t \Delta\nu \geq 1$)

Example

In a measurement of the wavelength of water waves, 10 waves crests are counted in a distance of 200 cm. Estimate the minimum uncertainty in the wavelength that might be obtained from this experiment.

ANS

- ***want to find $\Delta\lambda$, given λ and Δx***
- ***λ = 200 cm / 10 = 20 cm, Δx = 200 cm***
- ***use $\Delta x \Delta\lambda \geqslant \lambda^2$, $\Delta\lambda = (20\ \text{cm})^2 / 200\text{cm} = 2\ \text{cm}$***

Example

An frequency measurement device automatically displays the frequency of an input sinusoidal signal. To account for frequency variations, the frequency is measured and the display updated each second (i.e the frequency is *measured once a second). Can this device possibly measure an accuracy of up to 0.01 Hz?*

ANS

Based on $\Delta t \Delta \nu \geqslant 1$, a measurement of frequency in a time Δt = 1s must have an associated uncertainty of $\Delta\nu = 1/\Delta t$ = 1 Hz. Hence the device is theoretically not capable of measuring an accuracy better than 1 Hz, not to say 0.01 Hz

Wave groups and dispersion

- A pure wave is 'everywhere' as it extent from $-x$ to $+x$, not ***localised***
- However, waves of limited extent (which shall be an important ingredient to represent particle by matter wave in quantum mechanics) – wave groups – can be constructed by adding different wavelengths, amplitudes and phases chosen

Simplest example – phenomena of beats (two pure waves added together)

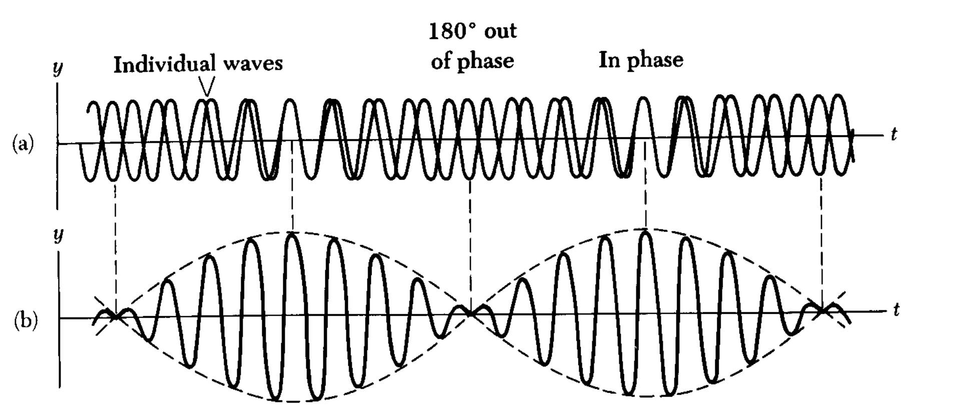


Figure 4.11 Beats are formed by the combination of two waves of slightly different frequencies traveling in the same direction. (a) The individual waves. (b) The combined wave has an amplitude (broken line) that oscillates in time. (From R. Resnick and D. Halliday, *Physics*, New York, Wiley, 1977; by permission of the publisher)

Two pure waves with slight difference in frequency and wave number $\Delta\omega = \omega_1 - \omega_2$, $\Delta k = k_1 - k_2$, are superimposed

$$y_1 = A\cos(k_1 x - \omega_1 t)\text{ , }\quad y_2 = A\cos(k_2 x - \omega_2 t)$$

The velocity of an individual plain wave is called the ***phase velocity***, v_p = $v\lambda = \omega/k$ *(by definition,* $\omega = 2\pi v$; k = $2\pi/\lambda$*)*

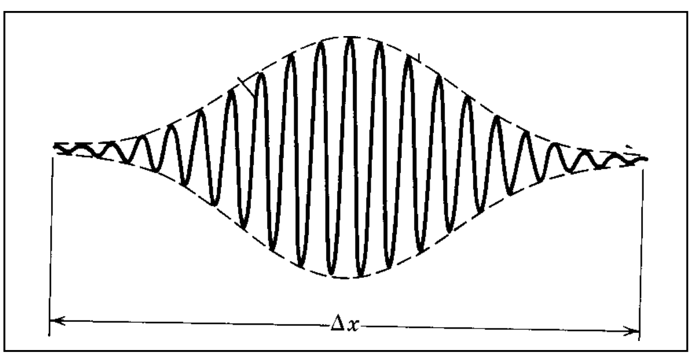
When superimposed, the resultant wave is a slowly varying envelop (the group wave) modulating a travelling wave

$$y = y_1 + y_2 = 2A\cos\frac{1}{2}\left(\{k_2 - k_1\}x - \{\omega_2 - \omega_1\}t\right)\cdot\cos\left\{\left(\frac{k_2 + k_1}{2}\right)x - \left(\frac{\omega_2 + \omega_1}{2}\right)t\right\}$$

group wave	phase wave
Δk, $\Delta\omega$	$k_p = \frac{k_2 + k_1}{2}$, $\omega_p = \frac{\omega_2 + \omega_1}{2}$
v_g = $\Delta\omega/\Delta k$	v_p = ω_p/k_p ($\approx$speed of individual waves if $\Delta\omega$ *and* Δk *are slight)*

The energy carried by the group wave is concentrated in regions in which the amplitude of the envelope is large

The speed with which the waves' energy is transported through the medium is the speed with which the envelope advances



A more `localised' group wave, a *wavepulse* can be constructed by adding more sine waves of different numbers k_i *and* possibly different amplitudes so that they interfere constructively over a small region Δx and outside this region they interfere destructively so that the resultant field approach zero

This is a generalisation of previous example (adding only two waves). In this case the speed of the wavepulse is given by the generalisation of $v_g = \Delta\omega/\Delta k$ for the two waves case to

$$v_g = \left.\frac{d\omega}{dk}\right]_{k_0}$$

where k_0 is the central wave number (sort of the average) of the many wave presents, while the individual component waves move with its phase velocity $v_p = \omega/k$ (phase velocity not defined for wavepulse but is meaningful only for a single component wave)

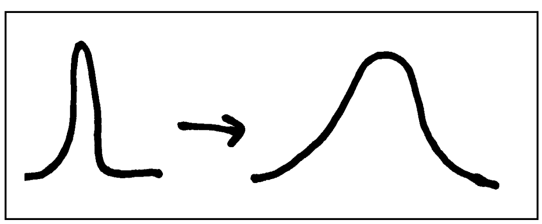
Relation between the phase and group velocity is

$$v_g = \left.\frac{d\omega}{dk}\right]_{k_0} = v_p\Big]_{k_0} + k\left.\frac{dv_p}{dk}\right]_{k_0}$$

Generally, for waves propagating in a medium, the individual phase speed is dependent on the wave number k – *dispersion* (e.g. light dispersion in prism)

Note that if $\frac{dv_p}{dk} = 0$, $v_p = v_g$. But generally, in a dispersive medium, $\frac{dv_p}{dk}$ is non zero but some positive or negative value. (Most medium are dispersive; only vacuum is not dispersive)

As a result of dispersion, an originally sharp pulse changes shape and becomes spread out (or dispersed)



When a group wave travels through a medium, the energy is transported at a speed - the group velocity v_g - that differs from the phase velocity of either of the component waves

Example

Newton show that the phase velocity of deep water waves having wavelength λ is given by $v_p = \sqrt{\frac{g\lambda}{2\pi}}$ *, where g is the acceleration of gravity. What is the velocity of a group of these waves?*

ANS

Since k = 2π/λ, we can write $v_p = \sqrt{g/k}$. Therefore

$$v_g = v_p\Big]_{k_0} + k\frac{dv_p}{dk}\Big]_{k_0} = \sqrt{\frac{g}{k_0}} - \frac{1}{2}\sqrt{\frac{g}{k_0}} = \frac{1}{2}\sqrt{\frac{g}{k_0}} \equiv \frac{1}{2}v_p\Big]_{k_0}$$

Waves and particles are important in physics because they represent the only modes of energy transport (interaction) between two points.

E.g we signal another person with a thrown rock (a particle), a shout (sound waves), a gesture (light waves), a telephone call (electric waves in conductors), or a radio message (electromagnetic waves in space).

Interactions take place only between

(i) particles and particles (e.g. in particle-particle collision) or

(ii) between waves and particle, in which a particle gives up all or part of its energy to generate a wave, or when all or part of the energy carried by a wave is absorbed by a nearby particle (e.g. a wood chip dropped into water, or an electric charge under acceleration, generates waves)

However, in contrast, two waves do not interact (as like in the case of particle-particle or particle-wave interactions) but they simply "**superimpose**", in which they pass through each other essentially unchanged, and their respective effects at every point in space simply add together according to the principle of superposition to form a resultant at that point -- a sharp contrast with that of two small, impenetrable particles

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