Particle nature of radiation

Wave nature of light (EM wave)

Light, classically treated as a wave phenomena, is described in Maxwell theory of electromagnetics as an alternating electric and magnetic fields that propagate in the form of a transverse wave (assuming plane wave travelling in the **z**-direction)

$\mathbf{E} = \mathbf{E}_0 \sin(kz - \omega t + \phi)$, $\mathbf{B} = \mathbf{B}_0 \sin(kz - \omega t + \phi)$

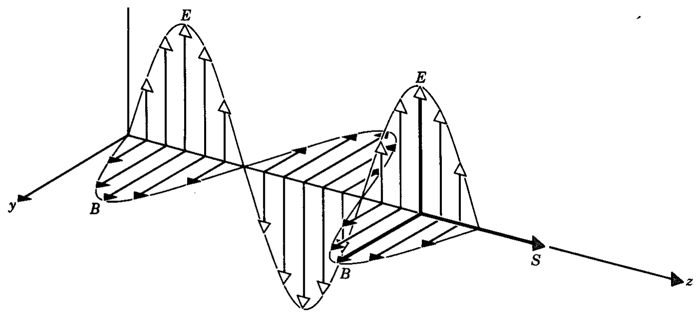


FIGURE 3.1 An electromagnetic wave traveling in the z direction. The electric field **E** lies in the xz plane and the magnetic field **B** lies in the yz plane.

Energy flux of the EM wave is described by the Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{B} = (E_0 B_0/\mu_0) \sin^2(kz - \omega t + \phi)\,\hat{\mathbf{z}}$$

(in unit of energy per unit time per unit area; **S** is perpendicular to the directions of both **E** and **B**)

The rate of energy transported by the EM wave across an orthogonal area *A* is

$$P = \frac{1}{\mu_0 c} E_0^2 A \sin^2(kz - \omega t + \phi)$$

(in unit of energy per unit time)

Since ω is large ($\sim 10^{15}$) we observe only the average behaviour of energy transported by an EM wave that

$$P_{ave} = \frac{1}{T}\int P\,dt = \frac{1}{2\mu_0 c} E_0^2 A$$

(constant in time)

EM wave display wave phenomena such as diffraction and interference

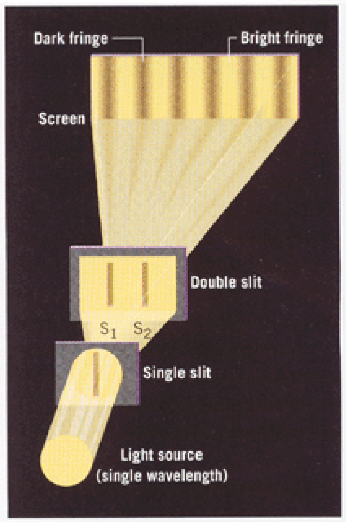


Figure 27.4 In Young's double-slit experiment, two slits S_1 and S_2 act as coherent sources of light. Light waves from these slits interfere constructively and destructively on the screen to produce, respectively, the bright and dark fringes. The slit widths and the distance between the slits have been exaggerated for clarity.

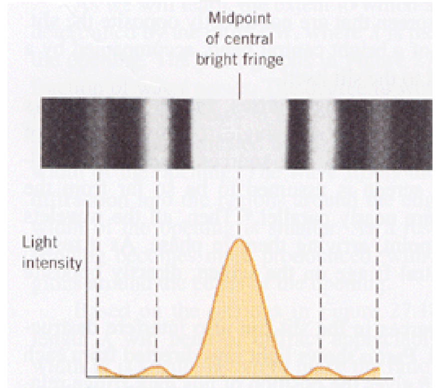


Figure 27.23 The photograph shows a single-slit diffraction pattern, with a bright and wide central fringe. The higher-order bright fringes are much less intense than the central fringe, as the graph indicates. (From Michel Cagnet, et al., *Atlas of Optical Phenomena*, Springer-Verlag, Berlin.)

Failure of wave theory

Classical Blackbody radiation

Hot body emits continuous spectrum of thermal radiation (radiation = EM radiation, e.g. IR, colour light, sunlight, = heat that is in radiation form)

An idealised hot object (i.e with its temperature $T > 0$ K) called blackbody,

- has surface that absorbs all thermal radiation (all wavelength) incident upon them, hence a perfect absorbers of thermal radiation
- ***reflects*** no radiation that falls upon their surface (hence property of the surface can then not been seen and is 'black')
- the thermal spectrum radiated depends only on the temperature (material, surface and geometry independent)

At thermodynamical equilibrium, a blackbody has no net transfer of energy (ie. energy absorbed and re-emitted are the same; or in other words, the absorbed energy is all re-radiated at the same rate), hence the temperature T remains constant (common sense)

Austrian, J. Stefan measured the thermal distribution spectrum (i.e. its radiance intensity R) of a blackbody at temperature T with a detector subtending a fixed angular width $d\theta$.

R = the distribution of the radiated EM energy as a function of emitted frequency, or equivalently, the wavelength) = energy radiated per unit time per unit area per unit wavelength interval.

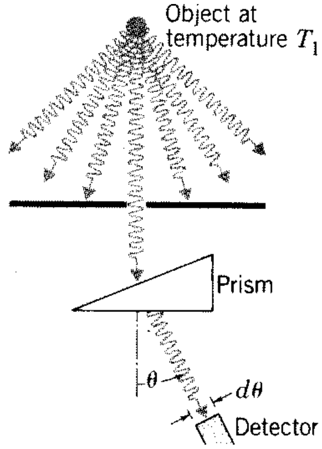


FIGURE 3.10 Measurement of the spectrum of thermal radiation. A device such as a prism is used to separate the wavelengths emitted by the object.

Different wavelength is deflected at different angle when the radiation traverses through the prism (dispersive medium). Hence, from the value of the angle θ the wavelength can be inferred. The differential angular width $d\theta$ corresponds to a fixed interval in wavelength, $d\lambda$

The measured values of $R\, d\lambda$ (which is just the calibrated intensity read off from the detector) plotted against different values of λ.

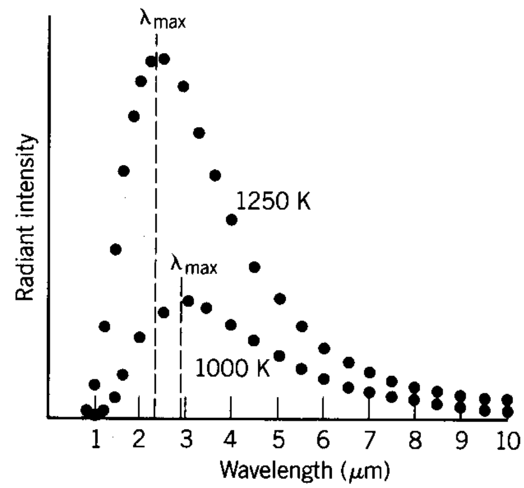


FIGURE 3.11 A possible result of the measurement of the radiant intensity over many different wavelengths. Each different temperature of the emitting body gives a different peak λ_{max}.

Experimentally, the following universally characteristic is found:

1) the thermal spectrum is independent of material and surface geometry, but depend only on the temperature. It is a function of wavelength and temperature, $R = R(\underline{\lambda, T})$.

2) $\lambda_{max} \propto 1/T$ (***known as Wein displacement law***).
   Experimentally, $\lambda_{max} T = 2.898 \times 10^{-3}$ m·K (explains the red hot phenomena)

3) The power radiated over all wavelengths, is proportional to the fourth power of *T*:

$$P(T) = \int_0^\infty R\, d\lambda = \sigma T^4$$

The proportional constant σ is called the Stefan-Boltzmann constant, σ = 5.6703×10^{-8} W/m^2·K^4. ***This is known as the Stefan's law***

NOW: how to use classical physics (thermodynamics (or more specifically, statistical physics) and wave theory of EM wave (i.e. the Maxwell theory)) to explain (i) the thermal spectrum as a function of wavelength, (ii) Wein's law and Stefan's law?

RAYLEIGH-JEANS calculated (using classical theories – statistical physics, classical thermodynamics and Maxwell theory) the dependence of R on temperature and the wavelength, that gives

$R \propto$ number of wave modes per unit volume at wavelength λ per unit wavelength × energy per wave,

$$R \propto \frac{4\pi c}{\lambda^4} \times \varepsilon$$

$\varepsilon = kT$ is energy of a mode of thermal radiation, wavelength independent, and takes on continuous values ranging from zero to infinity when the temperature varies continuously.
k = Boltzmann constant = 1.38×10^{-23} Joule/K;

RJ theory fails at ultraviolet limit because of the frequency independent form of $\varepsilon = kT$.

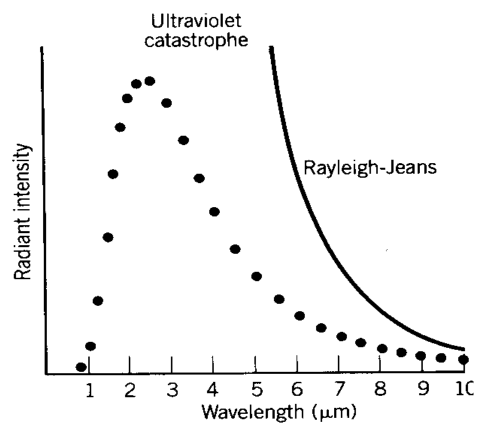


FIGURE 3.12 The failure of the classical Rayleigh-Jeans formula to fit the observed spectrum of radiant intensity. At long wavelengths the theory approaches the data, but at short wavelengths the classical formula fails miserably.

Ultraviolet catastrophe: the classical theory explanation of the blackbody radiation by Rayleigh-Jeans fails in the limit $\lambda \to 0$ (or equivalently, when frequency $\to \infty$), i.e. $R(\lambda) \to \infty$ at $\lambda \to 0$.

As a principle working basis for physical laws, any observable quantity must not be infinite. The theory that predict such is considered unsatisfying or incomplete, and must be supplanted by improved or alternative theory that can remove the infinity. Infinity in any physical observables is not acceptable in any physical theory.

Planck's postulate

To provide a consistent theoretical explanation to match the experimental data, Planck suggested

(P1) ε is made frequency (or equivalently, wavelength) dependent: $\varepsilon = \varepsilon(\lambda)$

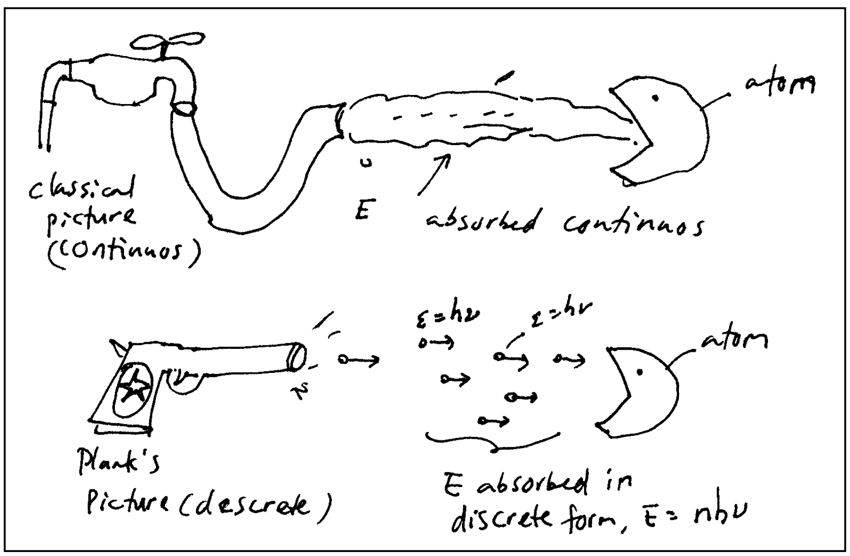
(P2) an atom could only absorb or reemit energy (carried by the radiation) in discrete bundles (called *quanta*) that are multiples of ε

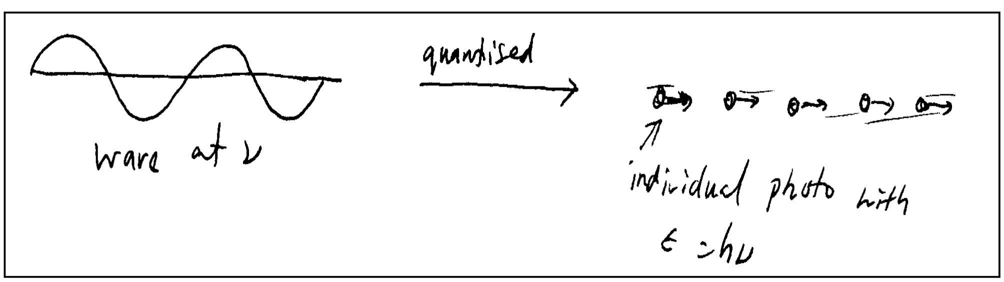
$$E = n\varepsilon = nh\nu = nhc/\lambda, \quad n = 1,2,3 \ldots$$

where n is the number of quanta. The energy of each of the quanta is determined by the frequency

$$\varepsilon = h\nu = hc/\lambda$$

h is a proportional constant, called Planck constant (to be determined experimentally). EM radiation, in this picture, cannot be regarded as wave anymore but have to be imagined as like discrete bundles of energy.





Based on the new assumption, Plank re-derived the spectrum of radiant intensity (using standard statistical mechanics) to be

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\left(\frac{hc}{\lambda}\right)\frac{1}{e^{hc/\lambda kT}} - 1\right]$$

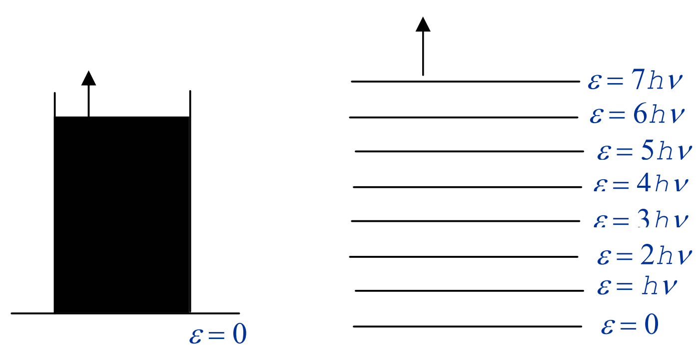
c.f. RJ, $R(\lambda) = \frac{8\pi c}{\lambda^4}kT$

Plank's law

(i) reduces to RJ's law at infrared limit (i.e. $\lambda \to 0$) correctly matches with experimental measurement of the thermal spectrum

(ii) can reproduce Wein's and Stefan's law correctly, in which it relates Stefan's constant to the Plank constant as per $\sigma = \left[\frac{2\pi^5 k^4}{15c^2 h^3}\right]$. From the measurement of σ in blackbody radiation experiment, the value of Planck constant is determined to be h = 6.626 × 10^{-34} Js.

In Plank's theory, oscillating particles and radiation fields at frequency ν can exchange energy only in integral multiples of the quantum energy $h\nu$. This is a revolutionary concept because this infers that at the atomic scale the energy levels of an atom are quantised (classically, the energies allowed in classical system are continuously distributed)



allowed energies in classical system – continuous (such as energy carried by a wave; total mechanical energy of an orbiting planet, etc.)

allowed energies in quantised system – discrete (such as energy levels in an atom, energies carried by a photon)

Photons – Particle-like properties of radiation

Planck's postulate necessitates a radical change of view on the nature of light. The classical picture that radiation is wave in nature now has to be replaced with that of a particle. The quantum of light is called ***photon***.

The particle-like properties of photon are empirically manifested in interactions between them with matter: photoelectricity, Compton effects, pair production/annihilation. These experiments (along with the blackbody radiation) cannot be satisfyingly explained if radiation is intrinsically of wave nature.

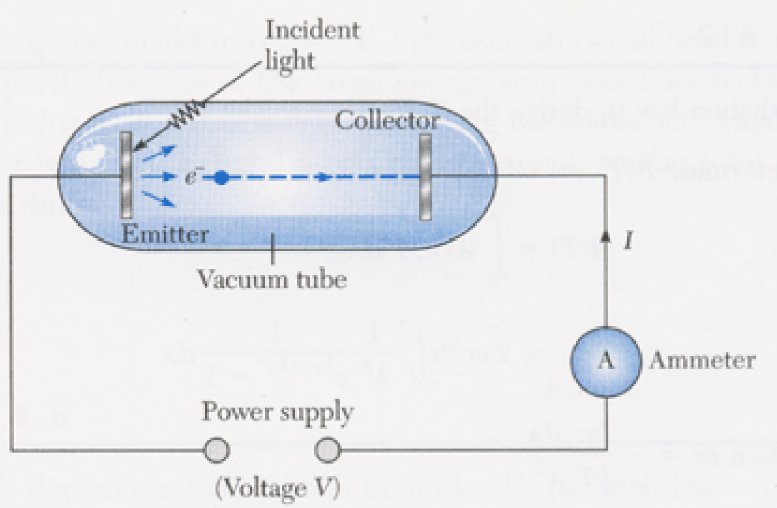


FIGURE 4.10 Photoelectric effect. Electrons, emitted when light shines on a surface, are collected, and the photocurrent I is measured. A negative voltage, relative to that of the emitter, can be applied to the collector. When this retarding voltage is sufficiently large, the emitted electrons are repelled, and the current to the collector drops to zero.

Photoelectric effects

- the ejection of electrons from a metal surface by the when hit by light
- cannot be explained by wave theory of light

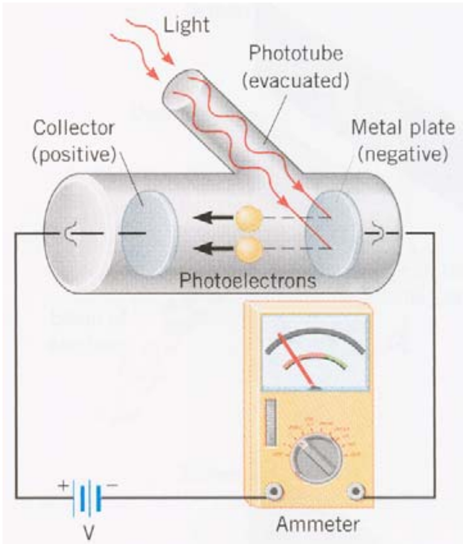


Figure 29.4 In the photoelectric effect, light with a sufficiently high frequency ejects electrons from a metal surface. These photoelectrons, as they are called, are drawn to the positive collector, thus producing a current.

In the experiment, monochromatic light hits the metal plate (photocathode), ejecting photoelectrons that are detected as electric current by the external circuit. Photoelectrons are attracted to the collecting anode by potential difference applied on the anode.

When the applied potential $V = -\phi_0$ photocurrent = 0. $-\phi_0$ is called the stopping potential.
When $V = 0$, the current is not zero because the photoelectrons carry kinetic energy. The fastest ejected photoelectron is given by $K_{max} = e\phi_0$.

In addition, theres exist a cut-off frequency below which no photoelectric effects occurs:

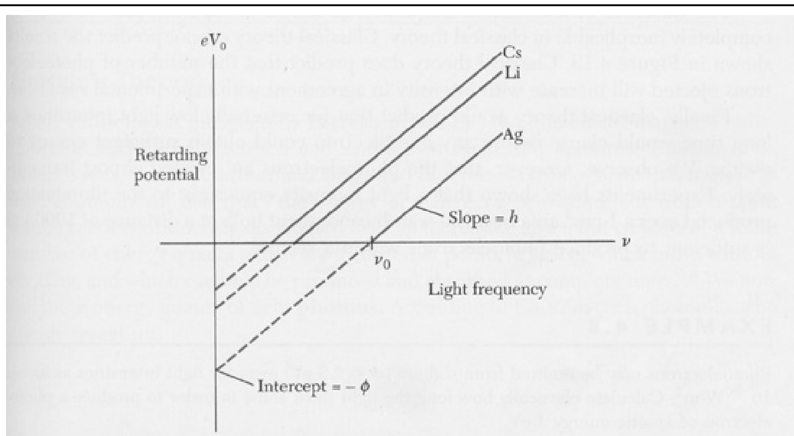


FIGURE 4.13 The retarding potential eV_0 (maximum electron kinetic energy) is plotted versus light frequency for three different emitter materials.

Three features unexplained by classical wave theory of light:

1) Experimentally, K_{max} is independent of intensity of the monochromatic light (The saturation curves of different intensities, I_0 and $2I_0$, $3I_0$ show a common stopping potential).

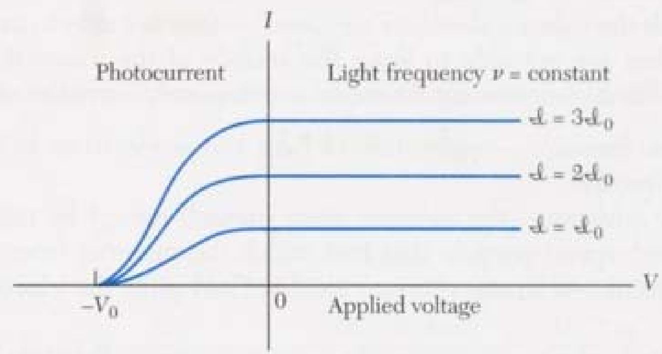


FIGURE 4.11 The photoelectric current I is shown as a function of the voltage V applied between the emitter and collector for a given frequency ν of light for three different light intensities. Notice that no current flows for a retarding potential more negative than $-V_0$ and that the photocurrent is constant for potentials near or above zero (this assumes that the emitter and collector are closely spaced or in spherical geometry to avoid loss of photoelectrons).

This is a puzzling behaviour if light is wave. For wave, as the intensity increases the stronger the energy carried by the radiation, hence the kinetic energy of photoelectrons ejected by the light should increase correspondingly. YET THE OBSERVATION IS OTHERWISE.

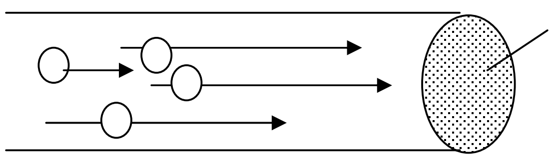
2) Existence of a characteristic cut-off frequency, ν_0. No photoelectric effects for different metallic surfaces if the frequency of the radiation used, ν, is less than ν_0.

   Wave theory predicts that photoelectric effect should occur for any frequency as long as the light is intense enough to give the energy to eject the photoelectrons. No cut-off frequency is predicted.

3) No detection time lag measured. Classical wave theory needs a time lag between the instance the light impinge on the surface with the instance the photoelectrons being ejected. Energy needs to be accumulated for the wave front before it has enough energy to eject photoelectrons. But, in the observation, photoelectricity is almost immediate.

Einstein's quantum theory of the photoelectricity (1905)

- A Noble-prize winning theory
- An idea taken from Planck but now formalised by Einstein
- Postulate that radiant energy is quantized into concentrated bundle called photon.
- In the wave picture, the energy flux carried by a beam of light is continous and is given by $S = \frac{E_0}{2\mu_0 c}$ (in unit of energy per unit time per unit area). Whereas in the photon picture, the energy flux of the beam is described in terms of 'photon number density, n_0 (in unit of number per unit volume), as per $S = n_0 ch\nu$. In this picture the total energy across a unit area in unit time (S) can be evaluated by 'counting' the total number of photon crossing the area within a unit time.



A beam of light with frequency ν crossing an unit area

Energy carried by the energy crossing the unit area is S (in unit of joule per area per unit time)

- Wave behaviour of light is a result of collective behaviour of very large numbers of photons

Assumptions by Einstein:

1) the energy of a single photon is $E = h\nu$

2) photoelectricity, one photon is completely absorbed by one atom in the photocathode. One electrons is then 'kicked out' by the absorbent atom. The kinetic energy for the ejected electron is $K = h\nu - W$
W is the worked required to (i) overcome the attraction from the atoms in the surface, (ii) cater for losses of kinetic energy due to internal collision of the electrons. When no internal kinetic energy loss (happen to electrons just below the surface), K is maximum:

$$K_{max} = h\nu - W_0 \tag{1}$$

W_0 = work function

What is the momentum of a photon? From SR, the total energy of a particle E is related to the momentum p and its rest mass m_0 via

$$E^2 = p^2c^2 + (m_0c^2)^2$$

For a photon, $m_0 = 0$. Hence, consistency of SR and Planck's postulation of light quanta requires that the momentum of a photosn be given by

$$p = E/c = h/\lambda.$$

Einstein theory manage to solve the three unexplained features:

(1) K_{max} is independent of light intensity. Doubling the intensity of light wont change K_{max} because the energy $h\nu$ of individual photons wont change, nor is W_0 (W_0 is the intrinsic property of a given metal surface)

(2) The cut-off frequency is explained. No detection of photoelectric current = photoelectrons from the photocathode do not reach the anode because the photoelectrons have got zero kinetic energy. In other words, if the energy (which will be \`absorbed' by the electron in the atom and used to overcome the attraction of the surface atom) of the impinging photon is less than W_0, the photoelectrons would not be able to leave the metal surface. This happens when

$$K_{max} = 0,$$
$$\Rightarrow W_0 = h\nu_0 \tag{2}$$

The above formula relates the cut-off frequency with the work function

A photon of the frequency ν_0 has just enough energy to eject the photoelectron and none extra to appear as kinetic energy

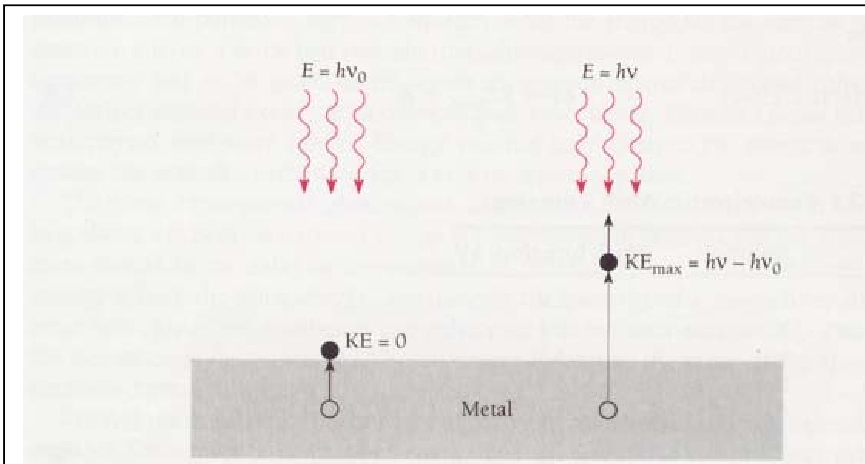


Figure 2.13 If the energy $h\nu_0$ (the work function of the surface) is needed to remove an electron metal surface, the maximum electron kinetic energy will be $h\nu - h\nu_0$ when light of frequen directed at the surface.

Table 3.1 Some Photoelectric Work Functions

Material	W(eV
Na	2.28
Al	4.08
Co	3.90
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

Since different matel have different work funtion, the cut-off frequency for different matel is also different.

(3) The required energy to eject photoelectrons is supplied in concentrated bundles of photons, not spread uniformly over a large area in the wave front. Any photon absorbed by the atoms in the target shall eject photoelectron immediately. Absorption of photon is a discrete process at quantum time scale (almost 'instantaneously'): it either got absorbed by the atoms, or otherwise.

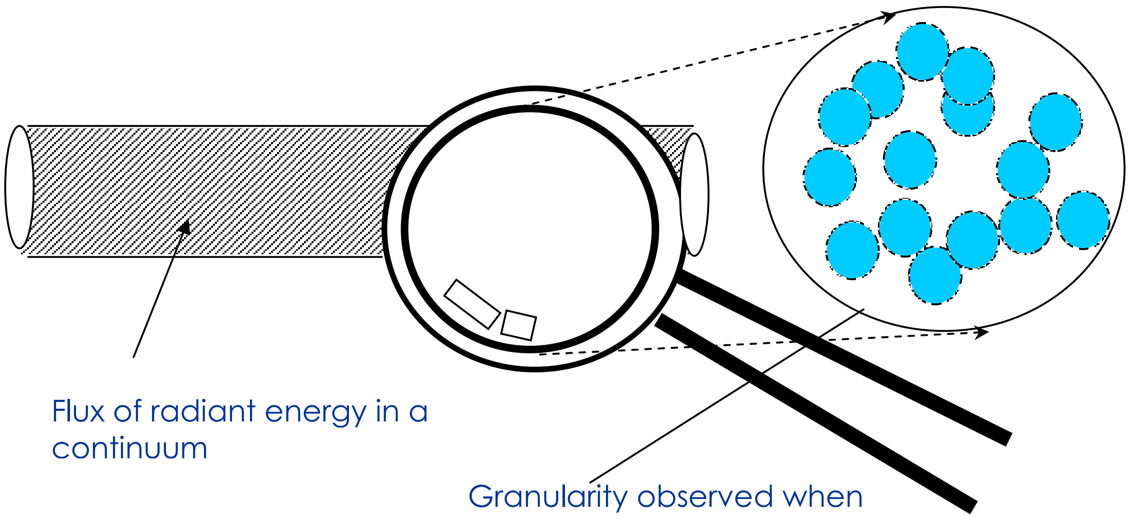
Experiment can measure $e\phi_0 (= K_{\max})$ for a given metallic surface (e.g. sodium) at different frequency of impinging radiation

$$e\phi_0 = h\nu - W_0$$

The slope in the graph of ϕ_0 verses frequency ν gives the value of $h/e = 4.1\times10^{-15}$ Vs: $h = 6.626 \times 10^{-34}$ Js

Particle properties of light is hardly observed since the radiation at even relatively low radiance will comprise of too large amount of photons.

This account for the extreme fineness of the granulation of radiation – hence why it is difficult to detect it in ordinary experiment. This is analogous to detecting individual molecules of a stream of water which appears as a ``continuous'' flux.



Photoelectricity happens only to bounded electrons within the atoms or solid, but not to free electrons due to kinatical reasons (conservation of linear momentum)

Example

(a) What are the energy and momentum of a photon of red light of wavelength 650nm? (b) What is the wavelength of a photon of energy 2.40 eV? (in atomic scale we usually express energy in eV, momentum in unit of eV/c, length in nm; the combination of constants, hc, is conveniently expressed in

$$hc = (6.62\times10^{-34}\ \text{Js})\cdot(3\times10^{8}\ \text{m/s})$$
$$= [6.62\times10^{-34}\cdot(1.6\times10^{-19})^{-1}\text{eV}\cdot\text{s}]\cdot(3\times10^{8}\ \text{m/s})$$
$$= 1.24\text{eV}\cdot10^{-6}\text{m} = 1240\text{eV}\cdot\text{nm}$$

$$1\ \text{eV/c} = (1.6\times10^{-19})\text{J}/\ (3\times10^{8}\ \text{m/s}) = 5.3\times10^{-28}\ \text{Ns}$$

ANS

(a) $E = hc/\lambda = 1240\ \text{eV}\cdot\text{nm} / 650\ \text{nm} = 1.91\ \text{eV}$
$= 3.1\times10^{-19}\text{J}$

(b) $p = E/c = 1.91\ \text{eV/c}\ (= 1\times10^{-27}\ \text{Ns})$
$\lambda = hc/E = 1240\text{eV}\cdot\text{nm} /2.40\ \text{eV} = 517\ \text{nm}$

Application of photoelectricity: digital camera (CCD camera), IR sensor and human's eye

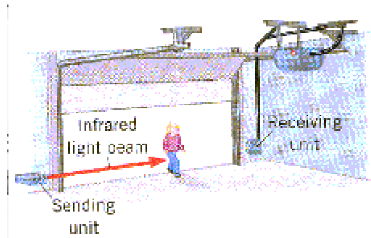


Figure 29.8 When an obstruction prevents the infrared light beam from reaching the receiving unit, the current in the receiving unit drops. This drop in current is detected by an electronic circuit that stops the downward movement of the door and then causes it to rise.

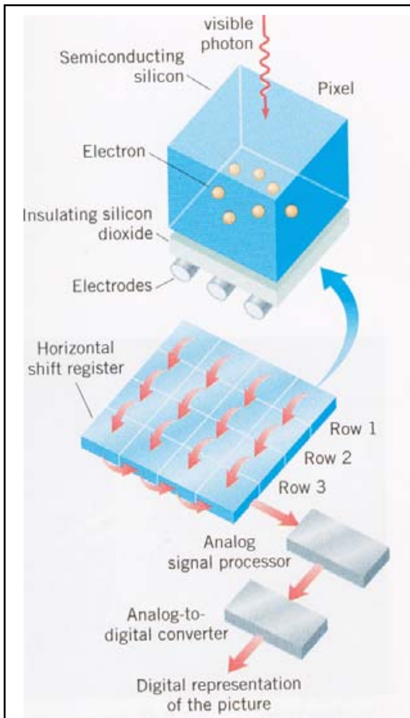
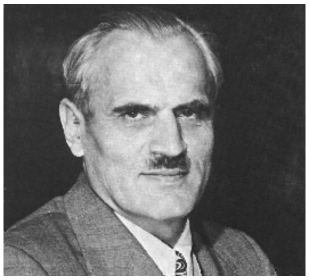


Figure 29.7 A CCD array can be used to capture photographic images using the photoelectric effect.

To summerise: In photoelectricity (PE), light behaves like particle rather than wave.

The Compton Effect (1923, Compton)



Compton, Arthur Holly (1892-1962), American physicist and Nobel laureate whose studies of X rays led to his discovery in 1922 of the so-called Compton effect. The Compton effect is the change in wavelength of high energy electromagnetic radiation when it scatters off electrons. The discovery of the Compton effect confirmed that electromagnetic radiation has both wave and particle properties, a central principle of quantum theory.

Beam of x-ray with sharp wavelength λ falls on graphite target. For various angle θ the scattered x-ray is measured as a function of their wavelength.

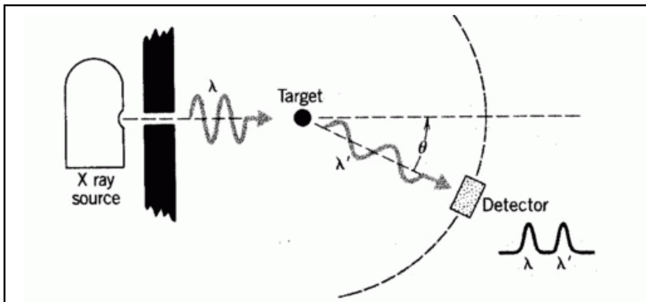


FIGURE 3.17 Schematic diagram of Compton scattering apparatus. The wavelength λ' of the scattered X rays is measured by the detector, which can be moved to different positions θ. The wavelength difference $\lambda' - \lambda$ varies with θ.

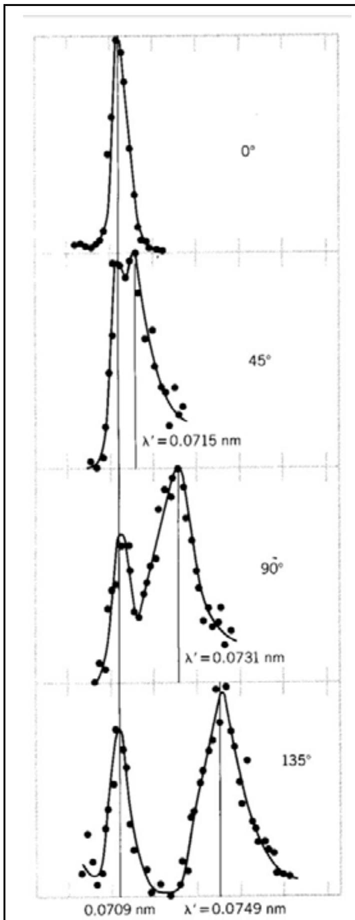


FIGURE 3.18 Compton's original results for X-ray scattering.

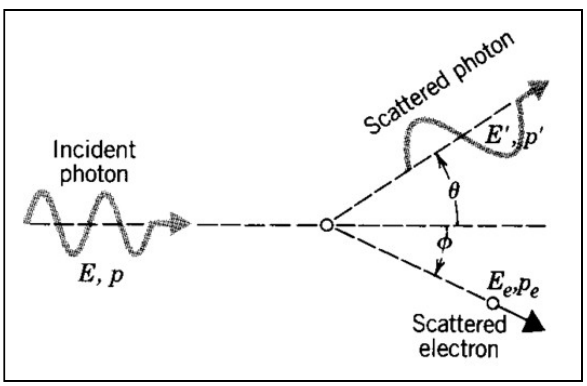
Experimental result shows that, although initially the incident beam consist only a single well-defined wavelength (λ) the scattered x-rays have intensity peaks at two wavelength (λ' in addition), where $\lambda' > \lambda$.

Unexplained by classical wave theory for radiation

Compton (and independently by Debye) explain this in terms of collision between collections of (particle-like) photon, each with energy $E = h\nu$, with the *free* electrons in the target graphite (imagine billard balls collision)

The `recoil' photons (scattered radiation) must have less energy because of energy transferred to the free electron when being scattered.

Relativistic dynamics of a photon scattering off a free but stationary electron



Conservation of momentum in parallel (i.e. x-direction)(relative to the direction of incident photon) and in orthogonal direction (y-direction):

y-direction: $p'\sin\theta = p_e\sin\phi$

(PY)

x-direction: $p_e\cos\phi = p - p'\cos\theta$

(PX)

Conservation of total relativistic energy:

$$E + m_ec^2 = E' + E_e$$

(RE)

algebra: $(PY)^2 + (PX)^2$, then substitute into $(RE)^2$ to eliminate E_e and p_e:

$$(E + m_ec^2 - E')^2 = c^2(p^2 - 2pp'\cos\theta + p'^2) + m_ec^2$$

which reduces to

$$\Delta\lambda \equiv \lambda' - \lambda = (h/m_ec)(1 - \cos\theta)$$

(DL)

$\lambda_c = h/m_ec = 0.0243$ Angstrom, is the Compton wavelength (for electron)

It is IMPORTANT to note that the wavelength of the x-ray used in the scattering is comparable to the Compton wavelength of the electron

Notice that $\Delta\lambda$ depend on θ only, not on the incident wavelength, λ.

For $\theta = 0^0$ → "grazing" collision ⇒ $\Delta\lambda = 0$

For $\theta = 180^0$ → "head on" collision ⇒ $\Delta\lambda = 2\lambda_c = 0.0243\overset{o}{A}$ (photon being reversed in direction)

The experimental result in Fig. 3.18 is perfectly explained by Eq.(DL)

Example

X-rays of wavelength 0.2400 nm are Compton scattered and the scattered beam is observed at an angle of 60 degree relative to the incident beam. Find (a) the wave length of the scattered x-rays, (b) the energy of the scattered x-ray photons, (c) the kinetic energy of the scattered electrons, and (d) the direction of travel of the scattered electrons.

(a) $\lambda' = \lambda + \lambda_c(1 - \cos\theta) = 0.2412$ nm

(b) $E' = hc/\lambda' = 1240$ eV•nm /0.2412 nm = 5141 eV

(c) *kinetic energy gained by the scattered electron = energy transferred by the incident photon during the scattering:* $K = hc/\lambda - hc/\lambda' = (5167 - 5141)\,\mathrm{eV} = 26\ \mathrm{eV}$

(d) $\tan\phi = p'\sin\theta\,/\,(p - p'\cos\theta) = E'\sin\theta\,/\,(E - E'\cos\theta) = 1.716$; hence $\phi = 59.7$ degree

Rayleigh scattering and Compton scattering

- There are two kinds of electrons in the graphite target: free electron and electron that is bounded to individual atom
- The sifted peak corresponds to scattering of stationary free electron with incident photon (for $K_e >>$ biding energy). This is just the Compton effect
- The unshifted peak corresponds to collision of photon with strongly bounded electron which is not free (incident photon energy << biding energy)
- Photon scattering off a bounded electron sees the mass of the whole atom instead of only that of a free electron. Hence the Compton shift, instead being given by Eq.(DL), the mass term is now give by $m_e \to M$ ($M >> m_e$) instead, such that $\Delta\lambda(M) << \Delta\lambda(m_e)$. Hence, the scattered λ' appears unmodified
- This is called Rayleigh scattering - explainable in classical theory
- In the limit $\lambda \to \infty$ (low incident photon energy), $\lambda >> \lambda_c$, Rayleigh scattering dominates (biding energy is large as seen by the incident photons)
- As $\lambda \to \lambda_c$ Compton scattering dominates

Note that Compton scattering occurs at energy scale which is much higher, ~ 0.1 MeV (typical energy of the x-ray photon, with wavelength ~ Compton wavelength of the electron. In comparison, photoelectricity occurs at a much lower energy scale, ~ eV.

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