The inverse of photoelectricity: X-rays

X-ray, discovered by Wilhelm Konrad Roentgen (1845-1923). He won the first Nobel prize in 1902. He refused to benefit financially from his work and died in poverty in the German inflation that followed the end of World War 1.

Photo Researchers, Inc./Omikron/Science Source

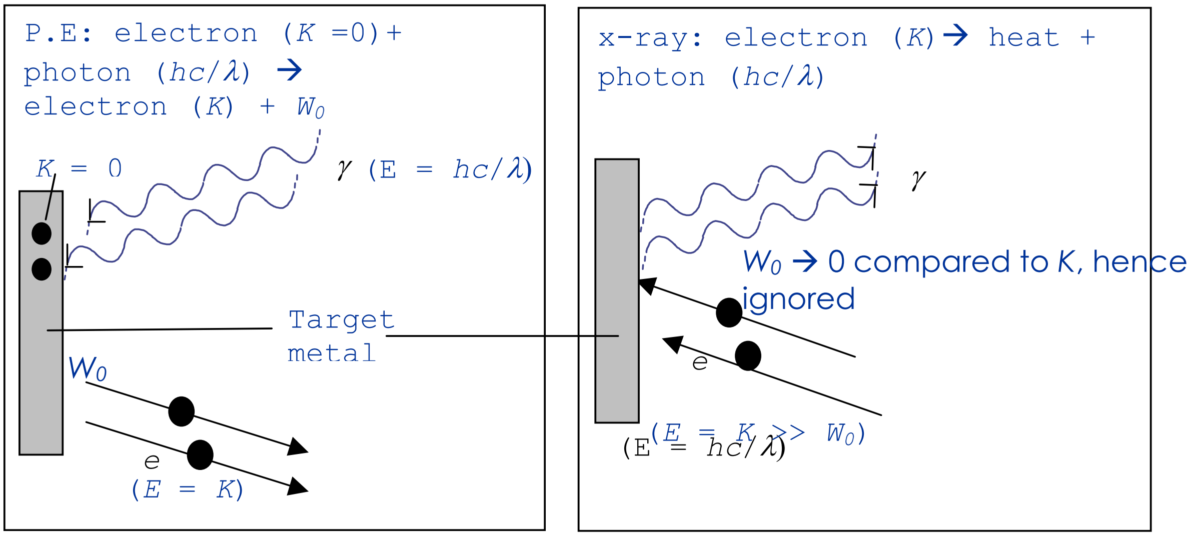
X-rays are simply EM radiation with very short wavelength, ~ 10 nm – 0.01 nm (hence energetic, according to $E = hc/\lambda$):

(i) travels in straight lines,
(ii) is unaffected by electric and magnetic fields
(iii) passes readily through opaque materials – highly penetrative
(iv) causes phosphorescent substances to glow
(v) exposes photographic plates

X-rays, consist of high-energy photons, is discovered accidentally when he observed that barium platinocyanide (a kind of salt) glowed when he switched on a nearby cathode-ray tube that was entirely covered with black cardboard. X-ray is highly penetrative.

X-rays (call 'x-ray' because its nature was unknown at that time).

In photoelectricity, energy is transferred from photons to kinetic energy of electrons. The inverse of this process produces x-rays.



X-ray production in experiments

- X-rays is produced when electrons, accelerated by an electric field in a vacuum cathode-ray tube, are impacted on the glass end of the tube. Part or all of the kinetic energy of a moving electron is converted into a x-ray photon. Due to conversion of energy from the impacting electrons to x-ray photons is not efficient, the difference between input energy, K_e and the output x-ray energy E_γ becomes heat
- Hence the target materials have to be made from metal that can stand heat and must have high melting point (such as Tungsten and Molybenum). Experimentally,

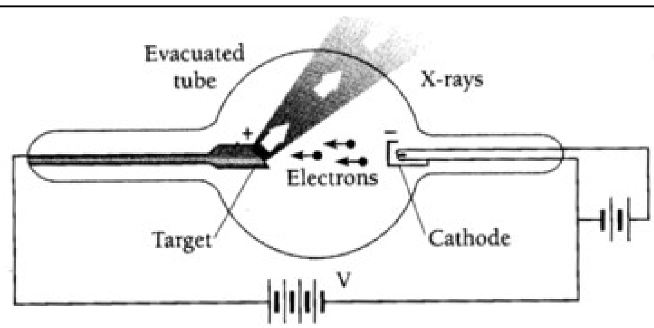
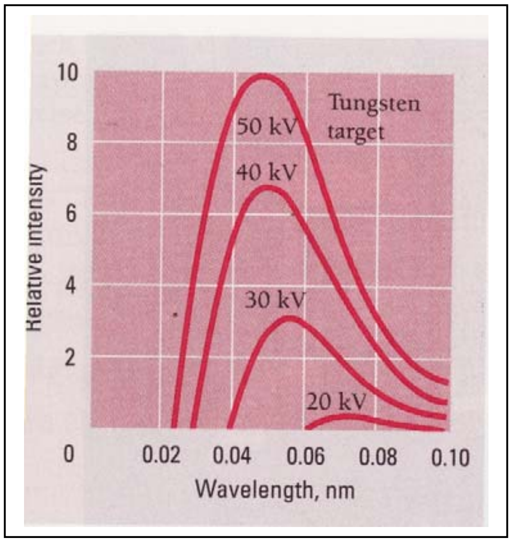


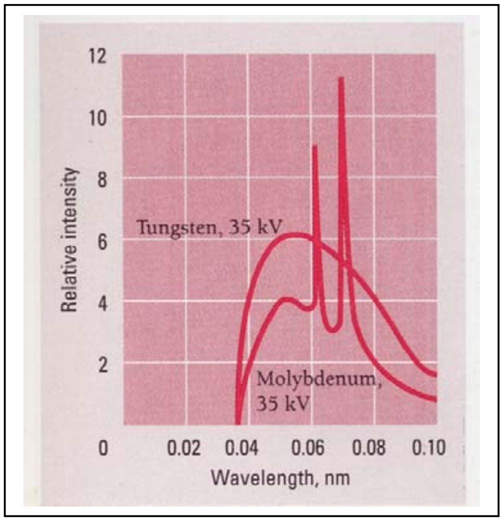
Figure 2.15 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.

- A cathode (the `pole' that emits negative charge) is heated by means of electric current to produce thermionic emission of the electrons from the target.
- A high potential difference *V* is maintained between the cathode and a metallic target.
- The thermionic electrons will get accelerated toward the latter.

The observed x-ray spectrum has a few important features:

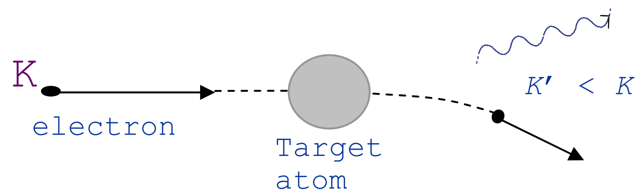


1) The spectrum is continuous
2) The existence of a minimum wavelength $\lambda_{\min}$ for a given *V*, below which no x-ray is observed
3) Increasing *V* decreases $\lambda_{\min}$
4) At a particular *V*, $\lambda_{\min}$ is the same for different target materials. Experimentally one finds that $\lambda_{\min}$ is inversely proportional to *V*, $\lambda_{\min} = \frac{1.24\times 10^{-6}}{V}$ Vm.



Classical explanation of feature (1) in the x-ray spectrum: Bremsstrahlung

Electromagnetic theory predicts that an accelerated electric charge will radiate EM waves. Hence, when a rapidly moving electron is brought to rest (either rapidly or gradually) radiation is produced. When the electrons strike the target they are slowed down and eventually come to rest because the make collisions with the atoms of the target. In such a collision, momentum is transferred to the atom, the electron slows down, and photon emitted. Radiation produced under these circumstances is given the German name bremsstrahlung ("braking radiation").



The energy of the photon emitted in the Bremsstrahlung is simply the difference between the initial and final kinetic energy of the electron:

$$h\nu = K - K'$$

The wavelength of the emitted photon is not uniquely determined as the K.E loss, $K - K'$ is not unique in one single collision. An electron usually makes many glancing collisions before brought to rest. Therefore the photons emitted will cover a continuous range of wavelength. Glancing collisions only leads to small loss of K.E., hence the wavelength of photon emitted in this case is very large (corresponds to small energy). This is the classical Bremsstrahlung explanation to feature (1) stated above.

Despite that classical Bremsstrahlung explains feature (1) well, it cannot explain the rest of the features. Features (2-4) can only be understood in the light of x-ray production being the inverse of photoelectricity, which assumes the quantum nature of photon.

Explanation of $\lambda_{\min}$ in the light of photon

In order to explain the existence of $\lambda_{\min}$ we must resort to the photon nature of light: The shortest wavelength of the emitted photon gains its energy, $E = hc/\lambda_{\min}$, from the maximal loss of the K.E. of an electron in a single collision. Hence the smallest wavelength of the emitted photon is given by which $K' = 0$ (loss all of its K.E):

$$\frac{hc}{\lambda_{\min}} = K = eV \quad , \text{ or } \quad \lambda_{\min} = \frac{hc}{eV} = \frac{1240\text{nm}\cdot\text{eV}}{eV} = \left(\frac{1.24\times10^{-6}}{V}\right)\text{m}\cdot\text{V}$$

(work function, W_0, is ignored as $W_0 << K$)

Plugging in some typical values, one finds that x-rays lie in the range of 0.01 to 10 nm.

Note that the work function W_0 is ignored due to the fact that they are only a few electronvolts, whereas the accelerating potentials that is used to produce x-ray in a x-ray vacuum tube, V, is in the range of 10,000 V. This explains why $\lambda_{\min}$ is the same for different target materials.

It is also important to note that the above picture is possible only if we view the x-ray to behave like a particle (i.e. photon). If it were `wave' the wavelength of the emitted x-ray, according to classical Bremstraahlung, would have been continuous instead of a `sharp' value, $\lambda_{\min}$.

Example

Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V.

Solution

$$\lambda_{\min} = \frac{1.24\times10^{-6}\,\text{V}\cdot\text{m}}{5.00\times10^{4}\,\text{V}} = 2.48\times10^{-11}\,\text{m} = 0.0248\text{ nm}$$

This wavelength corresponds to the frequency

$$\nu_{\max} = \frac{c}{\lambda_{\min}} = \frac{3\times10^{8}\,\text{m/s}}{2.48\times10^{-11}\,\text{m}} = 1.21\times10^{19}\,\text{Hz}$$

X-ray diffraction

X-ray wavelengths can be determined through Bragg's diffraction: x-ray is diffracted by the crystal planes that are of the order of the wavelength of the x-ray, ~ 0.1 nm.

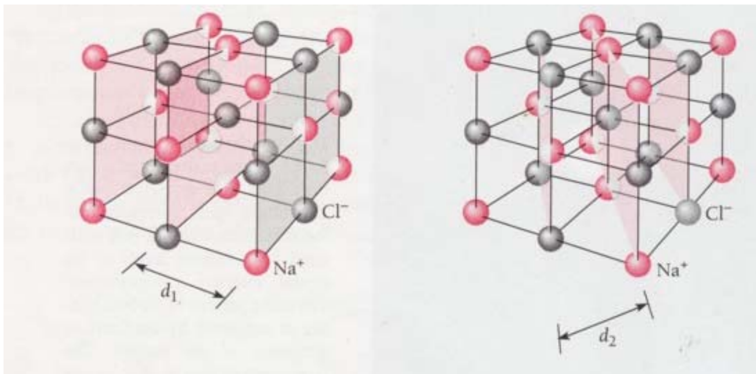
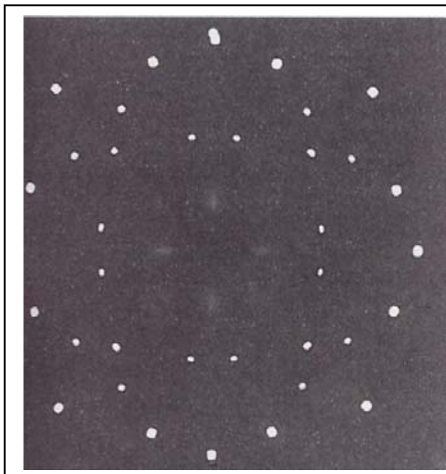


Figure 2.19 Two sets of Bragg planes in an NaCl crystal.

Consider an example of Bragg's diffraction with NaCl crystal by x-ray.



The bright spots correspond to the directions where x-rays scattered from various layers (different Braggs planes) in the crystal interfere constructively.

The conditions that must be fulfilled for radiation scattered by crystal atoms to undergo constructive interference can be determined from the following consideration.

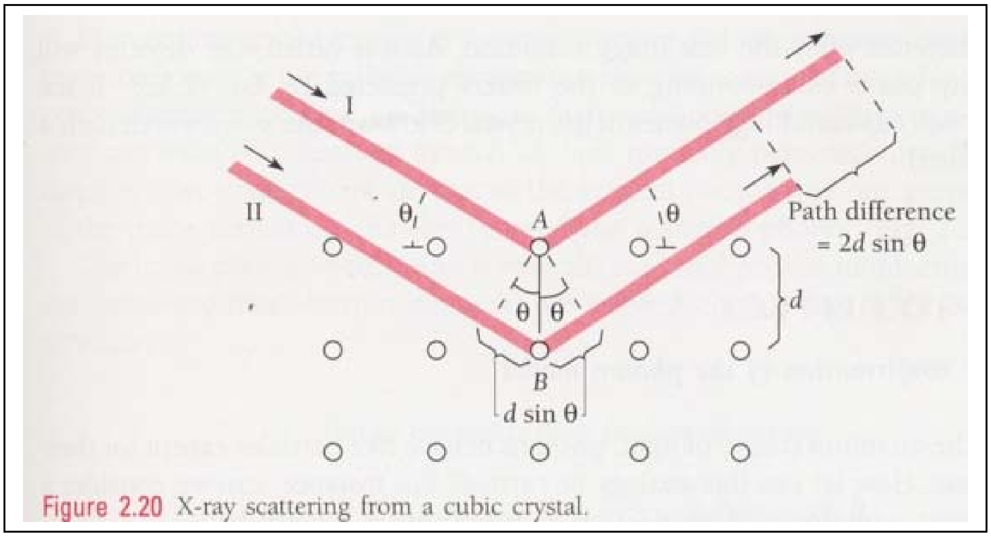


Figure 2.20 X-ray scattering from a cubic crystal.

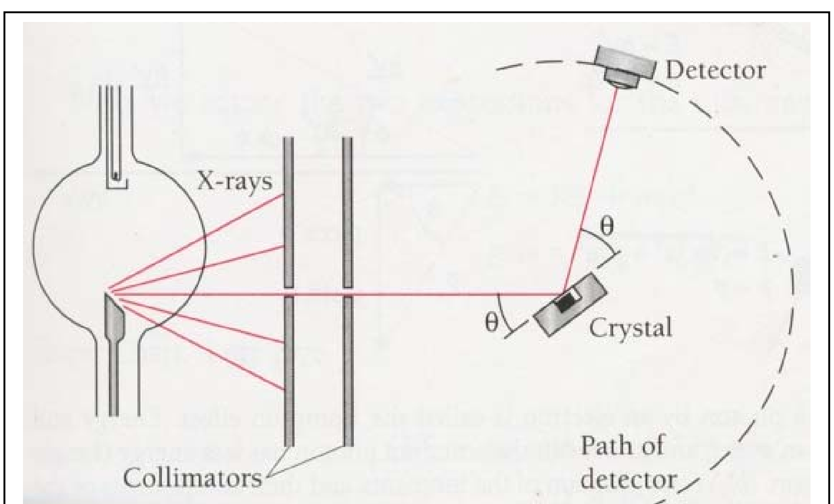
A x-ray with wavelength λ is incident upon a crystal at an angle θ with a family of Bragg planes whose spacing is d. The beam goes past atom A in the first plane and atom B in the next, and each of them scatters part of the beam in random directions. Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly λ, 2λ, 3λ and so on (beam I, II):

$$2d \sin \theta = n\lambda, \quad n = 1, 2, 3 \ldots$$

(Bragg's law for x-ray diffraction)

since ray II must travel the distance $2d \sin \theta$ farther than ray I.

The schematic design of an x-ray spectrometer based upon Bragg's analysis is shown in Fig. 2.2 1.



Example

A single crystal of table salt (NaCl) is irradiated with a beam of x-rays of unknown wavelength. The first Bragg's reflection is observed at an angle of 26.3 degree. Given that the spacing between the interatomic planes in the NaCl crystal to be 0.282 nm, what is the wavelength of the x-ray?

Solution

Solving Bragg's law for the $n = 1$ order,

$$\lambda = 2d \sin \theta = 2 \times 0.282 \text{ nm} \times \sin(26.3^\circ) = 0.25 \text{ nm}$$

Pair Production: Energy into matter

In photoelectric effect, a photon gives an electron all of its energy. In Compton effect, a photon give parts of its energy to an electron.

A photon can also materialize into an electron and a positron, which is a positively charged electron. In this process, called pair production, electromagnetic energy is converted into matter.

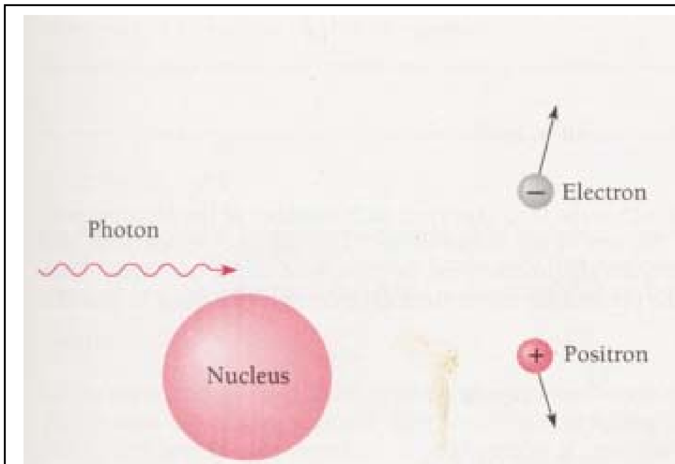


Figure 2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

In the pair production, charge conservation, total linear momentum all observed and the total relativistic energy must also be observed.

Due to kinematical consideration (i.e. energy and linear momentum conservations) pair production cannot occur in empty space. Pair production can only occur in the presence of a nucleus. Without the presence of a massive nucleus (that acts to recoil some of the the photon's linear momentum) in the proximity, linear momentum and energy could not be simultaneously conserved. We skip the kinematical details here.

The rest energy m_0c^2 of an electron or positron is 0.51 MeV, hence pair production requires a photon energy of at least 1.02 MeV.

Any additional photon energy becomes kinetic energy of the electron and positron, *K*.

$$\frac{hc}{\lambda} = 2m_0c^2 + K$$

Hence, to pair-produce an electron-positron pair, the minimum energy of a photon is $\lambda_{\min} = \frac{hc}{2m_0c^2} = 1.2 \times 10^{12}$ m. pm. Electromagnetic waves with such wavelengths are extremely energetic. These are called gamma rays and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.

Pair-annihilation

The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges. Both particles vanish simultaneously, with the lost masses becoming energies in the form of two gamma-ray photons:

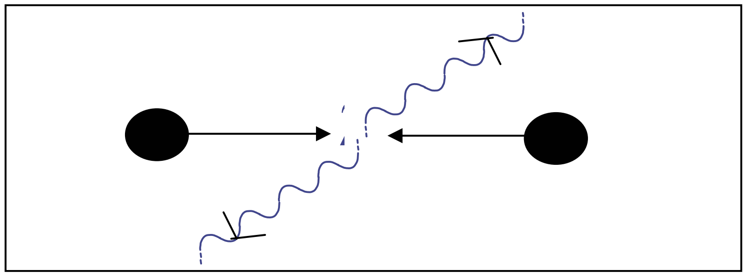
$$e^+ + e^- \rightarrow \gamma + \gamma$$

The energy of a gamma photon can be deduced from conservation of energy-momentum:

$$2m_0c^2 + K = \frac{hc}{\lambda} + \frac{hc}{\lambda}$$

The total rest mass of the positron and electron is equivalent to $2m_0c^2$ = 1.02 MeV, with *K* the total kinetic energy of the electron-positron pair before annihilation. Each resultant gamma ray photon has an energy $h\nu$ of 0.51 MeV plus half the kinetic energy of the particles

The gamma photons are always emitted in a back-to-back manner:



so both energy and linear momentum are always conserved. No nucleus or other particle is needed for this pair annihilation to take place.

Wave-particle duality

NOTE THAT:

- "Quantum nature of light" refers to the particle attribute of light

- "Quantum nature of particle" refers to the wave attribute of a particle

Light (so as other microscopic ``particle'' such as electron, see later chapters) is said to display "wave-particle duality" - it behave like wave in one experiment but as particle in others (c.f. a person with schizophrenia)

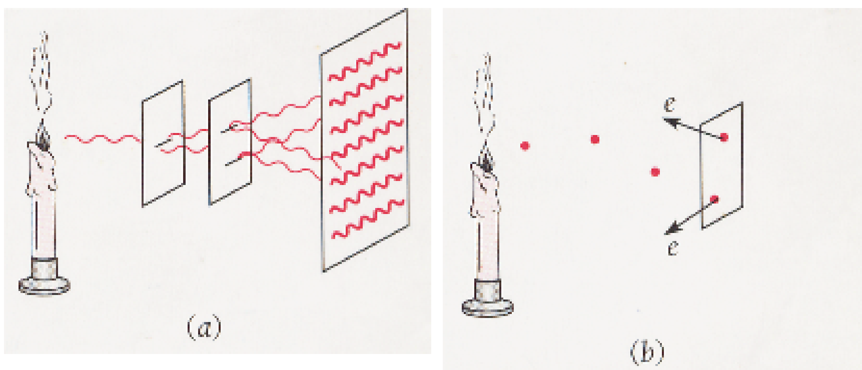


Figure 2.14 (*a*) The wave theory of light explains diffraction and interference, which the quantum theory cannot account for. (*b*) The quantum theory explains the photoelectric effect, which the wave theory cannot account

Wave-particle duality is essentially the manifestation of the quantum nature of things. This is an very weird picture quite contradicts to our conventional assumption with is deeply rooted on classical physics or intuitive notion on things.

Whether light displays wave or particle nature depends on the object it is interacting with, and also on the experimental set-up to observe it. If an experiment is set-up to observe the wave nature (such as in interference or diffraction experiment), it displays wave nature; if the experimental set-up has a **scale** that is corresponding to the quantum nature of radiation, then light will displays particle behaviour.

As an example of a ‘scale’ in a given experiment or a theory, let’s consider the Compton wavelength in Compton scattering.

Compton wavelength is the ***length scale*** which characterises the onset of quantum (corpuscular/particle) nature of light in its interaction with a particle. If the wavelength of light is much larger than the Compton wavelength of the particle it is interacting with (e.g. electron), light behaves like wave; If the wavelength of the radiation is comparable to the Compton wavelength of the interacting particle, light interacts like particle with the interacting particle

For microscopic particle (e.g. electron, proton, muon etc.) Compton wavelength also characterises the scale at which the quantum nature of particles starts to take over from classical physics (see later in the wave nature of particle)

Light is both wave and particle. The wave theory and the quantum theory complement of light each other.

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