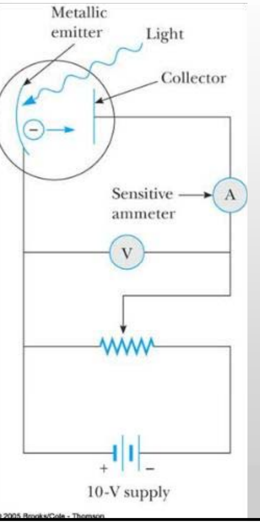
Photoelectricity

- Classically, light is treated as EM wave according to Maxwell equation
- However, in a few types of experiments, light behave in ways that is not consistent with the wave picture
- In these experiments, light behave like particle instead
- So, is light particle or wave? (recall that wave and particle are two mutually exclusive attributes of existence)
- This is a paradox that we will discuss in the rest of the course – wave particle duality

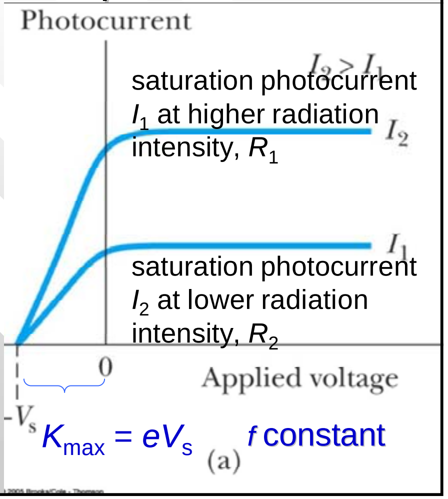
Photoelectric effect

- Photoelectrons are ejected from a metal surface when hit by radiation of sufficiently high frequency f (usually in the uv region)
- The photoelectrons are attracted to the collecting anode (positive) by potential difference applied on the anode and detected as electric current by the external circuits
- A negative voltage, relative to that of the emitter, can be applied to the collector.
- When this retarding voltage is sufficiently large the emitted electrons are repelled, and the current to the collector drops to zero (see later explanation).



Photocurrent *I* vs applied voltage at constant *f*

- No current flows for a retarding potential more negative than $-V_s$
- The photocurrent I saturates for potentials near or above zero
- Why does the I-v curve rises gradually from $-V_s$ towards more positive V before it flat off ?



Features of the experimental result

- When the external potential difference $V = 0$, the current is not zero because the photoelectrons carry some kinetic energy, K
- K range from 0 to a maximal value, K_{max}
- As V becomes more and more positive, there are more electrons attracted towards the anode within a given time interval. Hence the pthotocurrent, I, increases with V
- Saturation of I will be achieved when all of the ejected electron are immediately attracted towards the anode once they are kicked out from the metal plates (from the curve this happens approximately when $V \approx 0$ or larger

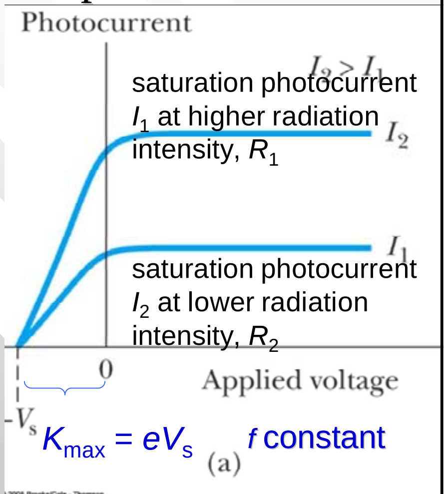
- On the other direction, when V becomes more negative, the photocurrent detected decreases in magnitude because the electrons are now moving against the potential
- K_{max} can be measured. It is given by eV_s, where V_s, is the value of $|V|$ when the current flowing in the external circuit = 0
- V_s is called the '**stopping potential**'
- When $V = -V_s$, e of the highest KE will be sufficiently retarded by the external electric potential such that they wont be able to reach the collector

$I_2 > I_1$ because more electrons are kicked out per unit time by radiation of larger intensity, R

- The photocurrent saturates at a larger value of I_2 when it is irradiated by higher radiation intensity R_2
- This is expected as larger R means energy are imparted at a higher rate on the metal surface

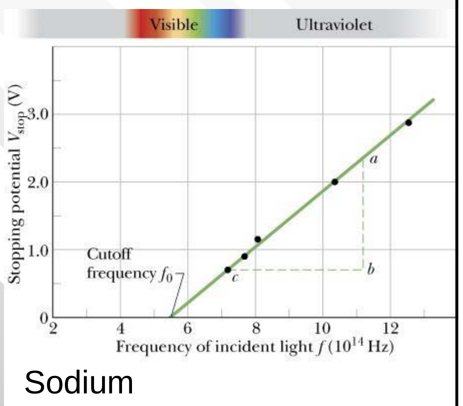
Stopping potential V_s is radiation intensity-independent

- Experimentalists observe that for a given type of surface:
- At constant frequency the maximal kinetic energy of the photoelectrons is measured to be a constant independent of the intensity of light.



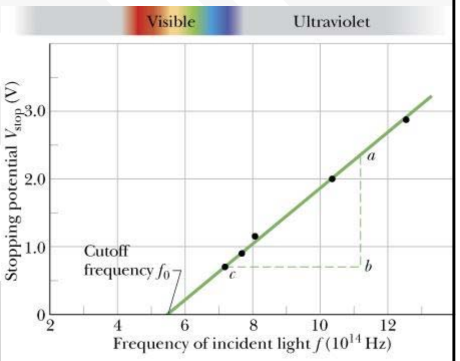
K_{max} of photoelectrons is frequency-dependent at constant radiation intensity

- One can also detect the stopping potential V_s for a given material at different frequency (at constant radiation intensity)
- K_{max} ($=eV_s$) is measured to linearly dependent on the radiation frequency $K_{max} \propto f$



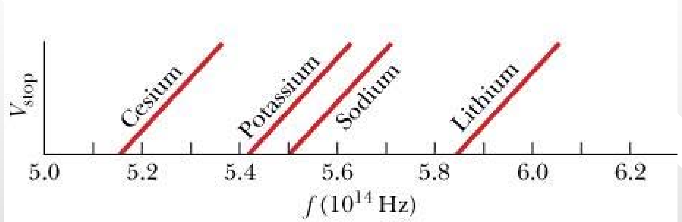
Cutoff frequency, f_0

- From the same graph one also found that there exist a **cut-off frequency**, f_0, below which no PE effect occurs no matter how intense is the radiation shined on the metal surface



Sodium

Different material have different cut-off frequency f_0



- For different material, the cut-off frequency is different

Classical physics can't explain PE

- The experimental results of PE pose difficulty to classical physicists as they cannot explain PE effect in terms of classical physics (Maxwell EM theory, thermodynamics, classical mechanics etc.)

Puzzle one

- If light were wave, the energy carried by the radiation will increases as the intensity of the monochromatic light increases
- Hence we would also expect K_{max} of the electron to increase as the intensity of radiation increases (because K.E. of the photoelectron must come from the energy of the radiation)
- YET THE OBSERVATION IS OTHERWISE.

Puzzle two

- Existence of a characteristic cut-off frequency, ν_0. (previously I use f_0)
- Wave theory predicts that photoelectric effect should occur for any frequency as long as the light is intense enough to give the energy to eject the photoelectrons.
- No cut-off frequency is predicted in classical physics.

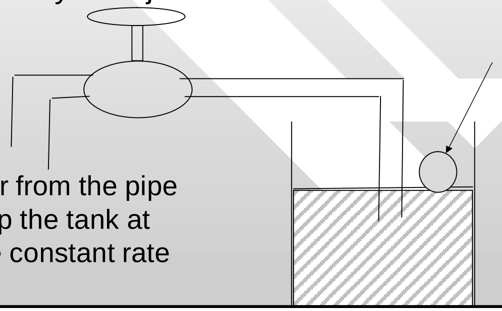
Puzzle three

- No detection time lag measured.
- Classical wave theory needs a time lag between the instance the light impinge on the surface with the instance the photoelectrons being ejected. Energy needs to be accumulated for the wave front, at a rate proportional to

  , $S = \frac{E_0}{2\mu_0 c}$

  before it has enough energy to eject photoelectrons.
- But, in the PE experiments, PE is almost immediate

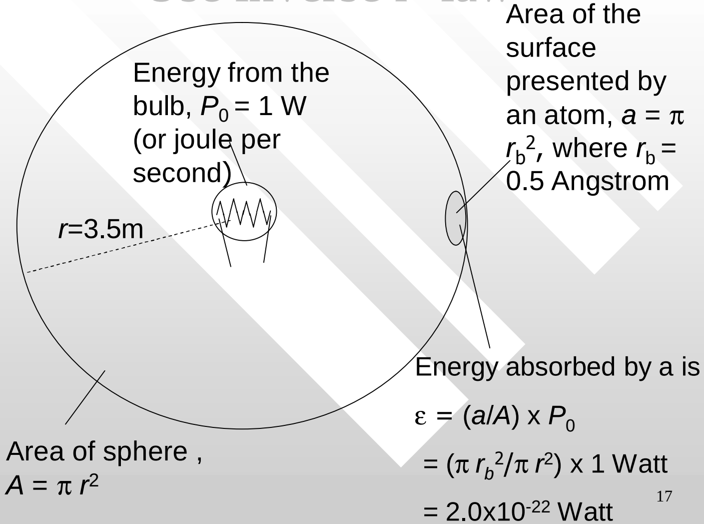
Cartoon analogy: in the wave picture, accumulating the energy required to eject an photoelectron from an atom is analogous to filling up a tank with water from a pipe until the tank is full. One must wait for certain length of time (time lag) before the tank can be filled up with water at a give rate. The total water filled is analogous to the total energy absorbed by electrons before they are ejected from the metal surface at



Wave theory and the time delay problem

- A potassium foil is placed at a distance r = 3.5 m from a light source whose output power P_0 is 1.0 W. How long would it take for the foil to soak up enough energy (=1.8 eV) from the beam to eject an electron? Assume that the ejected electron collected the energy from a circular area of the foil whose radius is 5.3 x 10^{-11} m

Use inverse r^2 law



Energy absorbed by a is

$\varepsilon = (a/A) \times P_0$

$= (\pi r_b^2/\pi r^2) \times 1$ Watt

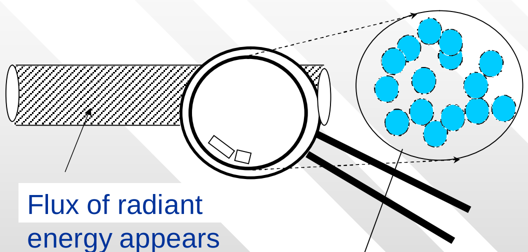
$= 2.0\times10^{-22}$ Watt

- Time taken for a to absorb 1.8 eV is simply 1.8 x 1.6 x 10^{-19} J / ε = 5000 s = 1.4 h!!!
- In PE, the photoelectrons are ejected almost immediately but not 1.4 hour later
- This shows that the wave model used to calculate the time lag in this example fails to account for the almost instantaneous ejection of photoelectron in the PE experiment

Einstein's quantum theory of the photoelectricity (1905)

- A Noble-prize winning theory (1905)
- To explain PE, Einstein postulates that the radiant energy of light is quantized into concentrated bundle. The discrete entity that carries the energy of the radiant energy is called photon
- Or, in quantum physics jargon, we say "photon is the quantum of light"
- Wave behaviour of light is a result of collective behaviour of very large numbers of photons

Photon is granular



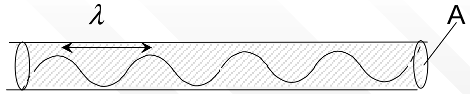
Flux of radiant energy appears like a continuum at macroscopic scale of intensity

Granularity of light (in terms of photon) becomes manifest when magnified

Wave and particle carries energy differently

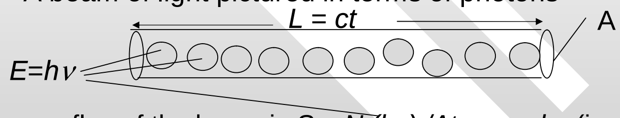
- The way how photon carries energy is in in contrast to the way wave carries energy.
- For wave the radiant energy is continuously distributed over a region in space and not in separate bundles
- (always recall the analogy of water in a hose and a stream of ping pong ball to help visualisation)

A beam of light if pictured as monochromatic wave (λ, ν)



Energy flux of the beam is $S = \frac{E_0}{2\mu_0 c}$ (in unit of joule per unit time per unit area), analogous to fluid in a host

A beam of light pictured in terms of photons



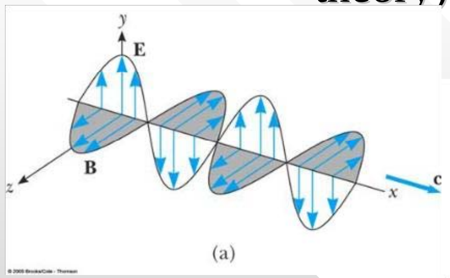
Energy flux of the beam is $S = N(h\nu)/At = n_0\, ch\nu$ (in unit of joule per unit time per unit area). N is obtained by 'counting' the total number of photons in the beam volume, $N = n_0 V = n_0$ x ($A\ ct$), where n_0 is the photon number density of the radiation (in unit of number per unit volume)

Einstein's 1st postulate

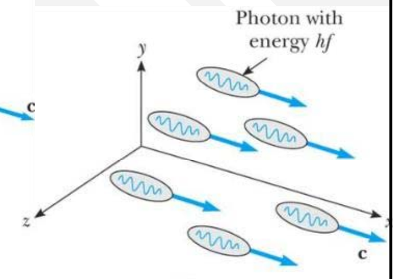
1. The energy of a single photon is $E = h\nu$. h is a proportional constant, called the Planck constant, that is to be determined experimentally.

- With this assumption, a photon will have a momentum given by $p = E/c = h/\lambda$.
- This relation is obtained from SR relationship $E^2 = p^2c^2 + (m_0c^2)^2$, for which the mass of a photon is zero.
- Note that in classical physics momentum is intrinsically a particle attribute not defined for wave. By picturing light as particle (photon), the definition of momentum for radiation now becomes feasible

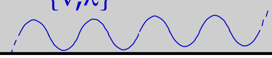
Light as photon (in Einstein theory) instead of wave (in Classical EM theory)



(a)



(b)



Example

- (a) What are the energy and momentum of a photon of red light of wavelength 650nm?
- (b) What is the wavelength of a photon of energy 2.40 eV?
- In atomic scale we usually express energy in eV, momentum in unit of eV/*c*, length in nm; the combination of constants, *hc*, is conveniently expressed in
- 1 eV = 1.6×10^{-19} J
- $hc = (6.62\times10^{-34}\ \mathrm{Js})\cdot(3\times10^{8}\ \mathrm{m/s})$
  $= [6.62\times10^{-34}\cdot(1.6\times10^{-19})^{-1}\mathrm{eV\cdot s}]\cdot(3\times10^{8}\ \mathrm{m/s})$
  $= 1.24\mathrm{eV}\cdot10^{-6}\mathrm{m} = 1240\mathrm{eV\cdot nm}$
- 1 eV/*c* = $(1.6\times10^{-19})\mathrm{J}/(3\times10^{8}\ \mathrm{m/s}) = 5.3\times10^{-28}$ Ns

solution

- (a) $E = hc/\lambda$
  $= 1240\ \mathrm{eV\cdot nm}/650\ \mathrm{nm}$
  $= 1.91\ \mathrm{eV}\ (= 3.1\times10^{-19}\mathrm{J})$
- (b) $p = E/c = 1.91\ \mathrm{eV}/c\ (= 1\times10^{-27}\ \mathrm{Ns})$
- (c) $\lambda = hc/E$
  $= 1240\mathrm{eV\cdot nm}/2.40\ \mathrm{eV}$
  $= 517$ nm

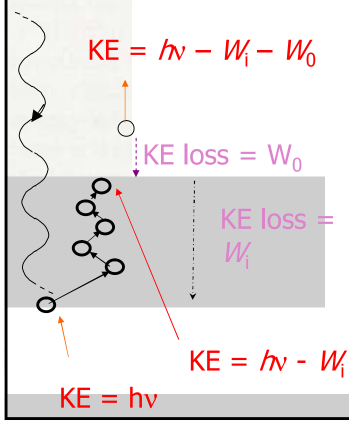
Einstein's 2nd postulate

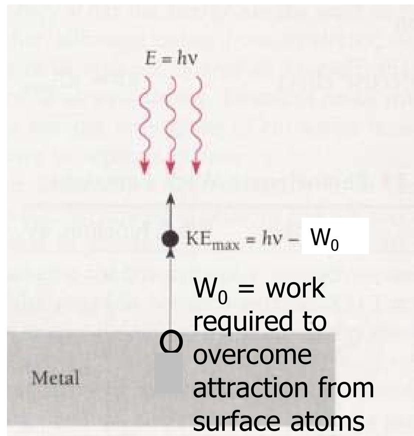
- In PE one photon is completely absorbed by one atom in the photocathode.
- Upon the absorption, one electron is 'kicked out' by the absorbent atom.
- The kinetic energy for the ejected electron is
  $K = h\nu - W$
- *W* is the worked required to
- (i) cater for losses of kinetic energy due to internal collision of the electrons (W_i),
- (ii) overcome the attraction from the atoms in the surface (W_0)
- When no internal kinetic energy loss (happens to electrons just below the surface which suffers minimal loss in internal collisions), *K* is maximum:
- $K_{max} = h\nu - W_0$

In general,

$K = h\nu - W$, where

$W = W_0 + W_i$





Einstein theory manage to solve the three unexplained features:

- First feature:
- In Einstein's theory of PE, $K_{max} = h\nu - W_0$
- Both $h\nu$ and W_0 do not depend on the radiation intensity
- Hence $K_{\max}$ is independent of irradiation intensity
- Doubling the intensity of light wont change K_{max} because only depend on the energy hν of individual photons and W_0
- W_0 is the intrinsic property of a given metal surface

Second feature explained

- **The cut-off frequency is explained**

- Recall that in Einstein assumption, a photon is completely absorbed by one atom to kick out one electron.
- Hence each absorption of photon by the atom transfers a discrete amount of energy by $h\nu$ only.
- If $h\nu$ is not enough to provide sufficient energy to overcome the required work function, W_0, no photoelectrons would be ejected from the metal surface and be detected as photocurrent

Cut-off frequency is related to work function of metal surface $W_0 = h\nu_0$

- A photon having the cut-off frequency ν_0 has just enough energy to eject the photoelectron and none extra to appear as kinetic energy.
- Photon of energy less than $h\nu_0$ has not sufficient energy to kick out any electron
- Approximately, electrons that are eject at the cut-off frequency will not leave the surface.
- This amount to saying that the have got zero kinetic energy: $K_{\max} = 0$
- Hence, from $K_{\max} = h\nu - W_0$, we find that the cut-off frequency and the work function is simply related by
  - $W_0 = h\nu_0$
- Measurement of the cut-off frequency tell us what the work function is for a given metal

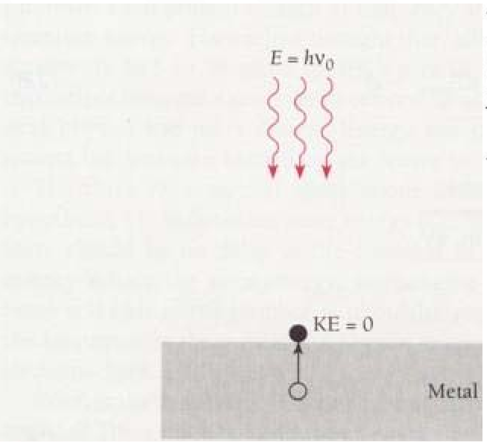


Table 3.1 Some Photoelectric Work Functions $W_0 = h\nu_0$

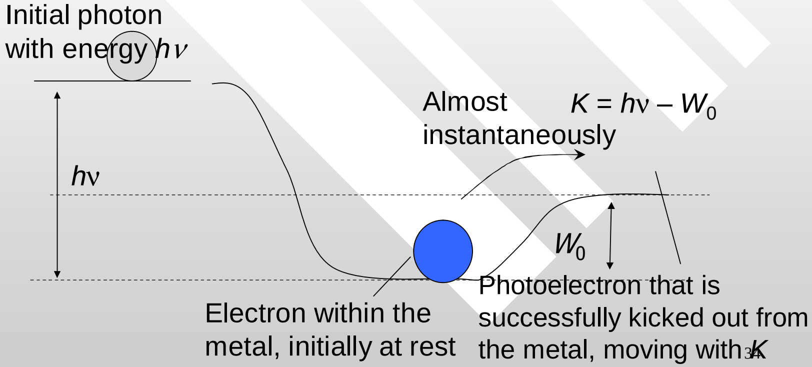
Material	W(eV
Na	2.28
Al	4.08
Co	3.90
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

Third feature explained

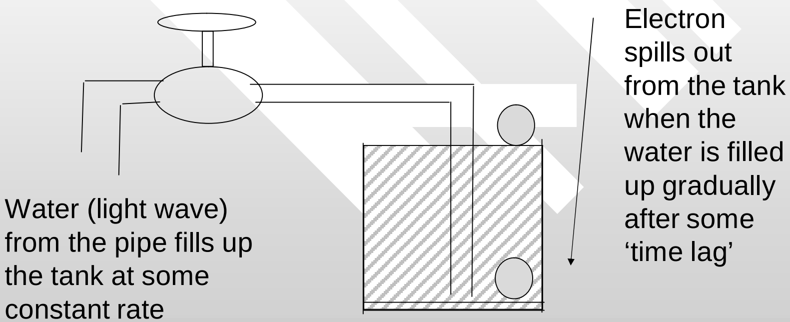
- The required energy to eject photoelectrons is supplied in concentrated bundles of photons, not spread uniformly over a large area in the wave front.
- Any photon absorbed by the atoms in the target shall eject photoelectron immediately.
- Absorption of photon is a discrete process at quantum time scale (almost ‘instantaneously’): it either got absorbed by the atoms, or otherwise.
- Hence no time lag is expected in this picture

A simple way to picture photoelectricity in terms of particle-particle collision:

Energy of photon is transferred during the instantaneous collision with the electron. The electron will either get kicked up against the barrier threshold of W_0 almost instantaneously, or fall back to the bottom of the valley if $h\nu$ is less than W_0



Compare the particle-particle collision model with the water-filling-tank model:

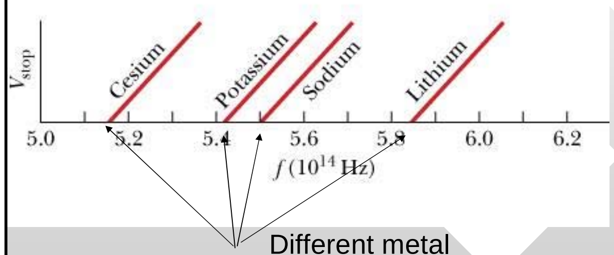


Experimental determination of Planck constant from PE

- Experiment can measure eV_s ($= K_{max}$) for a given metallic surface (e.g. sodium) at different frequency of impinging radiation
- We know that the work function and the stopping potential of a given metal is given by
- $eV_s = h\nu - W_0$

In experiment, we can measure the slope in the graph of V_s verses frequency ν for different metal surfaces. It gives a universal value of h/e = 4.1x10^{-15} Vs. Hence, h = 6.626 x 10^{-34} Js

$$V_s = (h/e)\nu - \nu_0$$



Different metal surfaces have different V_0

PYQ 2.16, Final Exam 2003/04

- Planck constant
- (i) is a universal constant
- (ii) is the same for all metals
- (iii) is different for different metals
- (iv) characterises the quantum scale
- **A.** I,IV **B.** I,II, IV **C.** I, III,IV
- **D.** I, III **E.** II,III
- **ANS: B, Machlup, Review question 8, pg. 496, modified**

PYQ 4(a,b) Final Exam 2003/04

- (a) Lithium, beryllium and mercury have work functions of 2.3 eV, 3.9 eV and 4.5 eV, respectively. If a 400-nm light is incident on each of these metals, determine
- (i) which metals exhibit the photoelectric effect, and
- (ii) the maximum kinetic energy for the photoelectron in each case (in eV)

Solution for Q3a

- The energy of a 400 nm photon is $E = hc/\lambda$ = **3.11 eV**
- The effect will occur only in **lithium***
- **Q3a(ii)**
- For lithium, $K_{max} = h\nu - W_0$

  = 3.11 eV – 2.30 eV

  = **0.81 eV**

*marks are deducted for calculating "K_{max}" for beryllium and mercury which is meaningless

PYQ 4(a,b) Final Exam 2003/04

- (b) Molybdenum has a work function of 4.2 eV.
- (i) Find the cut-off wavelength (in nm) and threshold frequency for the photoelectric effect.
- (ii) Calculate the stopping potential if the incident radiation has a wavelength of 180 nm.

Solution for Q4b

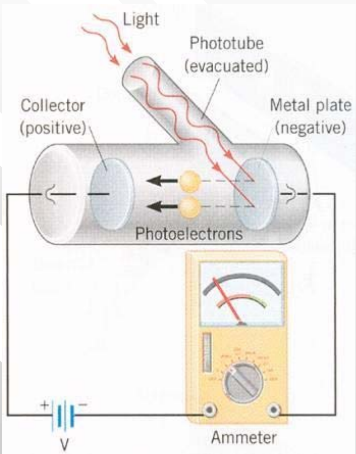
- **Q3a(ii)**
- Known $h\nu_{cutoff} = W_0$
- Cut-off wavelength = $\lambda_{cutoff} = c/\nu_{cutoff}$

  $= hc/W_0$ = 1240 nm eV / 4.2 eV = **295 nm**
- Cut-off frequency (or threshold frequency), ν_{cutoff}

  $= c / \lambda_{cutoff}$ = 1.01 x 1015 Hz
- **Q3b(ii)**
- Stopping potential $V_{stop} = (hc/\lambda - W_0) / e$ = (1240 nm·eV/180 nm – 4.2 eV)/e = **2. 7 V**

Example (read it yourself)

- Light of wavelength 400 nm is incident upon lithium (W_0 = 2.9 eV). Calculate
- (a) the photon energy and
- (b) the stopping potential, V_s
- (c) What frequency of light is needed to produce electrons of kinetic energy 3 eV from illumination of lithium?



Solution:

- (a) $E = h\nu = hc/\lambda$ = 1240eV·nm/400 nm = 3.1 eV
- (b) The stopping potential x e = Max Kinetic energy of the photon
- => $eV_s = K_{max} = h\nu - W_0$ = (3.1 - 2.9) eV
- Hence, V_s = 0.2 V
- i.e. a retarding potential of 0.2 V will stop all photoelectrons
- (c) $h\nu = K_{max} + W_0$ = 3 eV + 2.9 eV = 5.9 eV. Hence the frequency of the photon is

  $\nu = 5.9 \times (1.6 \times 10^{-19}\ \text{J}) / 6.63 \times 10^{-34}$ Js

  $= 1.42 \times 10^{15}$ Hz

PYQ, 1.12 KSCP 2003/04

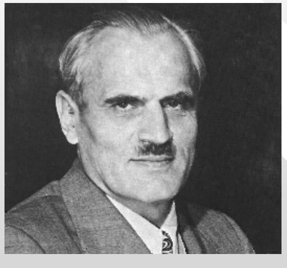
Which of the following statement(s) is (are) true?

- **I** The energy of the quantum of light is proportional to the frequency of the wave model of light
- **II** In photoelectricity, the photoelectrons has as much energy as the quantum of light which causes it to be ejected
- **III** In photoelectricity, no time delay in the emission of photoelectrons would be expected in the quantum theory
- **A. II, III** **B. I, III** **C. I, II, III** **D. I ONLY**
- **E.** Non of the above
- **Ans: B**
- Murugeshan, S. Chand & Company, New Delhi, pg. 136, Q28 (for I), Q29, Q30 (for II,III)

To summerise: In photoelectricity (PE), light behaves like particle rather than like wave.

Compton effect

- Another experiment revealing the particle nature of X-ray (radiation, with wavelength ~ 10^{-10} nm)

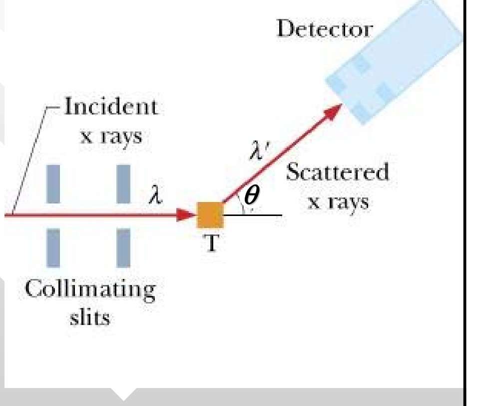


Compton, Arthur Holly (1892-1962), American physicist and Nobel laureate whose studies of X rays led to his discovery in 1922 of the so-called Compton effect.

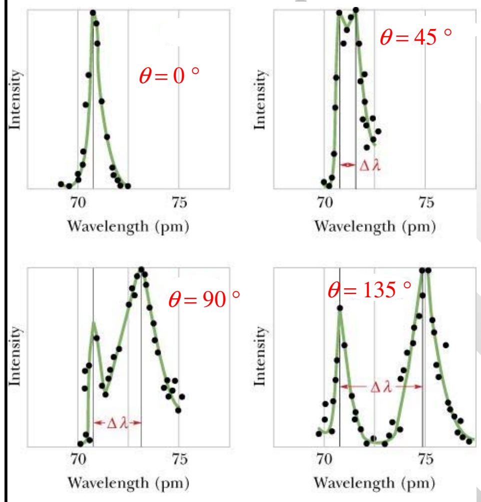
The Compton effect is the change in wavelength of high energy electromagnetic radiation when it scatters off electrons. The discovery of the Compton effect confirmed that electromagnetic radiation has both wave and particle properties, a central principle of quantum theory.

Compton's experimental setup

- A beam of x rays of wavelength 71.1 pm is directed onto a carbon target T. The x rays scattered from the target are observed at various angle θ to the direction of the incident beam. The detector measures both the intensity of the scattered x rays and their wavelength



Experimental data



Although initially the incident beam consists of only a single well-defined wavelength (λ) the scattered x-rays at a given angle θ have intensity peaks at two wavelength (λ' in addition), where $\lambda' > \lambda$

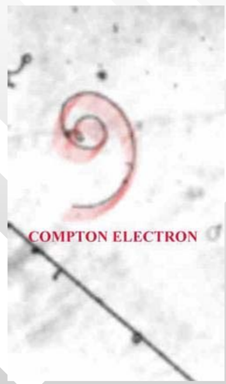
Compton shouldn't shift, according to classical wave theory of light

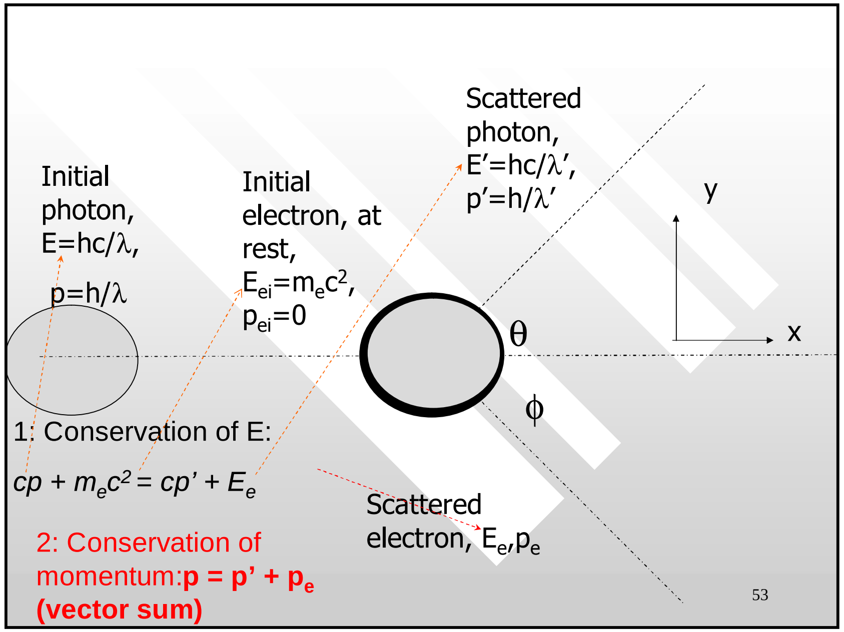
- **Unexplained by classical wave theory for radiation**
- **No shift of wavelength is predicted in wave theory of light**

Modelling Compton shift as "particle-particle" collision

- Compton (and independently by Debye) explain this in terms of collision between collections of (particle-like) photon, each with energy $E = h\nu$, with the *free* electrons in the target graphite (imagine billard balls collision)

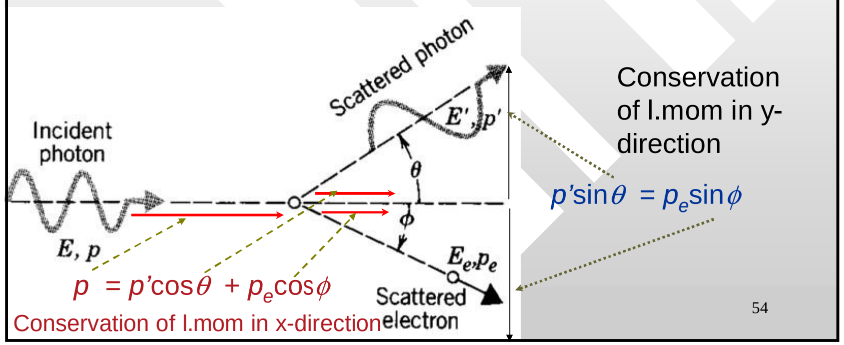
- ***Part of a bubble chamber picture (Fermilab'15 foot Bubble Chamber', found at the University of Birmingham). An electron was knocked out of an atom by a high energy photon.***





Conservation of momentum in 2-D

- **p** = **p′** + $\mathbf{p_e}$ (vector sum) actually comprised of two equation for both conservation of momentum in x- and y- directions



Some algebra…

Mom conservation in y: $p'\sin\theta = p_e\sin\phi$ (PY)

Mom conservation in x: $p - p'\cos\theta = p_e\cos\phi$ (PX)

Conservation of total relativistic energy:

$cp + m_ec^2 = cp' + E_e$ (RE)

$(PY)^2 + (PX)^2$, substitute into $(RE)^2$ to eliminate ϕ, p_e and E_e (and using $E_e^2 = c^2p_e^2 + m_e^2c^4$):

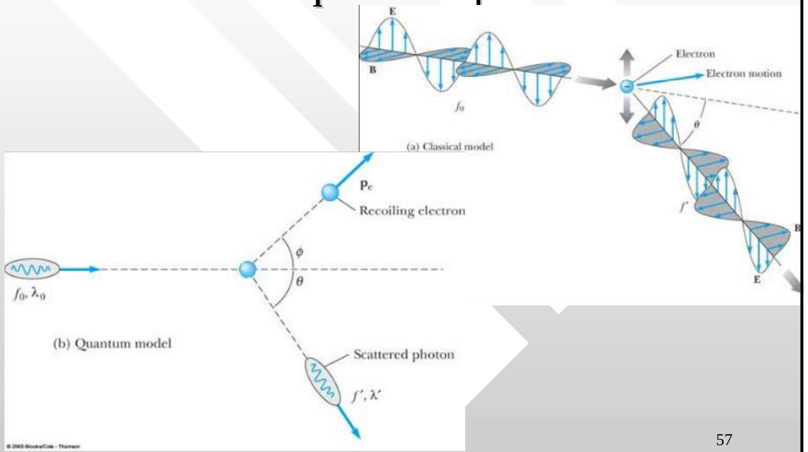
$$\Delta\lambda \equiv \lambda' - \lambda = (h/m_ec)(1 - \cos\theta)$$

Compton wavelength

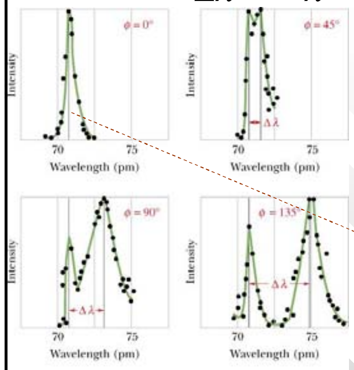
$\lambda_e = h/m_ec$ = 0.0243 Angstrom, is the Compton wavelength (for electron)

- Note that the wavelength of the x-ray used in the scattering is of the similar length scale to the Compton wavelength of electron
- The Compton scattering experiment can now be perfectly explained by the Compton shift relationship
  $\Delta\lambda \equiv \lambda' - \lambda = \lambda_e(1 - \cos\theta)$
  as a function of the photon scattered angle
- Be reminded that the relationship is derived by assuming light behave like particle (photon)

X-ray scattering from an electron (Compton scattering): classical versus quantum picture



$$\Delta\lambda \equiv \lambda' - \lambda = (h/m_e c)(1 - \cos\theta)$$

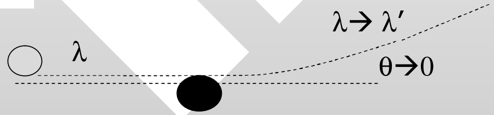


λ'=0.1795 nm

Notice that $\Delta\lambda$ depend on θ only, not on the incident wavelength, λ..

Consider some limiting behaviour of the Compton shift:

For $\theta = 0^0 \rightarrow$ "grazing" collision => $\Delta\lambda = 0$



For $\theta \rightarrow 180^0$ "head-on" collision => $\Delta\lambda = \Delta\lambda_{max}$

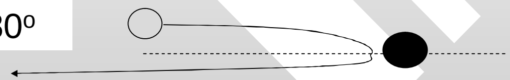
$\theta \rightarrow 180^0$ photon being reversed in direction

$\Delta\lambda_{max} = \lambda_{max}' - \lambda = (h/m_e c)(1 - \cos 180°)$

- $= 2\lambda_c = 2(\ 0.00243\text{nm})$

initially λ

θ=180°



After collision

$\lambda'_{max} = \lambda + \Delta\lambda_{max}$

PYQ 2.2 Final Exam 2003/04

Suppose that a beam of 0.2-MeV photon is scattered by the electrons in a carbon target. What is the wavelength of those photon scattered through an angle of 90°?

A. 0.00620 nm

B. 0.00863 nm

C. 0.01106 nm

D. 0.00243 nm

E. Non of the above

Solution

First calculate the wavelength of a 0.2 MeV photon:

$E = hc/\lambda = 1240$ eV·nm/λ = 0.2 MeV

λ =1240 nm / 0.2 x 10^6 = 0.062 nm

From Compton scattering formula, the shift is

$\Delta\lambda = \lambda' - \lambda = \lambda_c (1 - \cos 90°) = \lambda_c$

Hence, the final wavelength is simply

$\lambda' = \Delta\lambda + \lambda = \lambda_c + \lambda =$ 0.00243nm + 0.062 nm = 0.00863 nm

ANS: B, Schaum's 3000 solved problems, Q38.31, pg. 712

Example

- X-rays of wavelength 0.2400 nm are Compton scattered and the scattered beam is observed at an angle of 60 degree relative to the incident beam.
- Find (a) the wave length of the scattered x-rays, (b) the energy of the scattered x-ray photons, (c) the kinetic energy of the scattered electrons, and (d) the direction of travel of the scattered electrons

solution

$\lambda' = \lambda + \lambda_c (1 - \cos\theta)$

= 0.2400nm+0.00243nm(1–cos60°)

= 0.2412 nm

$E' = hc/\lambda'$

= 1240 eV·nm /0.2412 nm

= 5141 eV

kinetic energy gained by the scattered electron = energy transferred by the incident photon during the scattering:

$K = hc/\lambda - hc/\lambda'$ =(5167–5141)eV = 26 eV

Note that we ignore SR effect here because K << rest mass of electron, m_e = 0.5 MeV

By conservation of momentum in the x- and y- direction:

$p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$; $p'_\gamma \sin\theta = p_e \sin\phi$;

$\tan\phi = p_e \sin\phi / p_e \cos\phi = (p'_\gamma \sin\theta)/ (p_\gamma - p'_\gamma \cos\theta)$

$=(E'_\gamma \sin\theta)/ (E_\gamma - E'_\gamma \cos\theta)$

$= (5141 \sin 60^0 / [5167–5141 (\cos 60^0] = 0.43 = 1.71$

Hence, ϕ = 59.7 degree

PYQ 3(c), Final exam 2003/04

- (c) A 0.0016-nm photon scatters from a free electron. For what scattering angle of the photon do the recoiling electron and the scattered photon have the same kinetic energy?
- **Serway solution manual 2, Q35, pg. 358**

Solution

- The energy of the incoming photon is
  $E_i = hc/\lambda$ = **0.775 MeV**
- Since the outgoing photon and the electron each have half of this energy in kinetic form,
- $E_f = hc/\lambda'$ = 0.775 MeV / 2 = 0.388 MeV and
  $\lambda' = hc/E_f$ = 1240 eV· nm / 0.388 MeV = 0.0032 nm
- The Compton shift is
  $\Delta\lambda = \lambda' - \lambda$ = (0.0032 – 0.0016) nm = 0.0016 nm
- By $\Delta\lambda = \lambda_c (1 - \cos\theta)$
- $= (h/m_e c)\ (1 - \cos\theta)$ 0.0016 nm
- $= 0.00243$ nm $(1 - \cos\theta)$

  $\theta = 70^o$

PYQ 1.10 KSCP 2003/04

Which of the following statement(s) is (are) true?

- **I.** Photoelectric effect arises due to the absorption of electrons by photons
- **II.** Compton effect arises due to the scattering of photons by free electrons
- **III.** In the photoelectric effect, only part of the energy of the incident photon is lost in the process
- **IV**.*I*n the Compton effect, the photon completely disappears and all of its energy is given to the Compton electron
- **A. I,II** **B. II,III,IV** **C. I, II, III**
- **D. III,IV** **Ans: E**
- **[I = false; II = true; III = false; IV = false]**
- Murugeshan, S. Chand & Company, New Delhi, pg. 134, Q13,

X-ray: The inverse of photoelectricity

- X-ray, discovered by Wilhelm Konrad Roentgen (1845-1923). He won the first Nobel prize in 1902. He refused to benefit financially from his work and died in poverty in the German inflation that followed the end of World War 1.

X-rays are simply EM radiation with very short wavelength, ~ 0.01 nm –10 nm

Some properties:

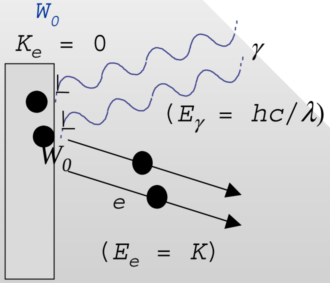
- energetic, according to $E = hc/\lambda$ ~ 0.1 – 100 keV (c.f. E ~ a few eV for visible light)
- travels in straight lines
- is unaffected by electric and magnetic fields
- passes readily through opaque materials – highly penetrative
- causes phosphorescent substances to glow
- exposes photographic plates

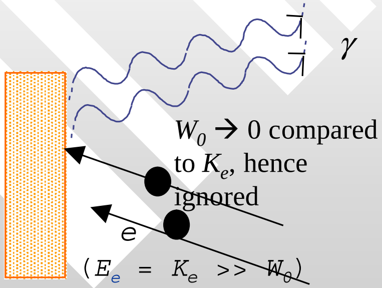
In photoelectricity, energy is transferred from photons to kinetic energy of electrons. The inverse of this process produces x-rays

P.E:

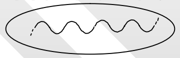
electron (K_e=0) + photon (hc/λ) → electron (K_e) + W_0

x-ray: electron (K_e) → heat + photon (hc/λ)



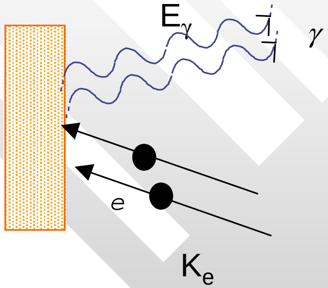


PE and x-rays production happen at different energy scale

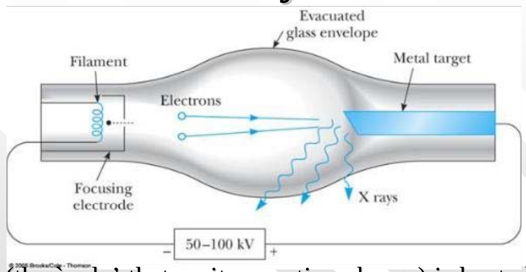
- However, both process occur at disparately different energy scale
- Roughly ,for PE, it occurs at eV scale with ultraviolet radiation 
- For x-ray production, the energy scale involved is much higher - at the order of 100 eV - 100 keV 

X-ray production

- X-rays is produced when electrons, accelerated by an electric field in a vacuum cathode-ray tube, are impacted on the glass end of the tube
- Part or all of the kinetic energy of a moving electron is converted into a x-ray photon

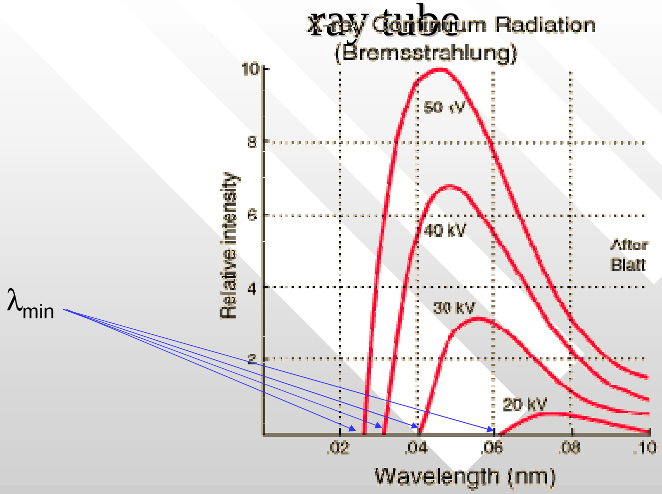


The x-ray tube



- A cathode (the `pole' that emits negative charge) is heated by means of electric current to produce thermionic emission of the electrons from the target
- A high potential difference *V* is maintained between the cathode and a metallic target
- The Thermionic electrons will get accelerated toward the latter
- The higher the accelerating potential *V*, the faster the electron and the shorter the wavelengths of the x-rays

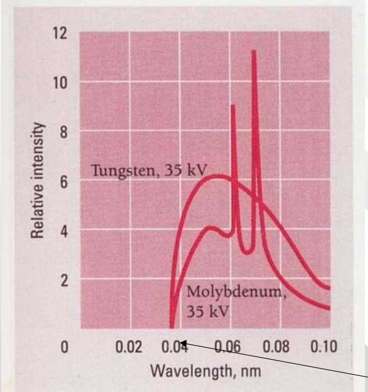
Typical x-ray spectrum from the x-ray tube



Important features of the x-ray spectrum

1. The spectrum is continuous
2. The existence of a minimum wavelength λ_{min} for a given *V*, below which no x-ray is observed
3. Increasing *V* decreases λ_{min} .

$\lambda_{\min} \propto 1/V$, the same for all material surface



- At a particular V, $\lambda_{\min}$ is approximately the same for different target materials. Experimentally one finds that lmin is inversely proportional to V,

$$\lambda_{\min} = \left(\frac{1.24\times10^{-6}}{V}\right)\mathrm{m}\cdot\mathrm{V}$$

The peaks in the spectrum are due to the electronic transition occurring between the adjacent shells (orbit) in the atom. We would not discuss them further here.

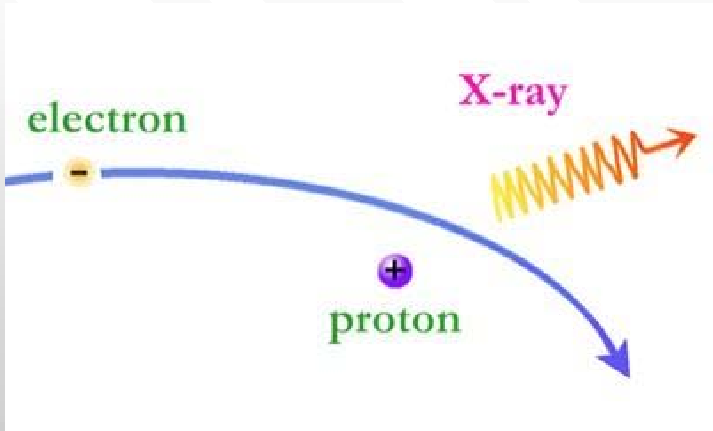
X-ray production heats up the target material

- Due to conversion of energy from the impacting electrons to x-ray photons is not efficient, the difference between input energy, K_e and the output x-ray energy E_γ becomes heat
- Hence the target materials have to be made from metal that can stand heat and must have high melting point (such as Tungsten and Molybdenum)

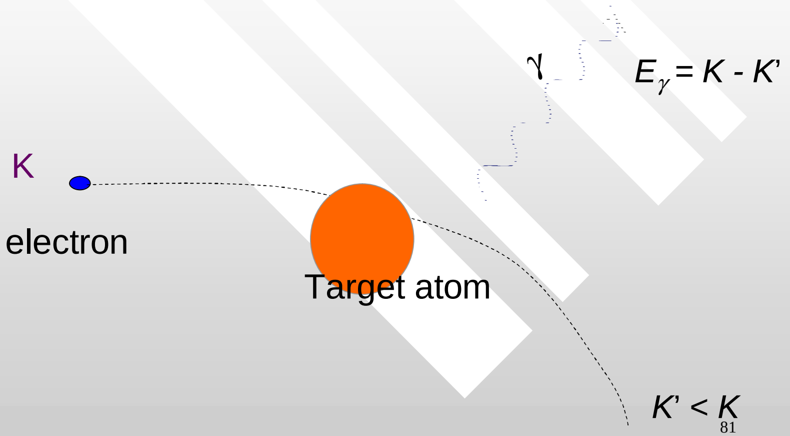
Classical explanation of continuous x-ray spectrum:

- The continuous X-ray spectrum is explained in terms of **Bremsstrahlung:** radiation emitted when a moving electron “tekan brake”
- According to classical EM theory, an accelerating (decelerating) electric charge will radiate EM radiation
- Electrons striking the target get slowed down and brought to eventual rest because of collisions with the atoms of the target material
- Within the target, many electrons collides with many atoms for many times before they are brought to rest
- Each collision causes some non-unique losses to the kinetic energy of the Bremsstrahlung electron
- As a net effect of the collective behavior by many individual collisions, the radiation emitted (a result due to the lost of KE of the electron) forms a continuous spectrum

Bremsstrahlung



Bremsstrahlung, simulation



Bremsstrahlung canot explain λ_{min}

- Notice that in the classical **Bremsstrahlung** process the x-ray radiated is continuous and there is no lower limit on the value of the wavelength emitted. Hence, the existence of λ_{min} is not explained with the classical **Bremsstrahlung** mechanism. All range of λ from 0 to a maximum should be possible in this classical picture.

λ_{min} can only be explained by assuming light as photons but not as EM wave

Energy of the x-ray photon in the quantum picture

- According to Einstein assumption on the energy of a photon, the energy of the photon emitted in the Bremsstrahlung is simply the difference between the initial and final kinetic energy of the electron:

$$h\nu = K - K'$$

- The shortest wavelength of the emitted photon gains its energy, $E = h\,\nu_{max} = hc/\lambda_{min}$ corresponds to the maximal loss of the K.E. of an electron in a single collision (happen when $K' = 0$ in a single collision)
- This (e.g. the maximal lose on KE) only happens to a small sample of collisions. Most of the other collisions loss their KE gradually in smaller amount in an almost continuous manner.

Theoretical explanation of the experimental Value of λ_{min}

- K (of the Bremsstrahlung electron) is converted into the photon with $E = hc/\lambda_{min}$
- Experimentally K is caused by the external potential V that accelerates the electron before it bombards with the target, hence

  $K = eV$
- Conservation of energy requires

$$K = eV = hc/\lambda_{min}$$

- or, $\lambda_{min} = hc/eV = (1240\ \text{nm}\cdot\text{eV})/eV = (1240\text{V}/V)$ nm

which is the value measured in x-ray experiments

Why is λ_{min} the same for different material?

- The production of the x-ray can be considered as an inverse process of PE
- Hence, to be more rigorous, the conservation of energy should take into account the effects due to the work potential of the target material during the emission of x-ray process, W_0
- However, so far we have ignored the effect of W_0 when we were calculating the relationship between λ_{min} and K
- This approximation is justifiable because of the following reason:
- The accelerating potentials that is used to produce x-ray in a x-ray vacuum tube, V, is in the range of 10,000 V
- Whereas the work function W_0 is only of a few eV
- Hence, in comparison, W_0 is ignored wrp to eV
- This explains why is the same for different target materials

Example

- Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V
- Solution:

$$\lambda_{min} = \frac{hc}{eV} = \frac{1.24\times10^{-6}\,\text{V}\cdot\text{m}}{5.00\times10^{4}\,\text{V}} = 2.48\times10^{-11}\,\text{m} = 0.0248\text{nm}$$

This wavelength corresponds to the frequency

$$\nu_{max} = \frac{c}{\lambda_{min}} = \frac{3\times10^{8}\,\text{m/s}}{2.48\times10^{-11}\,\text{m}} = 1.21\times10^{19}\,\text{Hz}$$

PYQ 1. 9 Final Exam 2003/04

- To produce an x-ray quantum energy of 10^{-15} J electrons must be accelerated through a potential difference of about
- A. 4 kV
- B. 6 kV
- C. 8 kV
- D. 9 kV
- E. 10 kV
- **ANS: B, OCR ADVANCED SUBSIDIARY GCE PHYSICS B (PDF), Q10, pg. 36**

Solution:

The energy of the x-rays photon comes from the external accelerating potential, V

$E_\lambda = eV$

$$V = E_\lambda / e = 1\times10^{-15}\,\text{J}/e = \left(\frac{1\times10^{-15}}{1.6\times10^{-19}}\right)\text{eV}/e = 6250\text{V}$$

PYQ 1.9 KSCP 2003/04

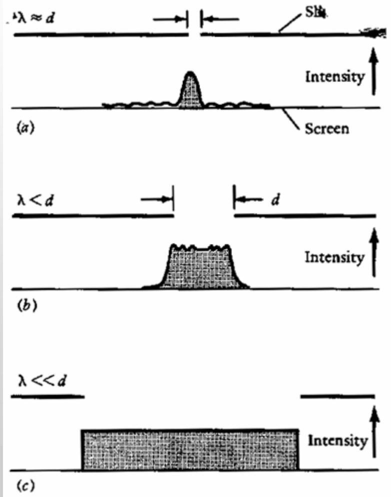
Which of the following statement(s) is (are) true?

- **I.** γ-rays have much shorter wavelength than *x*-rays
- **II.** The wavelength of *x*-rays in a *x*-ray tube can be controlled by varying the accelerating potential
- **III**. *x*-rays are electromagnetic waves
- **IV.** *x*-rays show diffraction pattern when passing through crystals
- **A. I,II** **B. I,II,III,IV** **C. I, II, III**
- **D. III.IV** **E.** Non of the above
- **Ans: B** Murugeshan, S. Chand & Company, New Delhi, pg. 132, Q1.(for I), pg. 132, Q3 (for II), pg. 132, Q4 (for III,IV)

X-ray diffraction

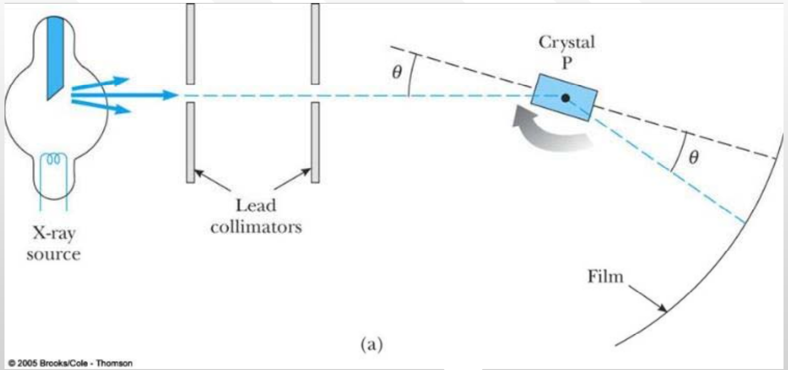
- X-ray wavelengths can be determined through diffraction in which the x-ray is diffracted by the crystal planes that are of the order of the wavelength of the x-ray, ~ 0.1 nm
- The diffraction of x-ray by crystal lattice is called 'Bragg's diffraction'
- It is also used to study crystal lattice structure (by analysing the diffraction pattern)

Condition for diffraction

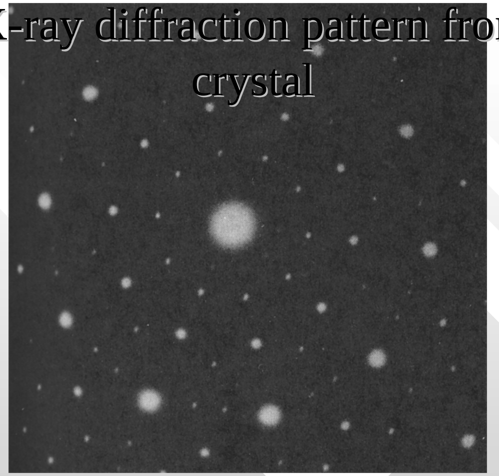


- Note that as a general rule in wave optics, diffraction effect is prominent only when the wavelength and the 'hole/obstacle are comparable in their length scale

Experimental setup of Bragg's diffraction



X-ray diffraction pattern from crystal



The bright spots correspond to the directions where x-rays scattered from various layers (different Braggs planes) in the crystal interfere constructively.

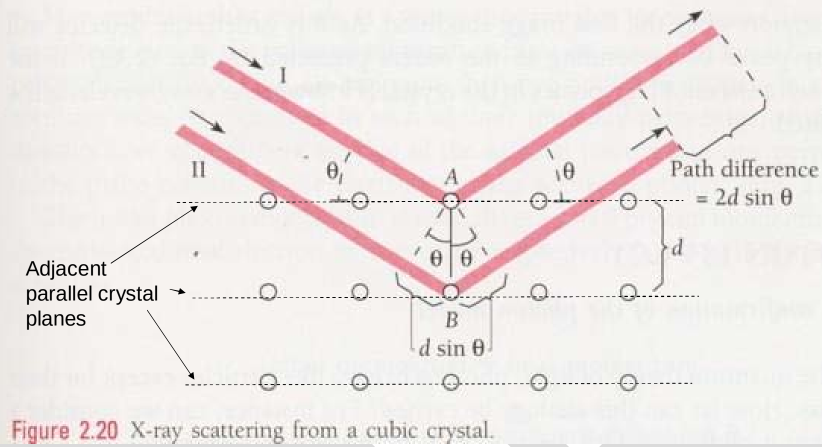


Figure 2.20 X-ray scattering from a cubic crystal.

Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly λ, 2 λ,3 λ and so on (beam I, II):

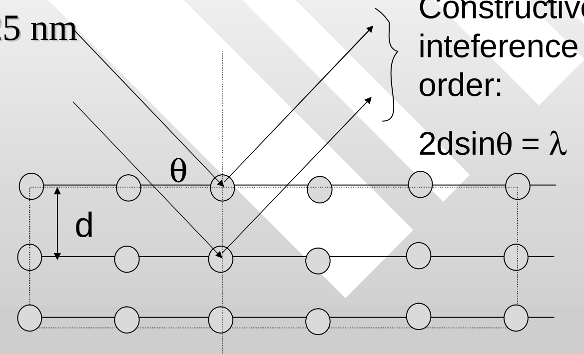
$2d \sin \theta = n \lambda$, n = 1, 2, 3 …**Bragg's law for x-ray diffraction**

Example

- **A single crystal of table salt (NaCl) is irradiated with a beam of x-rays of unknown wavelength. The first Bragg's reflection is observed at an angle of 26.3 degree. Given that the spacing between the interatomic planes in the NaCl crystal to be 0.282 nm, what is the wavelength of the x-ray?**

Solution

- Solving Bragg's law for the *n = 1* order,
  $\lambda = 2d \sin \theta$ = 2 x 0.282 nm x sin (26.3°) = 0.25 nm

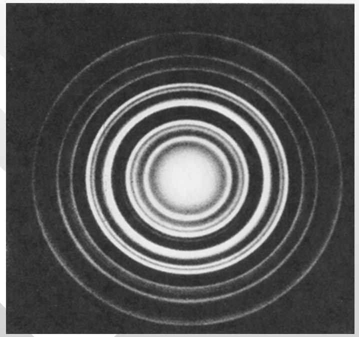


Constructive inteference of n=1 order:

$2d\sin\theta = \lambda$

If powder specimen is used (instead of single crystal)

- We get diffraction ring due to the large randomness in the orientation of the planes of scattering in the power specimen



PYQ 6 Test I, 2003/04

- X-ray of wavelength 1.2 Angstrom strikes a crystal of *d*-spacing 4.4 Angstrom. Where does the diffraction angle of the second order occur?
- **A.**16°   **B.** 33°   **C.**55 °
- **D.** 90°   **E.** Non of the above
- Solution: $n\lambda = 2d \sin\theta$
- $\sin\theta = n\lambda/2d$ = 2 x 2.2 / (2 x 4.4) = 0. 5

  $\theta = 30$ °
- ANS: B, Schaum's 3000 solved problems, Q38.46, pg. 715

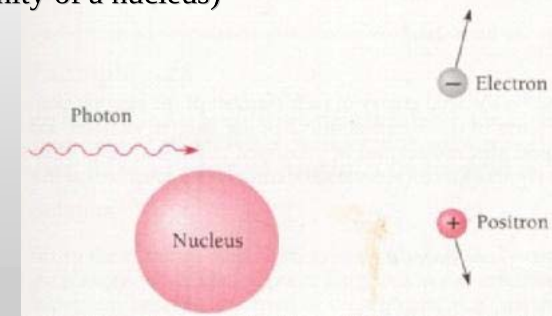
I hope you didn't come by bus!

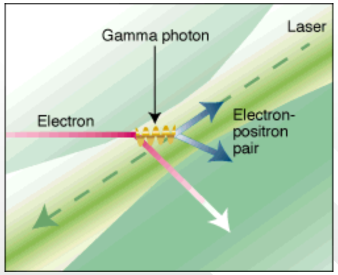
Pair Production: Energy into matter

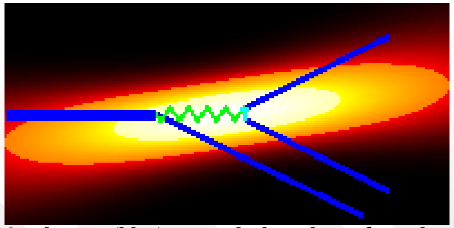
- In photoelectric effect, a photon gives an electron all of its energy. In Compton effect, a photon give parts of its energy to an electron
- A photon can also materialize into an electron and a positron
- Positron = anti-electron, positively charged electron with the exactly same physical characteristics with electron except opposite in charge and spin
- In this process, called pair production, electromagnetic energy is converted into matter
- Creation of something (electron-positron pair) out of nothing (pure EM energy) triggered by strong external EM field

Pictorial visualisation of pair production

- In the process of pair production, a photon of sufficient energy is converted into electron-positron pair. The conversion process must occur only in the presence of some external EM field (such as near the vicinity of a nucleus)







An electron (blue) enters the laser beam from the left, and collides with a laser photon to produce a high-energy gamma ray (wiggly yellow line). The electron is deflected downwards. The gamma ray then collides with four or more laser photons to produce an electron-positron pair

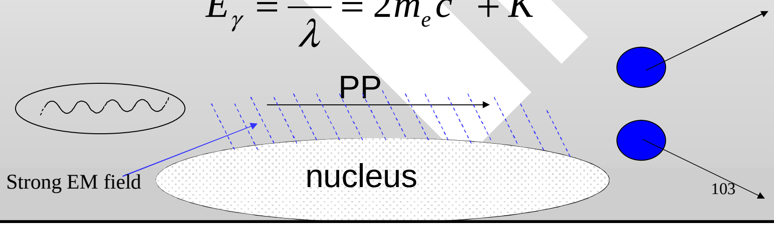
Conservational laws in pair-production

- The pair-production must not violate some very fundamental laws in physics:
- Charge conservation, total linear momentum, total relativistic energy are to be obeyed in the process
- Due to kinematical consideration (energy and linear momentum conservations) pair production cannot occur in empty space
- PP Must occur in the proximity of a nucleus where strong EM field is present to mediate interactions between the photon and the nucleus
- The presence of the nucleus is necessary to absorb part of the momentum transfer delivered by the incident photon so that momentum is conserved in the process of pair creation

Energy threshold

- Due to conservation of relativistic energy, pair production can only occur if E_γ is larger than 2 m_e = 2 x 0.51 MeV = 1.02 MeV
- Any additional photon energy becomes kinetic energy of the electron and positron, K

$$E_\gamma = \frac{hc}{\lambda} = 2m_e c^2 + K$$



Example

- What is the minimal wavelength of a EM radiation to pair-produce an electron-positron pair?
- Solutions: minimal photon energy occurs if the pair have no kinetic energy after being created, $K = 0$
- Hence,

$$\lambda_{\min} = \frac{hc}{2m_e c^2} = \frac{1240\text{nm}\cdot\text{eV}}{2\cdot 0.51\text{MeV}} = 1.21\times10^{-12}\,\text{m}$$

These are very energetic EM radiation called gamma rays and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.

Electron-positron creation

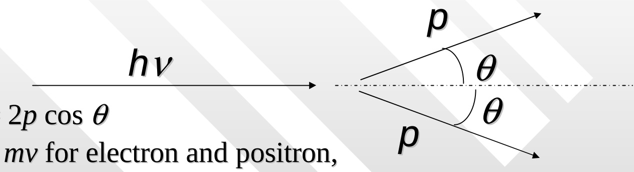
- ***Part of a bubble chamber picture (Fermilab'15 foot Bubble Chamber', found at the University of Birmingham). The curly line which turns to the left is an electron. Positron looks similar but turn to the right The magnetic field is perpendicular to the picture plan***

Solution

- Conservation of energy must me fulfilled,

$$h\nu = 2mc^2$$

- Conservation of linear momentum must be fulfilled:



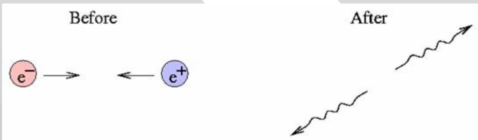
- $\Rightarrow h\nu/c = 2p \cos\theta$
- Since $p = mv$ for electron and positron,
- $\Rightarrow h\nu = 2c\,(mv)\cos\theta = 2mc^2\,(v/c)\cos\theta$
- Because $v/c < 1$ and $\cos\theta \leqslant 1$, $h\nu < 2mc^2$
- But conservation of energy requires $h\nu = 2mc^2$. Hence it is impossible for pair production to conserve both energy and momentum unless some other object (such as a nucleus) in involved in the process to carry away part of the initial of the photon momentum

Pair-annihilation

- The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges

$$e^+ + e^- \rightarrow \gamma + \gamma$$

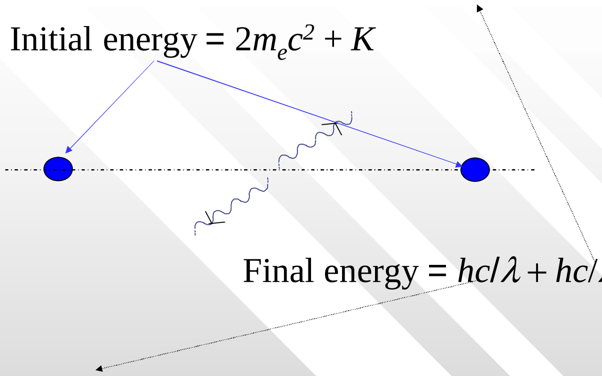
- Both particles vanish simultaneously, with the lost masses becoming energies in the form of two gamma-ray photons
- Positron and electron annihilate because they are anti particles to each other



Pair annihilation

- ***Part of a bubble-chamber picture from a neutrino experiment performed at the Fermilab (found at the University of Birmingham). A positron in flight annihilate with an electron. The photon that is produced materializes at a certain distance, along the line of flight, resulting a new electron-positron pair (marked with green)***

Initial energy = $2m_ec^2 + K$



Final energy = $hc/\lambda + hc/\lambda$

Conservation of relativistic energy:

$2m_ec^2 + K = 2\,hc/\lambda$

Energy and linear momentum are always conserved in pair annihilation

- The total relativistic energy of the e⁻-e+ pair is
  - $E = 2m_ec^2 + K = 1.02$ MeV + K
- where K the total kinetic energy of the electron-positron pair before annihilation
- Each resultant gamma ray photon has an energy

  $h\nu = 0.51$ MeV + $K/2$
- Both energy and linear momentum are automatically conserved in pair annihilation (else it wont occur at all)
- The gamma photons are always emitted in a back-to-back manner due to kinematical reasons (conservation of linear momentum)
- No nucleus or other particle is needed for pair annihilation to take place
- Pair annihilation always occurs whenever a matter comes into contact with its antimatter

As a tool to observe anti-world

- What is the characteristic energy of a gamma-ray that is produced in a pair-annihilation production process? What is its wavelength?
- Answer: 0.51 MeV, $\lambda_{annih} = hc/0.51$ MeV = 0.0243 nm
- The detection of such characteristic gamma ray in astrophysics indicates the annihilation of matter-antimatter in deep space

PYQ 4, Test I, 2003/04

- An electron and a positron collide and undergo pair-annihilation. If each particle is moving at a speed of 0.8c relative to the laboratory before the collision, determine the energy of each of the resultant photon.
- **A.** 0.85MeV **B.** 1.67 MeV
- **C.** 0.51 MeV **D.** 0.72MeV
- **E.** Non of the above

Solution

Total energy before and after anniliation must remain the same: *i.e*. the energy of each electron is converted into the energy of each photon. Hence the energy of each photon is simple equal to the total relativistic energy of each electron travelling at 0.8*c* :

$E_\gamma = E_e = \gamma m_e c^2$

where $\gamma = 1/\sqrt{1-(0.8)^2} = 1.678$

Hence $E_\gamma = 1.678 \times 0.51$ MeV = 0.85 MeV

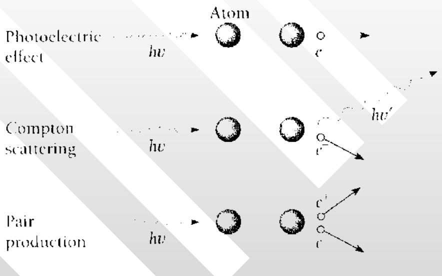
- ANS: A, Cutnell, Q17, pg. 878, modified

What is the wavelength of the gamma ray in a proton-antiproton annihilation?

$\lambda_{annih} = hc/M_p c^2$ = 1240 nm·eV/937 MeV = 1.3×10^{-15} m

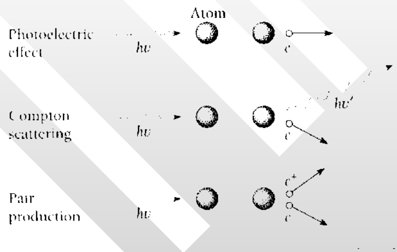
Photon absorption

- Three chief "channels" photons interact with matter are: photoelectric effect, Compton scattering and Pair-production
- In all of these process, photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material



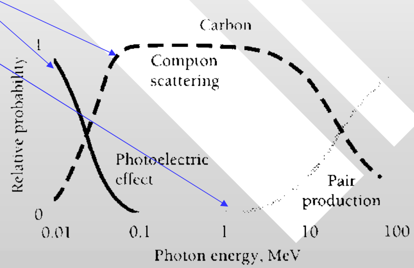
Photon absorption

- The probability of a photon undergoes a given channel of interaction with matter depends on
- (1) Photon energy, and
- (2) Atomic number of the absorbing material



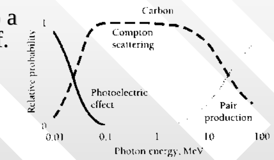
Relative probabilities of photon absorption mechanisms

- For a fixed atomic number (say Carbon, A = 12)
- At low energy photoelectric effect dominates. It diminishes fast when E_γ approaches tens of keV
- At E_γ = a few tens of keV, Compton scattering start to take over
- Once E_γ exceeds the threshold of $2m_ec^2$ = 1.02 MeV, pair production becomes more likely. Compton scattering diminishes as energy increases from 1 MeV



Relative probabilities between different absorbers

- Compare with Lead absorber (much higher *A*):
- Photoelectric effect remains dominant up to a higher energy of a few hundreds of keV (c.f. Carbon of a few tens of keV)
- This is because the heavier the nucleus the better it is in absorbing the momentum transfer that occurs when the energetic photon impart its momentum to the atom
- Compton scattering starts to appears after a much higher energy of 1 MeV (c.f. a few tens of keV for Carbon).
- This is because a larger atomic number binds an electron stronger, rendering the electron less 'free'> In this case, to Compton scatter off an "free" electron the photon has to be more energetic
- (recall that in Compton scattering, only free electrons are scattered by photon).



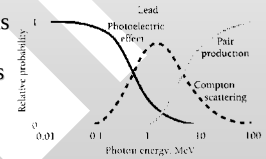
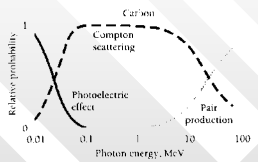


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

Relative probabilities of pair creation between different absorbers

- The energy at which pair production takes over as the principle mechanism of energy loss is called the crossover energy
- The crossover energy is 10 MeV for Carbon, 4 for Lead
- The greater atomic number, the lower the crossover energy
- This is because nuclear with larger atomic number has stronger electric field that is necessary to trigger pair-creation



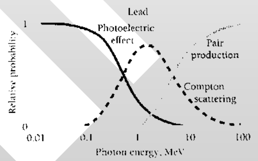


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

What is a photon

- Like an EM wave, photons move with speed of light *c*
- They have zero mass and rest energy
- The carry energy and momentum, which are related to the frequency and wavelength of the EM wave by $E=h\nu$ and $p=h/\lambda$
- They can be created or destroyed when radiation is emitted or absorbed
- They can have particle-like collisions with other particles such as electrons

Contradictory nature of photon

- In Photoelectric effect, Compton scatterings, inverse photoelectric effect, pair creation/annihilation, light behaves as particle. The energy of the EM radiation is confined to localised bundles production/annihilation
- In Young's Double slit interference, diffraction, light behave as waves. In the wave picture of EM radiation, the energy of wave is spread smoothly and continuously over the wavefronts
- Both the wave and particle explanations of EM radiation are obviously mutually exclusive

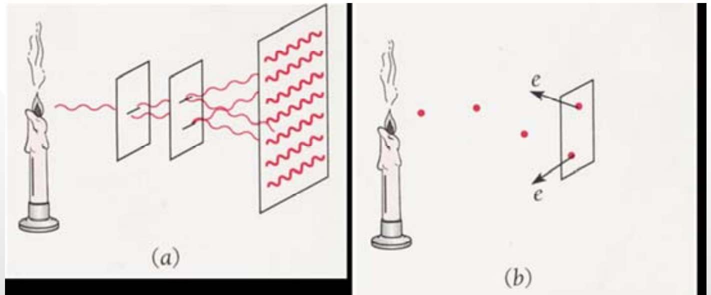
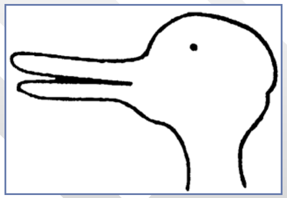


Figure 2.14 (a) The wave theory of light explains diffraction and interference, which the quantum theory cannot account for. (b) The quantum theory explains the photoelectric effect, which the wave theory cannot account

The identity of photon depends on how the experimenter looks at it

The face of a young or an old woman?



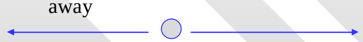
Is this a rabbit or a duck?

Gedanken experiment with remote light source

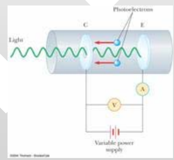
- The same remote light source is used to simultaneously go through two experimental set up
- In the left experiment, the EM radiation behaves as wave; the right one behave like particle
- This is weird: the "light source" from 100 M light years away seems to "know" in which direction to aim the waves and in which direction to aim the particles

Light source from 100 M light years away

Double slit experiment



Photoe lectric experi ment



Interference pattern observed

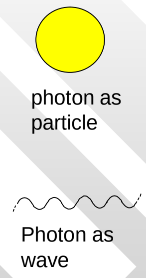
Photoelectron observed

So, is light wave of particle?

- So, it is not *either* particle or wave but *both* particles and waves
- However, both typed of nature cannot be simultaneously measured in a single experiment
- The light only shows one or the other aspect, depending on the kind of experiment we are doing
- Particle experiments show the particle nature, while a wave-type experiment shows the wave nature

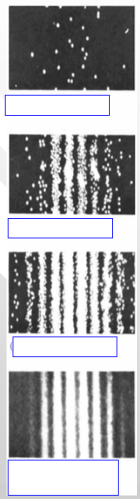
Coin a simile of wave-particle duality

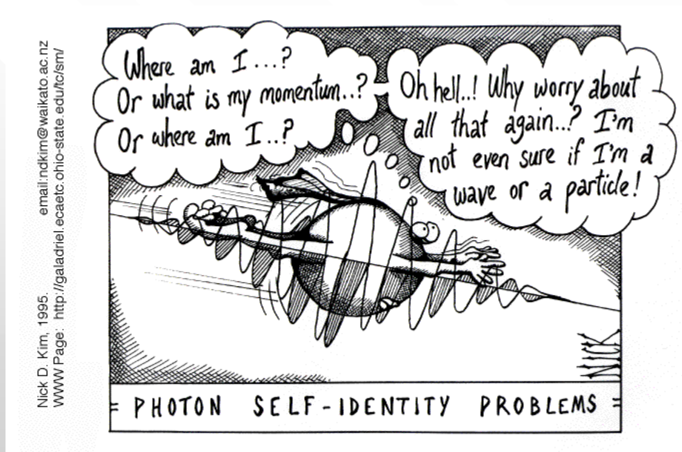
- It's like a coin with two faces. One can only sees one side of the coin but not the other at any instance
- This is the so-called wave-particle duality
- Neither the wave nor the particle picture is wholly correct all of the time, that both are needed for a complete description pf physical phenomena
- The two are *complementary* to another



Interference experiment with a single photon

- Consider an double slit experiment using an extremely weak source (say, a black body filament) that emits only one photon a time through the double slit and then detected on a photographic plate by darkening individual grains.
- When one follows the time evolution of the pattern created by these individual photons, interference pattern be observed
- At the source the light is being emitted as photon (radiated from a dark body) and is experimentally detected as a photon which is absorbed by an individual atom on the photographic plate to form a grain
- In between, we must interpret the light as electromagnetic energy that propagates smoothly and continuously as a wave
- However, the wave nature between the emission and detection is not directly detected. Only the particle nature are detected in this procedure.
- The correct explanation of the origin and appearance of the interference pattern comes from the wave picture, and the correct interpretation of the evolution of the pattern on the screen comes from the particle picture;
- Hence to completely explain the experiment, the two pictures must somehow be taken together – this is an example for which *both pictures are complimentary to each other*





Both light and material particle display wave-particle duality

- Not only light manifest such wave-particle duality, but other microscopic material particles (e.g. electrons, atoms, muons, pions well).
- In other words:
- Light, as initially thought to be wave, turns out to have particle nature;
- Material particles, which are initially thought to be corpuscular, also turns out to have wave nature (next topic)