ZCT 104/3E Modern Physics

2nd semester, academic session 2005/06, School of Physics, USM, Penang, Malaysia

Course webpage:

www.fizik.usm.my/tlyoon/teaching/ZCT104_05_06/

mirror site:
http://www.usm.my/phy/tlyoon/teaching/ZCT104_05_06/

Exam Grading:

Grading will be weighted: Coursework (in the forms of 2 midterm tests) will contribute 30 marks, while final exam 70 marks, totaling (30 + 70 =100) marks.

Lecturer: Yoon Tiem Leong
Contact: Room 115, School of Physics, USM. Tel:04-6533674
Email: tlyoon@usm.my

Course Meeting Times:
1) Monday, 11.00-11.50 am, SK3
2) Wednesday, 9.00-9.50 am, SK4
3) Friday, 3.00 - 3.50 pm, SK3

Tutorials: Friday, (alternate with lectures)

Course Materials

Hard copy of lecture notes plus problem sets will be available for sale in the photocopy shop at the Main library basement. If you like to view the simulation and colour pages not available in the hard copy version, powerpoint lecture notes are also downloadable in the course webpage. Past year questions plus their solutions (tests, final exam and KSCP papers) and problem sheets (i.e. tutorial sheets) are also available on the course webpage.

Course Description

This course is intended to cover some of the standard concepts in modern physics since 1900. It includes special theory of relativity, wave-particle duality of light and material particles, introductory quantum theory of atoms and introductory quantum mechanics. The course aims to lay the foundational concepts for students who would take up papers on quantum mechanics at a higher level.

Course Duration

This course is offered in the second semester for science students in USM -- a 14-week term at USM that runs from 26 Dec 2005 until 7 April 2006.

Course Prerequisites

Since ZCT 104 is conducted in English, students must prepare to take the challenge to deal with language barrier (if relevant). Despite requiring no formal prerequisites (prasyarat kursus), students are assumed to be familiar with elementary calculus, differential equations, and Newtonian mechanics. Most importantly, students are expected to exercise independence throughout the learning process. This course demands ones to think critically to comprehend some rather counter intuitive physics ideas.

Consultation hours

There is no specific timeslots allocated for consultation with the lecturer as he of dedicated willingness to offer consultation and advice to students who wish to engage discussion with him. The principle of the lecturer is that:

as long as the students are showing enthusiasm to learn, he will be willing to offer his time for discussion. However, in order to avoid inconvenience students are advised to call up (ext 3674) or email him (tlyoon@usm.my) before rushing into his office. His door is always open to any one who are keen to explore physics.

General Comments

Modern physics is one of the most interesting subject offered to USM undergraduates. Most of the concepts introduced, such as Einstein's notion that space and time is a relative concept, and that microscopic particles are intrinsically behaving like waves (as expounded in quantum theory), are both intellectually intriguing and somewhat counter-intuitive.

Textbooks

The following textbooks are required or strongly recommended. There exist many good textbooks on the topics of modern physics. I have decided to select the following as my main reference texts. Lecture material are written based on them. It is strongly advised that students should not be contented with the lecture material supplied from the lecturer alone. They should make reference to these suggested texts and do the reading on a consistent manner. You gonna prepare to think in an intellectual manner in order to comprehend the essential concepts I wish to convey in this course. To people who are expecting to make only mechanical memorisation and pass with flying colour, please be prepared for disappointment.

Main Text:

1. Modern Physics, 2nd ed., by Kenneth Krane, John. Wiley & Sons.
2. Concepts of Modern Physics, 6th ed., by Arthur Beiser, McGraw-Hill.
3. Modern Physics, 3rd ed., by Serway, Moses and Moyer, Thomson 2005.
4. Understanding Physics, by Karen Cummings et. al., John Wiley and Sons, 2004 (used for special theory of relativity only)

Others references:

Advanced texts for hard-core physics enthusiasts:

- Introduction to special relativity, by Robert Resnick, John Wiley & sons (readable and well explained, suggested for serious students.
- Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles by Robert Eisberg, Robert Resnick, John Wiley & Sons; 2nd edition

Suggestion for the pleasure of understanding (not totally useful for exam but precious for intellectual pursuit)

- Relativity: The Special and the General Theory. Dover Publications (2001). A classic book by the creator itself, Einstein, for a clear explanation that everyone can understand. Minimum equations.
- The Feynman Lecture on Physics Volume III, by Richard Feynman, Addison Wesley Longman (June, 1970). (Read how Feynman expounds excellently on the basics of quantum mechanics.)

Problem Sets (tutorial sheets)

A total of 5 problem sets will be prepared for you. Problem sets are an integral part of this course. It simply isn't possible to learn physics unless you sit down and work through the problems and concepts on your own. Formally there will be some assigned tutors to provide guidance and counseling to the students on the subject. However, it is recognized that students also learn a great deal from talking to and working with each other. Therefore I encourage each student to make his/her own attempt on every problem. Discussion among yourselves shall definitely help a lot in understanding the course content more thoroughly.

Our typical Malaysian students are used to spoon-feeding. As a matter of fact I am strongly disapproved of such unhealthy attitude. To encourage proactive and independent learning attitude (and to deprive your privilege to be a copy-cat and a bagger of spoon-feedings), NO solution sheets shall be provided to you. We will only discuss the solutions *on-the-spot* during the tutorial meetings. You would not be asked to pass up your tutorial answers for marking and grading. As an adult you are no longer treated as kinder garden kids. In view of such forceful arrangement, you are expected to make active discussions with your fellow course mates or your tutors in the attempts to solve the tutorial questions.

Grading:

Grading will be weighted: Coursework (in the forms of 2 midterm tests) will contribute 30 marks, while final exam 70 marks, totaling (30 + 70 =100) marks.

Coursework assessment: Two tests (20 objectives for each test) will be arranged during the term (check the dates from the calendar). They carry 2x15 marks = 30 marks.

Exam Format:

The exam questions will appear in dual language: English + Malay.

Final exam: A 3-hour final exam covering material throughout the lecture material will be conducted at the end of the semester. It comprises of 40 objective questions (40%) and 3 structured questions (3x20% = 60%), all are to be answered. In particular, the objectives (in the tests as well as in the final exam) will comprise of conceptual type questions. Structured questions in the final exam will be of calculation type. You may like to take a look at the past year questions to get some idea of the exam format (click here). The format of past year questions differ somewhat from the format mentioned above.

Calendar and lecture notes for ZCT 104, Semester II, 2005/06

Note: The planned schedule is tentative and subjected to possible modification.
Note: * The extra class scheduled at Wednesday night, noted with a star (*), is a tentative arrangement, subjected to last-minute change.

Topic and No. of lecture	Scheduled date	Extra reading materials
Special theory of Relativity (38 pg) 8 lectures	26 Dec 05 (Mon) - No lecture 28 Dec 05 (Wed)-Lecture 1 30 Dec 05 (Fri)-Lecture 2 2 Jan 06 (Mon)-Holidays, no lecture 4 Jan 06 (Wed)-Lecture 3 6 Jan 06 (Fri)-tutorial 9 Jan 06 (Mon)-Lecture 4 11 Jan 06 (Wed)-Lecture 5 13 Jan 06 (Fri)-tutorial 16 Jan 06 (Mon)-Lecture 6 18 Jan 06 (Wed)-Lecture 7 20 Jan 06 (Fri)-Test I 23 Jan 06 (Mon)-Lecture 8	The development of Physics and Modern Physics Michelson Morley experiment Ether Relativity Time
EM waves and Black Body (17 pg) 3.5 lecture	25 Jan 06 (Wed)-Lecture 9 27 Jan 06 (Fri)-tutorial 30 Jan 06 (Mon)-CNY Holidays, no lecture 1 Feb 06 (Wed)-CNY Holidays, no lecture 3 Feb 06 (Fri)-CNY Holidays, no tutorial 6 Feb 06 (Mon)-Lecture 10 8 Feb 06 (Wed)-Lecture 11 10 Feb 06 (Fri)-tutorial 13 Feb 06 (Mon)-Midterm break 15 Feb 06 (Wed)-Midterm break 17 Feb 06 (Fri)-Midterm break 20 Feb 06 (Mon)-Lecture 12	Electromagnetic waves: photon photoelectric effectX-rays:
Particle properties of waves (33 pg) 7 lectures	22 Feb 06 (Wed)-Lecture 13 24 Feb 06 (Fri)-tutorial 27 Feb 06 (Mon)-Lecture 14 1 Mar 06 (Wed)-Lecture 15 3 Mar 06 (Fri)-tutorial 6 Mar 06 (Mon)-Lecture 16 8 Mar 06 (Wed)-Lecture 17 8 Mar 06 (Wed)-Lecture 18 (extra class, 8.00 pm, at SK3)* 10 Mar 06 (Fri)-tutorial	Quantum theory
The wavelike properties of particles (16 pg) 3.5 lectures	13 Mar 06 (Mon)-Lecture 19 15 Mar 06 (Wed)-Lecture 20 17 Mar 06 (Fri)-tutorial 20 Mar 06 (Mon)-Lecture 21 22 Mar 06 (Wed)-Lecture 22	Quantum Atom
Atomic models (21 pg) 4.5 lectures	22 Mar 06 (Wed)-Lecture 23 (extra class, 8.00 pm, at SK3)* 24 Mar 06 (Fri)-Test II 27 Mar 06 (Mon)-Lecture 24 29 Mar 06 (Wed)-Lecture 25 29 Mar 06 (Wed)-Lecture 26 (extra class, 8.00 pm, at SK3)* 31 Mar 06 (Fri)-tutorial	
Introductory QM 3.5 lectures	3 Apr 06 (Mon)-Lecture 27 5 Apr 06 (Wed)-Lecture 28 5 Apr 06 (Wed)- Lecture 29 (extra class, 8.00 pm, at SK3)* 7 Apr 06 (Fri)-Lecture 30	Introduction to quantum mechanics

Meeting times (for the whole semester):
Lectures + extra classes = 26 times + 4 times = 30 times
Tutorial + tests = (9 + 2) times, on every Fridays, except the first and last Friday of the semester
Missed lectures + missed tutorials = (4 + 1) times = 5 times
Total = 30 + 11 - 5 = 42 times

---

142 x 4 pages of lecture notes, total 30 lectures.
142 x 4 /30 = 4.7 x 4 = 19 pages power point notes per lecture

ZCT 104

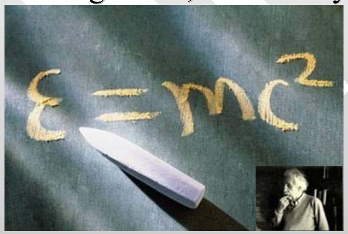
MODERN PHYSICS

ACADEMIC SESSION 2005/06
(SECOND SEMESTER)

LECTURE NOTES
(POWERPOINT)

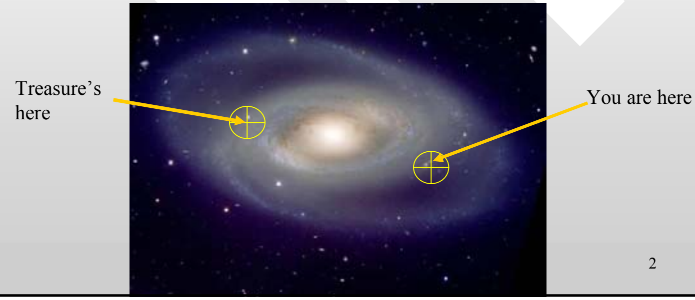
Special theory of Relativity

Notes based on
Understanding Physics
by Karen Cummings et al., John Wiley & Sons



An open-chapter question

- Let say you have found a map revealing a huge galactic treasure at the opposite edge of the Galaxy 200 ly away.
- Is there any chance for you to travel such a distance from Earth and arrive at the treasure site by traveling on a rocket within your lifetime of say, 60 years, given the constraint that the rocket cannot possibly travel faster than the light speed?

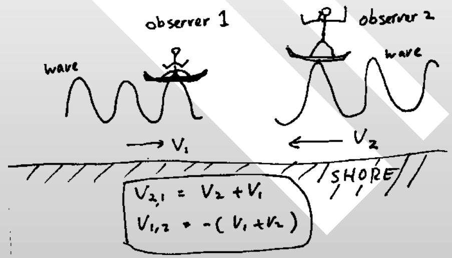


Relative motions at ordinary speed

- Relative motion in ordinary life is commonplace
- E.g. the relative motions of two cars (material objects) along a road
- When you observe another car from within your car, can you tell whether you are at rest or in motion if the other car is seen to be "moving?"

Relative motion of wave

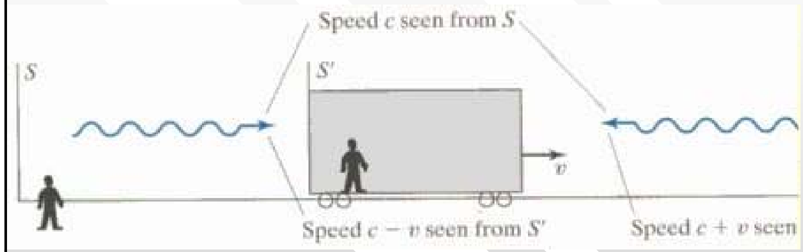
- Another example: wave motion
- Speed of wave measured by Observer 1 on wave 2 depends on the speed of wave 1 wrp (with respect) to the shore: $v_{2,1} = v_2 - v_1$



Query: can we surf light waves?

- Light is known to be wave
- If either or both wave 1 and wave 2 in the previous picture are light wave, do they follow the addition of velocity rule too?
- Can you surf light wave ? (if so light shall appear *at rest* to you then)

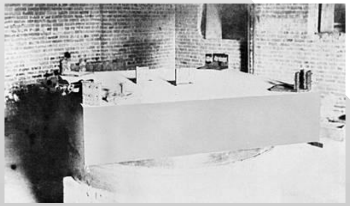
In other word, does light wave follows Galilean law of addition of velocity?



Frame S' travels with velocity *v* relative to S. If light waves obey Galilean laws of addition velocity, the speeds of the two opposite light waves would be different as seen by S'. But does light really obey Galilean law of addition of velocity?

The negative result of Michelson-Morley experiment on Ether

- In the pre-relativity era, light is thought to be propagating in a medium called ether -
- an direct analogy to mechanical wave propagating in elastic medium such as sound wave in air
- If exist, ether could render measurable effect in the apparent speed of light in various direction
- However Michelson-Morley experiment only find negative result on such effect
- A great puzzlement to the contemporary physicist: what does light wave move relative to?



How could we know whether we are at rest or moving?

- Can we cover the windows of our car and carry out experiments inside to tell whether we are at rest or in motion?
- NO

In a "covered" reference frame, we can't tell whether we are moving or at rest

- Without referring to an external reference object, whatever experiments we conduct in a constantly moving frame of reference (such as a car at rest or a car at constant speed) could not tell us the state of our motion (whether the reference frame is at rest or is moving at constant velocity)

9

《尚書經．考靈曜》

- 「地恒動不止，而人不知，譬如人在大舟中，閉牖而坐，舟行而不覺也。」
- "The Earth is at constant state of motion yet men are unaware of it, as in a simile: if one sits in a boat with its windows closed, he would not aware if the boat is moving" in "*Shangshu jing*", 200 B.C

10

Physical laws must be invariant in any reference frame

- Such an inability to deduce the state of motion is a consequence of a more general principle:
- There must be no any difference in the physical laws in any reference frame with constant velocity
- (which would otherwise enable one to differentiate the state of motion from experiment conducted in these reference frame)
- Note that a reference frame at rest is a special case of reference frame moving at constant velocity ($v = 0 =$ constant)

11

The Principle of Relativity

- All the laws of Physics are the same in every reference frame

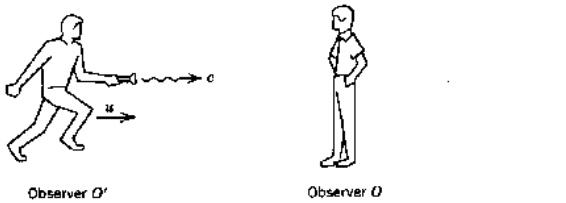
12

Einstein's Puzzler about running fast while holding a mirror

- Says Principle of Relativity: Each fundamental constants must have the same numerical value when measured in any reference frame (*c, h, e,*m_e*, etc*)
- (Otherwise the laws of physics would predict inconsistent experimental results in different frame of reference – which must not be according to the Principle)
- Light always moves past you with the same speed *c*, no matter how fast you run
- Hence: you will not observe light waves to slow down as you move faster

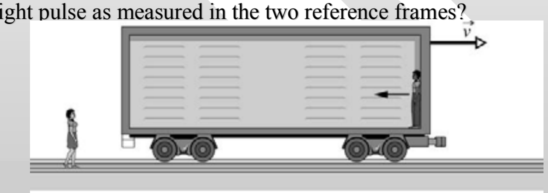
Constancy of the speed of light

Figure 1-2
Two observers in relative motion. O is at rest and O' moves toward O at constant speed u. O and O' agree on the speed of light coming from the source carried by O'.



Reading Exercise (RE) 38-2

- While standing beside a railroad track, we are startled by a boxcar traveling past us at half the speed of light. A passenger (shown in the figure) standing at the front of the boxcar fires a laser pulse toward the rear of the boxcar. The pulse is absorbed at the back of the box car. While standing beside the track we measure the speed of the pulse through the open side door.
- (a) Is our measured value of the speed of the pulse greater than, equal to, or less than its speed measured by the rider?
- (b) Is our measurement of the distance between emission and absorption of the light pulse great than, equal to, or less than the distance between emission and absorption measured by the rider?
- (c) What conclusion can you draw about the relation between the times of flight of the light pulse as measured in the two reference frames?



Touchstone Example 38-1: Communication storm!

- A sunspot emits a tremendous burst of particles that travels toward the Earth. An astronomer on the Earth sees the emission through a solar telescope and issues a warning. The astronomer knows that when the particle pulse arrives it will wreak havoc with broadcast radio transmission. Communications systems require ten minutes to switch from over-the-air broadcast to underground cable transmission. What is the maximum speed of the particle pulse emitted by the Sun such that the switch can occur in time, between warning and arrival of the pulse? Take the sun to be 500 light-seconds distant from the Earth.

Solution

- It takes 500 seconds for the warning light flash to travel the distance of 500 light-seconds between the Sun and the Earth and enter the astronomer's telescope. If the particle pulse moves at half the speed of light, it will take twice as long as light to reach the Earth. If the pulse moves at one-quarter the speed of light, it will take four times as long to make the trip. We generalize this by saying that if the pulse moves with speed v/c, it will take time to make the trip given by the expression:

  Δt_{pulse} = 500 s/ (v_{pulse}/c)
- How long a warning time does the Earth astronomer have between arrival of the light flash carrying information about the pulse the arrival of the pulse itself? It takes 500 seconds for the light to arrive. Therefore the warning time is the difference between pulse transit time and the transit time of light:

  $\Delta t_{warning} = \Delta t_{pulse}$ - 500 s.
- But we know that the minimum possible warning time is 10 min = 600 s.
- Therefore we have
- 600 s = 500 s / (v_{pulse}/c) – 500 s,
- which gives the maximum value for v_{puls} if there is to he sufficient time for warning:

  v_{puls} = 0.455 c. (Answer)
- Observation reveals that pulses of particles emitted from the sun travel much slower than this maximum value. So we would have much longer warning time than calculated here.

Relating Events is science

- Science: trying to relate one event to another event
- E.g. how the radiation is related to occurrence of cancer; how lightning is related to electrical activities in the atmosphere etc.
- Since observation of events can be made from different frames of reference (e.g. from an stationary observatory or from a constantly moving train), we must also need to know how to predict events observed in one reference frame will look to an observer in another frame

Some examples

- How is the time interval measured between two events observed in one frame related to the time interval measured in another frame for the same two events?
- How is the velocity of a moving object measured by a stationary observer and that by a moving observer related?

Defining events

- So, before one can work out the relations between two events, one must first precisely define what an event is

Locating Events

- ***An event is an occurrence that happens at a unique place and time***
- Example: a collision, and explosion, emission of a light flash
- An event must be sufficiently localised in space and time
- e.g. your birthday: you are born in the General Hospital PP at year 1986 1st April 12.00 am)

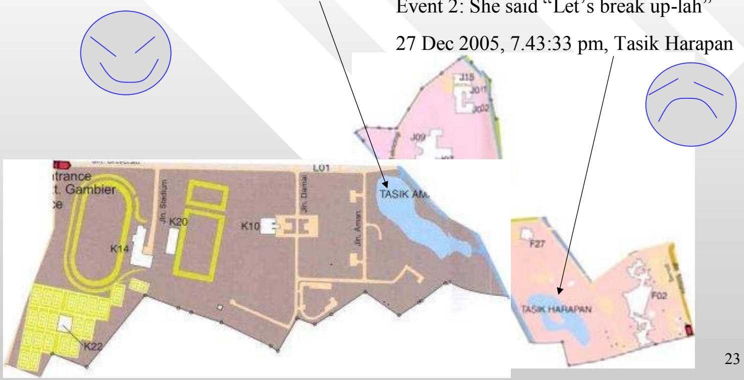
Example of two real-life events

Event 1: She said "I love you"

July 1Dec, 12.01:12 am, Tasik Aman

Event 2: She said "Let's break up-lah"

27 Dec 2005, 7.43:33 pm, Tasik Harapan

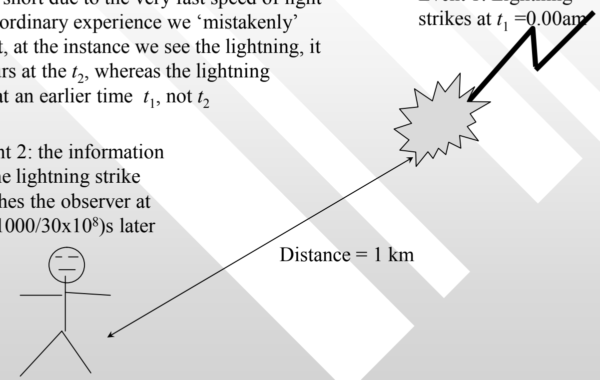


Subtle effect to locate an event: delay due to finiteness of light speed

- In our (erroneous) "common sense" information are assumed to reach us instantaneously as though it is an immediate action through a distance without any delay
- In fact, since light takes finite time to travel, locating events is not always as simple it might seems at first

An illustrative example of delay while measuring an event far away

t_2 is very short due to the very fast speed of light *c*. In our ordinary experience we 'mistakenly' think that, at the instance we see the lightning, it also occurs at the t_2, whereas the lightning actually at an earlier time t_1, not t_2



Reading Exercise 38-4

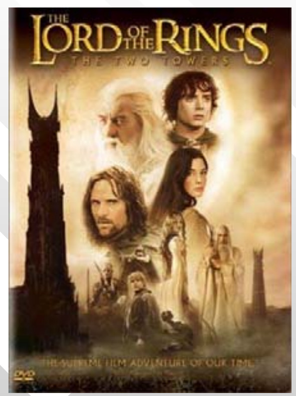
- When the pulse of protons passes through detector A (next to us), we start our clock from the time t = 0 microseconds. The light flash from detector B arrives hack at detector A at a time t = 0.225 microsecond later.
- (a) At what time did the pulse arrive at detector B?
- (b) Use the result from part (a) to find the speed at which the proton pulse moved, as a fraction of the speed of light.

Redefining Simultaneity

- Hence to locate an event accurately we must take into account the factor of such time delay
- An intelligent observer is an observer who, in an attempt to register the time and spatial location of an event far away, takes into account the effect of the delay factor
- (In our ordinary daily life we are more of an unintelligent observer)
- For an intelligent observer, he have to redefine the notion of "simultaneity" (example 38-2)

Example 38-2: Simultaneity of the Two Towers

- Frodo is an intelligent observer standing next to Tower A, which emits a flash of light every l0 s. 100 km distant from you is the tower B, stationary with respect to you, that also emits a light flash every 10 s. Frodo wants to know whether or not each flash is emitted from remote tower B simultaneous with (at the same time as) the flash from Frodo's own Tower A. Explain how to do this with out leaving Frodo position next to Tower A. Be specific and use numerical values.



Solution

- Frodo are an intelligent observer, which means that he know how to take into account the speed of light in determining the time of a remote event, in this case the time of emission of a flash by the distant Tower B. He measures the time lapse between emission of a flash by his Tower A and his reception of flash from Tower B.
- If this time lapse is just that required for light move from Tower B to Tower A, then the two emissions occur the same time.
- The two beacons are 100 km apart. Call this distance L. Then the time t for a light flash to move from B to A is
- $t = L/c = 10^5$ m/3 × 10^8 m/s = 0.333 ms. (ANS)
- If this is the time Frodo records between the flash nearby beacon A and reception of the flash from distant beacon then he is justified in saying that the two Towers emit flashes simultaneously in his frame.

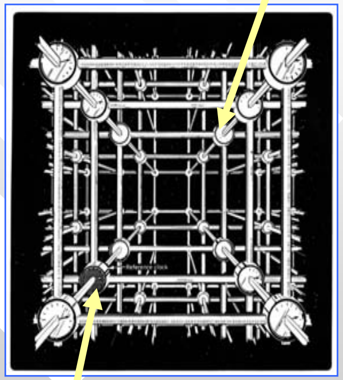
The problem of an intelligent observer

- For an intelligent observer, in order to take care of the time delay effect, she needs a precise way to determine the ***local*** time and (spatial) location of an event
- But how could she jot down the ***local time*** of every event since (i) she is not at the same spot at which the event occurs (ii) she does not already know the location of every event she wants to measure?

Latticework Clocks

- Conceptually an intelligent observer set up an latticewrok of meter sticks and clocks
- Events are then measured by reading the distance of the lattice site (where events occur) from an reference location
- However we need to synchronise all the clocks so that they all read the same reading at the beginning as seen from the reference lattice site
- Since the distance of all the lattice is known we can use light to synchronise all the lattice clock

Event occurs here

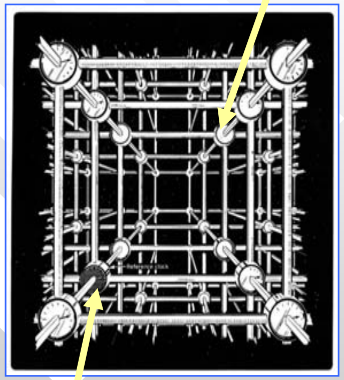


Reference lattice site

Latticework Clocks

- Station at every lattice site an timekeepers
- At 00.00am on the reference clock, send out light pulse
- Once a timekeeper at a lattice site receives the light, she sets the local clock to read (00.00 + l/c) am, where l is the distance from the reference point
- Then all the clocks in the same time would read the same time by the intelligent observer at the reference point
- (In fact they all read the same time from any other points in the lattice)
- All the clocks are said to have been synchronised with respect to this lattice

Event occurs here



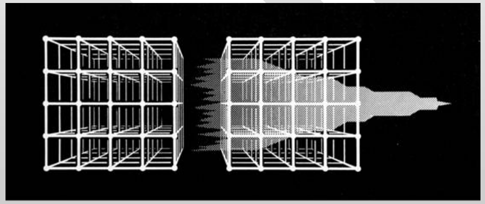
Reference lattice site

Ready to register events

- Within the frame of latticework, an intelligent observer can now start to register the location and time of an event that occur during an experiment, with the time-delay effect well taken care of
- Analysing the results of that experiment means relating events by collecting event data from all recording clocks in the lattice and analysing these date ***at*** some reference location
- (Note: To collect data of the time an event that occurs at some lattice site some distance away from the reference point, the intelligent observer don't travel to the site of the event to read the clock there. She collects the data of the time the event by reading the reading of the clock from a distance (i.e *from* the reference point).

Laboratory and Rocket Frames

- Events can be measured in any frames of reference; non is privileged over the other
- Laboratory frame (at rest) and Rocket frame (one that coasts at constant velocity)
- E.g. a "pom-chat" event.
- To the driver in the car (the rocket frame) the event occurs at the front tyre
- For a lepak kaki at a mamak store, the pom-chat event occurs just in front of the mamak stor
- The upshot is: the same event is observed by two observers in two frame of reference, both of which have equal status in observing and recording the pom-chat

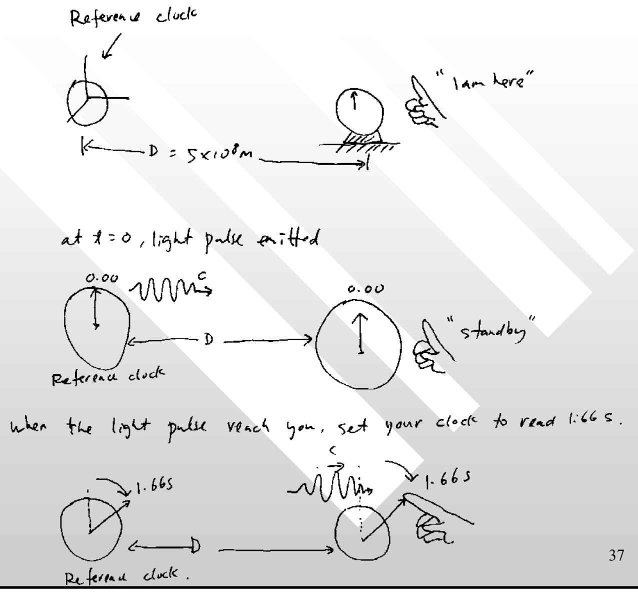


Example 38-3: Synchronizing Clocks

- You are stationed at a latticework clock with the coordinates $x = 3 \times 10^8$ m, $y = 3 \times 10^8$ m and $z = 0$ m. The reference clock at coordinates $x = y = z = 0$ emits a reference flash at exactly midnight on its clock. You want your clock to be synchronized with (set to the same time as) the reference clock. To what time do you immediately set your clock when you receive the reference flash?

Solution

- Your distance *D* from the reference clock is
- *D* = [(3 × 108 m)2 +(4 × 108 m)2] ½ = 5 × 108 m
- The time Δ*t* that it takes the reference flash to reach you is therefore
- Δ*t* = *D*/*c* = (5 × 108 m)/(3 × 108 m)= 1.66 s.
- So when you receive the reference flash, you quickly set your clock to 1.66 seconds after midnight.

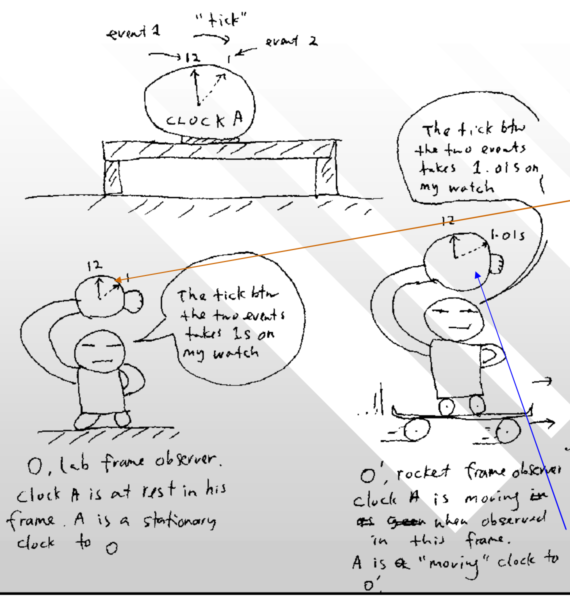


37

Time dilation as direct consequence of constancy of light speed

- According to the Principle of Relativity, the speed of light is invariant (i.e. it has the same value) in every reference frame (constancy of light speed)
- A direct consequence of the constancy of the speed of light is time stretching
- Also called time dilation
- Time between two events can have different values as measured in lab frame and rocket frames in relative motion
- "Moving clock runs slow"

38



"The time between two clock ticks measured on a clock at rest (as seen by O) is always less than the time between the same two ticks measured in a reference frame in which the clock is moving (as seen by O')"

39

Experimental verification of time stretching with pions

- Pion's half life $t_{1/2}$ is 18 ns.
- Meaning: If N_0 of them is at rest in the beginning, after 18 ns, N_0 /2 will decay
- Hence, by measuring the number of pion as a function of time allows us to deduce its half life
- Consider now N_0 of them travel at roughly the speed of light c, the distance these pions travel after $t_{1/2}$=18 ns would be $ct_{1/2}$ ≈5.4 m.
- Hence, if we measure the number of these pions at a distance 5.4 m away, we expect that N_0 /2 of them will survive
- However, experimentally, the number survived at 5.4m is much greater than expected
- The flying poins travel tens of meters before half of them decay
- How do you explain this? the half life of these pions seems to have been stretched to a larger value!
- Conclusion: in our lab frame the time for half of the pions to decay is much greater than it is in the rest frame of the pions!

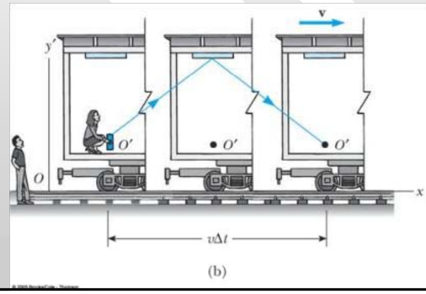
40

RE 38-5

- Suppose that a beam of pions moves so fast that at 25 meters from the target in the laboratory frame exactly half of the original number remain undecayed. As an experimenter, you want to put more distance between the target and your detectors. You are satisfied to have one-eighth of the initial number of pions remaining when they reach your detectors. How far can you place your detectors from the target?
- ANS: 75 m

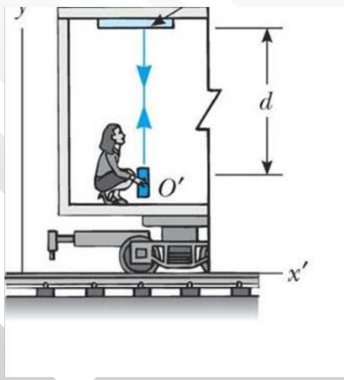
A Gedanken Experiment

- Since light speed c is invariant (i.e. the same in all frames), it is suitable to be used as a clock to measure time and space
- Use light and mirror as clock – light clock
- A mirror is fixed to a moving vehicle, and a light pulse leaves O′ at rest in the vehicle. O′ is the rocket frame.
- Relative to a lab frame observer on Earth, the mirror and O′ move with a speed v.



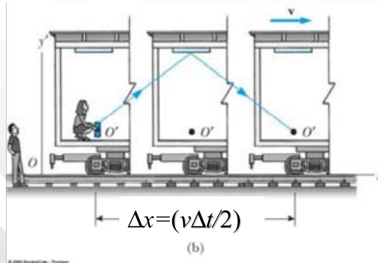
In the rocket frame

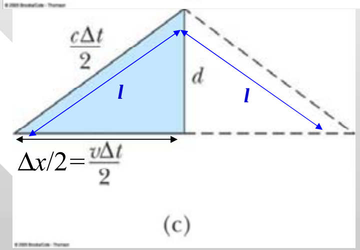
- The light pulse is observed to be moving in the vertical direction only
- The distance the light pulse traversed is $2d$
- The total time travel by the light pulse to the top, get reflected and then return to the source is $\Delta\tau = 2d/c$



In the lab frame

- However, O in the lab frame observes a different path taken by the light pulse – it's a triangle instead of a vertical straight line
- The total light path is longer $= 2l$
- $l^2 = (c\Delta t/2)^2$
  $= d^2 + (\Delta x/2)^2$
  $= d^2 + (v\Delta t/2)^2$

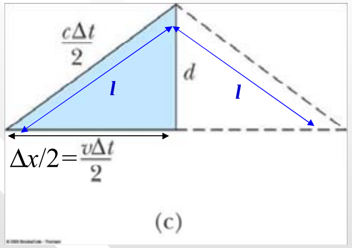




Light triangle

- We can calculate the relationship between Δt, $\Delta\tau$ and v:
- $l^2=(c\Delta t/2)^2=d^2+(v\Delta t/2)^2$ (lab frame)
- $\Delta\tau = 2d/c$ (Rocket frame)
- Eliminating d,

$$t=\frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=\gamma\tau \qquad \gamma=\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\geq 1$$



Time dilation equation

- Time dilation equation $t=\frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=\gamma\tau$
- Gives the value of time $\Delta\tau$ between two events occur a time Δt apart in some reference frame
- Lorentz factor $\gamma=\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\geq 1$
- Note that as $v << c$, $\gamma\approx 1$; as $v \rightarrow c$, $\gamma\rightarrow\infty$
- Appears frequently in SR as a measure of relativistic effect: $\gamma\approx 1$ means little SR effect; $\gamma >> 1$ is the ultra-relativistic regime where SR is most pronounce

RE 38-6

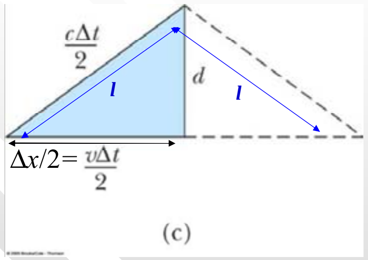
- A set of clocks is assembled in a stationary boxcar. They include a quartz wristwatch, a balance wheel alarm clock, a pendulum grandfather clock, a cesium atomic clock, fruit flies with average individual lifetimes of 2.3 days. A clock based on radioactive decay of nuclei, and a clock timed by marbles rolling down a track. The clocks are adjusted to run at the same rate as one another. The boxcar is then gently accelerated along a smooth horizontal track to a final velocity of 300 km/hr. At this constant final speed, which clocks will run at a different rate from the others as measured in that moving boxcar?

The Metric Equation

- From the light triangle in lab frame and the vertical light pulse in the rocket frame:
- $l^2=(c\Delta t/2)^2=d^2+(\Delta x/2)^2$;
- $d = c\Delta\tau/2$

$\Rightarrow (c\Delta t/2)^2=(c\Delta\tau/2)^2+(\Delta x/2)^2$

- If all the terms that refer to the lab frame are on the right: $(c\Delta\tau)^2=(c\Delta t)^2-(\Delta x)^2$



"the *invariant space-time interval*"

- We call the RHS, $s^2 \equiv (c\Delta t)^2-(\Delta x)^2$ "*invariant space-time interval squared*" (or sometimes simply "*the space-time interval*")
- In words, the space-time interval reads:
- s^2 = ($c\times$time interval between two events as observed in the frame)^2 - (distance interval between the two events as observed in the frame)^2
- We can always calculate the space-time intervals for any pairs of events
- The interval squared s^2 is said to be an *invariant* because it has the same value as calculated by all observers
- Obviously, in the light-clock gadanken experiment, the space-time interval of the two light pulse events $s^2 \equiv (c\Delta t)^2-(\Delta x)^2 = (\Delta\tau)^2$ is positive because $(\Delta\tau)^2 > 0$
- The space-time interval for such two events being positive is deeply related to the fact that such pair of events are causally related
- The space-time interval of such event pairs is said to be '*time-like*' (because the time component in the interval is larger in magnitude than the spatial component)
- Not all pairs of events has a positive space-time interval
- Pairs of events with a negative value of space-time interval is said to be "space-like", and these pairs of event cannot be related via any causal relation

RE 38-8

- Points on the surfaces of the Earth and the Moon that face each other are separated by a distance of 3.76×10^8 m.
- (a) How long does it take light to travel between these points?
- A firecraker explodes at each of these two points; the time between these explosion is one second.
- (b) What is the invariant space-time interval for these two events?
- Is it possible that one of these explosions caused the other explosion?

Solution

(a) Time taken is

$t = L / c = 3.76\times10^8$ m/ 3×10^8 m/s = 1.25 s

(b) $s^2= (ct)^2 - L^2$

$= (3\times10^8 \text{ m/s} \times 1.25 \text{ s})^2 - (3.76\times10^8 \text{ m})^2 = -7.51 \text{ m}^2$

(space-like interval)

(c) It is known that the two events are separated by only 1 s. Since it takes 1.25 s for light to travel between these point, it is impossible that one explosion is caused by the other, given that no information can travel fast than the speed of light.

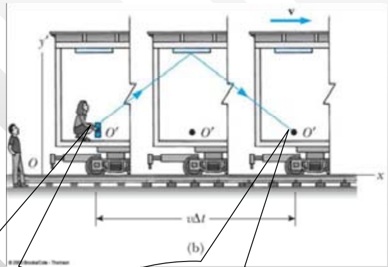
Alternatively, from (b), these events are separated by a space-like space-time interval. Hence it is impossible that the two explosions have any causal relation because

Proper time

- Imagine you are in the rocket frame, O', observing two events taking place at the same spot, separated by a time interval $\Delta\tau$ (such as the emission of the light pulse from source (EV1), and re-absorption of it by the source again, (EV2))
- Since both events are measured on the same spot, they appeared at rest wrp to you
- The time lapse $\Delta\tau$ between the events measured on the clock at rest is called the proper time or wristwatch time (one's own time)

Improper time

- In contrast, the time lapse measured by an observer between two events not at the same spot, i.e. $\Delta x \neq 0$, are termed *improper time*
- E.g., the time lapse, Δt, measured by the observer O observing the two events of light pulse emission and absorption in the train is improper time since both events appear to occur at different spatial location according to him.



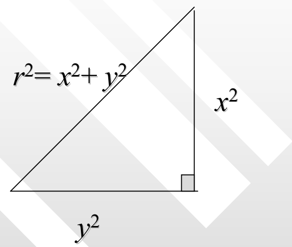
Event 1 occurs here at $x = 0$

Event 2 occurs here at $x = v\Delta t$

Space and time are combined by the metric equation: Space-time

$s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 =$ invariant$=(\Delta\tau)^2$

- The metric equation says $(c\Delta t)^2-(\Delta x)^2$ = invariant $=(\Delta\tau)^2$ in all frames
- *It combines space and time in a single expression on the RHS!!*
- Meaning: Time and space are interwoven in a fabric of space-time, and is not independent from each other anymore (we used to think so in Newton′s absolute space and absolute time system)



The space-time invariant is the 1+1 dimension Minkowsky space-time

analogous to the 3-D Pythagoras theorem with the hypotenuse $r^2 = x^2 + y^2$. However, in the Minkowsky space-time metric, the space and time components differ by an relative minus sign

s^2 relates two different measures of time between the *same* two events

$$s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 = \text{invariant} = (\Delta\tau)^2$$

- (1) the time recorded on clocks in the reference frame in which the events occur at different places (improper time, Δt), and
- (2) the wristwatch time read on the clock carried by a traveler who records the two events as occurring a the same place (proper time, $\Delta\tau$)
- In different frames, Δt and Δx measured for the same two events will yield different values in general. However, the interval squared, $(c\Delta t)^2-(\Delta x)^2$ will always give the same value, see example that ensues

Example of calculation of space-time interval squared

- In the light-clock gedanken experiment: For O′, he observes the proper time interval of the two light pulse events to be $\Delta\tau$. For him, $\Delta x' = 0$ since these events occur at the same place
- Hence, for O′,
- $s'^2 = (c \times \text{time interval observed in the frame})^2 - (\text{distance interval observed in the frame})^2$
- $= (c\Delta\tau)^2 - (\Delta x')^2 = (c\Delta\tau)^2$
- For O, the time-like interval for the two events is $s^2 = (c\Delta t)^2 - (\Delta x)^2 = (c\gamma\Delta\tau)^2 - (v\Delta t)^2 = (c\gamma\Delta\tau)^2 - (v\gamma\Delta\tau)^2 = \gamma^2 (c^2 - v^2)\Delta\tau^2 = c^2\Delta\tau^2 = s'^2$

What happens at high and low speed

$$t = \gamma\tau \qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \geq 1$$

- At low speed, $v << c$, $\gamma \approx 1$, and $\tau \approx t$, not much different, and we can't feel their difference in practice
- However, at high speed, proper time becomes much larger than improper time in comparison
- A journey that takes, say, 10 years to complete, according to a traveler on board (this is his proper time), looks like as if they take 10γ yr according to Earth observers.

Space travel with time-dilation

- A spaceship traveling at speed $v = 0.995c$ is sent to planet 100 light-year away from Earth
- How long will it takes, according to a Earth's observer?
- $\Delta t = 100\text{ ly}/0.995c = 100.05\text{yr}$
- But, due to time-dilation effect, according to the traveler on board, the time taken is only
  $\Delta\tau = \Delta t/\gamma = \Delta t\sqrt{(1-0.995^2)} = 9.992$ yr, not 100.05 yr as the Earthlings think
- So it is still possible to travel a very far distance within one's lifetime ($\Delta\tau \approx 50$ yr) as long as γ (or equivalently, v) is large enough

Nature's Speed Limit

- Imagine one in the lab measures the speed of a rocket v to be larger than c.
- As a consequence, according to $\tau = t\sqrt{1-\left(\frac{v}{c}\right)^2}$

  the proper time measurement $\Delta\tau$ in the rocket would be proportional to an imaginary number, $i = \sqrt{(-1)}$
- This is unphysical (and impossible) as no real time can be proportional to an imaginary number
- Conclusion: no object can be accelerated to a speed greater than the speed of light in vacuum, c
- Or more generally, no information can propagate faster than the light speed in vacuum, c
- Such limit is the consequence required by the logical consistency of SR

Time dilation in ancient legend

- 天上方一日，人间已十年
- One day in the heaven, ten years in the human plane

RE 38-7

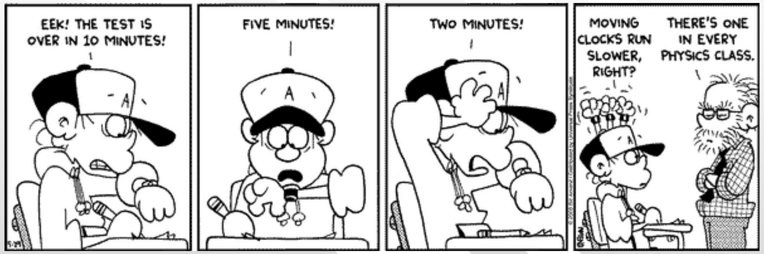
- Find the rocket speed v at which the time $\Delta\tau$ between ticks on the rocket is recorded by the lab clock as $\Delta t = 1.01\Delta\tau$

Satellite Clock Runs Slow?

- An Earth satellite in circular orbit just above the atmosphere circles the Earth once every $T = 90$ min. Take the radius of this orbit to be $r = 6500$ kilometers from the center of the Earth. How long a time will elapse before the reading on the satellite clock and the reading on a clock on the Earth's surface differ by one microsecond?
- For purposes of this approximate analysis, assume that the Earth does not rotate and ignore gravitational effects due the difference in altitude between the two clocks (gravitational effects described by general relativity).

Solution

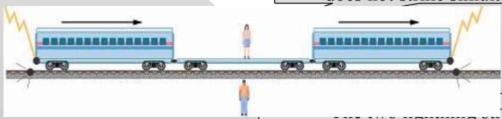
- First we need to know the speed of the satellite in orbit. From the radius of the orbit we compute the circumference and divide by the time needed to cover that circumference:
- $v = 2\pi r/T = (2\pi\times 6500\text{ km})/(90\times 60\text{ s}) = 7.56$ km/s
- Light speed is almost exactly $c = 3\times 105$ km/s. so the satellite moves at the fraction of the speed of light given by
- $(v/c)^2 = [(7.56\text{ km/s})/(3\times 10^5\text{ km/s})]^2 = (2.52\times 10^5)^2 = 6.35\times 10^{-10}$
- The relation between the time lapse Δt recorded on the satellite clock and the time lapse Δt on the clock on Earth (ignoring the Earth's rotation and gravitational effects) is given by .
- We want to know the difference between Δt and Δt i.e. Δt - Δt:
- We are asked to find the elapsed time for which the satellite clock and the Earth clock differ in their reading by one microsecond
- Rearrange the above equation to read
- This is approximately one hour. A difference of one microsecond between atomic clock is easily detectable.



Two events that are simultaneous in one frame are not necessarily simultaneous in a second frame in uniform relative motion

- This is due to the invariance of the space-time invariant in all frames, (i.e. the invariant must have the same value for all frames)

No, I don't agree. These two lightning does not strike simultaneously



...ikes simultaneously

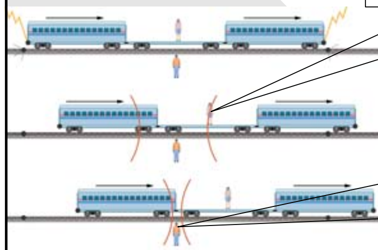
How invariance of space-time interval explains disagreement on simultaneity by two observers

- Consider a pair of events with space-time interval
  $s^2=(c\Delta t)^2-(\Delta x)^2=(c\Delta t')^2-(\Delta x')^2$
- where the primed and un-primed notation refer to space and time coordinates of two frames at relative motion (lets call them O and O′)
- Say O observes two simultaneous event in his frame (i.e. $\Delta t = 0$) and are separate by a distance of (Δx), hence the space-time interval is $s^2 = -(\Delta x)^2$
- The space-time interval for the same two events observed in another frame, O′, $s^2 = (c\Delta t')^2 - (\Delta x')^2$ must read the same, as $-(\Delta x)^2$
- Hence, $(c\Delta t')^2 = (\Delta x')^2 - (\Delta x)^2$ which may not be zero on the RHS. i.e. $\Delta t'$ is generally not zero. This means in frame O′, these events are not observed to be occurring simultaneously

Einstein Train illustration

- Lightning strikes the front and back of a moving train, leaving char marks on both track and train. Each emitted flash spreads out in all directions.

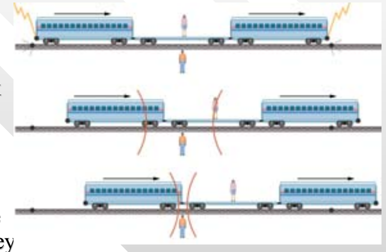
I am equidistant from the front and back char marks on the train. Light has the standard speed in my frame, and equal speed in both direction. The flash from the front of the train arrived first, therefore the flash must have left the front of the train first. The front lightning bolt fell first before the rear light bolt fell. I conclude that the two strokes are not simultaneous.



I stand by the tracks halfway between the char marks on the track. The flashes from the strokes reach me a the same time and I am equidistance from the char marks on the track. I conclude that two events were simultaneous

RE 38-9

- Susan, the rider on the train pictured in the figure is carrying an audio tape player. When she received the light flash from the front of the train she switches on the tape player, which plays very loud music. When she receives the light flash from the back end of the train, Susan switches off the tape player. Will Sam, the observers on the ground be able to hear the music?
- Later Susan and Sam meet for coffee and examine the tape player. Will they agree that some tape has been wound from one spool to the other?
- The answer is: …



Touchstone Example 38-5: Principle of Relativity Applied

- Divide the following items into two lists, On one list, labeled SAME, place items that name properties and laws that are always the same in every frame. On the second list, labeled MAY BE DIF FERENT. place items that name properties that can be different in different frames:
- a. the time between two given events
- b. the distance between two given events
- c. the numerical value of Planck’s constant h
- d. the numerical value of the speed of light c
- e. the numerical value of the charge e on the electron
- f. the mass ni of an electron (measured at rest)
- g. the elapsed time on the wristwatch of a person moving between two given events
- h. the order ot elements in the periodic table
- i. Newton’s First Law of Motion (“A particle initially at rest remains at rest, and …”)
- j. Maxwell’s equations that describe electromagnetic fields in a vacuum
- k. the distance between two simultaneous events

Solution

THE SAME IN ALL FRAMES

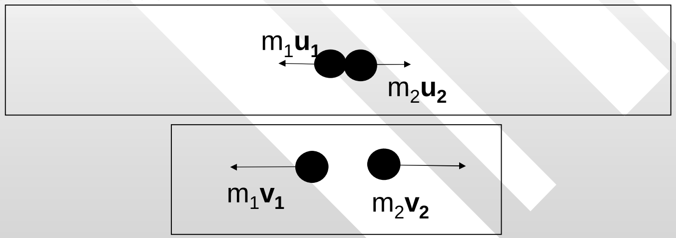
- c. numerical value of *h*
- d. numerical value of *c*
- e. numerical value of *e*
- f. mass of electron (at rest)
- g. wristwatch time between two
- h. order of elements in the periodic table
- i. Newton’s First Law of Motion
- j. Maxwell’s equations
- k. distance between two simultaneous events

MAY BE DIFFERENT IN DIFFERENT FRAMES

- a. time between two given events
- b. distance between two give events

Relativistic Dynamics

- Where does $E=mc^2$ comes from?
- By Einstein′s postulate, the observational law of linear momentum must also hold true in all frames of reference



Conservation of linear momentum classically means

$m_1\mathbf{u_1} + m_2\mathbf{u_2} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$

Modification of expression of linear momentum

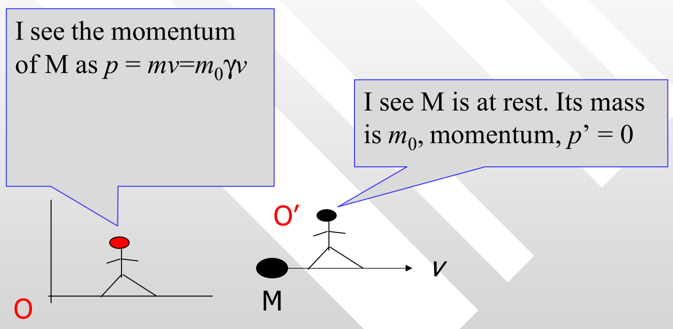
- Classically, $p = mv$. In the other frame, $p' = m'v'$; the mass m' (as seen in frame O') is the same as m (as seen in O frame) – this is according to Newton's mechanics
- However, simple consideration will reveal that in order to preserver the consistency between conservation of momentum and the LT, the definition of momentum has to be modified such that m' is not equal to m.
- That is, the mass of an moving object, m, is different from its value when it's at rest, m_0

In other words...

- In order to preserve the consistency between Lorentz transformation of velocity and conservation of linear momentum, the definition of 1-D linear momentum, classically defined as $p_{classical} = mv$, has to be modified to

  $p_{sr} = mv = \gamma m_0 v$ (where the relativisitic mass $m = \gamma m_0$ is not the same the rest mass m_0
- Read up the text for a more rigorous illustration why the definition of classical momentum is inconsistent with LT

Pictorially...



Two kinds of mass

- Differentiate two kinds of mass: rest mass and relativistic mass
- m_0 = rest mass = the mass measured in a frame where the object is at rest. The rest mass of an object must be the same in all frames (not only in its rest frame).
- Relativistic mass $m = \gamma m_0$. The relativistic mass is speed-dependent

Behaviour of p_{SR} as compared to $p_{classic}$

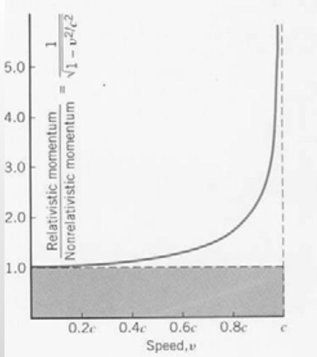


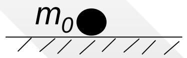
Figure 28.7 This graph shows how the ratio of the magnitude of the relativistic momentum to the magnitude of the nonrelativistic momentum increases as the speed of an object approaches the speed of light.

- Classical momentum is constant in mass, $p_{classic} = m_0 v$
- Relativistic momentum is $p_{SR} = m_0 \gamma v$
- $p_{SR} / p_{classic} = \gamma \rightarrow \infty$ as $v \rightarrow c$
- In the other limit, $v << c$

  $p_{SR} / p_{classic} = 1$

Example

The rest mass of an electron is $m_0 = 9.11 \times 10^{-31}$kg.



If it moves with $v = 0.75\ c$, what is its relativistic momentum?

$p = m_0 \gamma u$

Compare the relativistic p with that calculated with classical definition

Solution

- The Lorentz factor is

  $\gamma = [1-(v/c)^2]^{-1/2} = [1-(0.75c/c)^2]^{-1/2} = 1.51$
- Hence the relativistic momentum is simply

  $p = \gamma\, m_0 \times 0.75c$

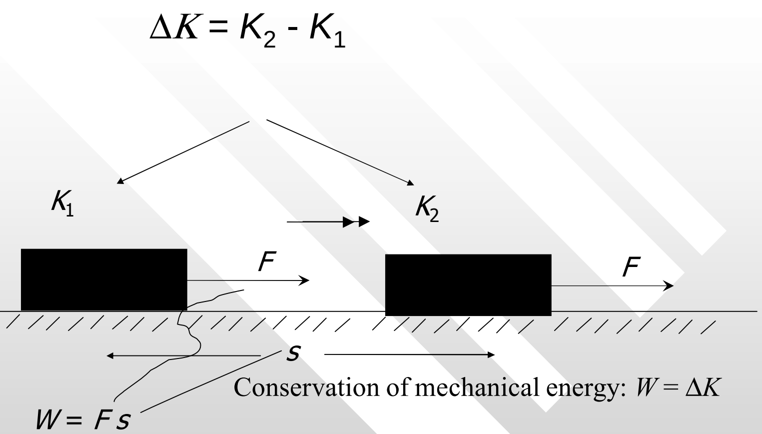
  $= 1.51 \times 9.11 \times 10^{-31}\text{kg} \times 0.75 \times 3 \times 10^8$ m/s

  $= 3.1 \times 10^{-22}$ Ns
- In comparison, classical momentum gives $p_{classical} = m_0 \times 0.75c = 2.5 \times 10^{-22}$ Ns – about 34% lesser than the relativistic value

Work-Kinetic energy theorem

- Recall the law of conservation of mechanical energy:

  Work done by external force on a system, W = the change in kinetic energy of the system, ΔK



Conservation of mechanical energy: $W = \Delta K$

The total energy of the object, $E = K + U$. Ignoring potential energy, E of the object is solely in the form of kinetic energy. If $K_1 = 0$, then $E = K_2$. But in general, U also needs to be taken into account for E.

- In classical mechanics, mechanical energy (kinetic + potential) of an object is closely related to its momentum and mass
- Since in SR we have redefined the classical mass and momentum to that of relativistic version
- m_{class}(cosnt, $= m_0$) → $m_{SR} = m_0\gamma$
- $p_{class} = m_{class}v$ → $p_{SR} = (m_0\gamma)v$
- we must also modify the relation btw work and energy so that the law conservation of energy is consistent with SR
- E.g, in classical mechanics, $K_{class} = p^2/2m = 2mv^2/2$. However, this relationship has to be supplanted by the relativistic version

$$K_{class} = mv^2/2 \rightarrow K_{SR} = E - m_0c^2 = \gamma m_0c^2 - m_0c^2$$

- We will like to derive K in SR in the following slides

Force, work and kinetic energy

- When a force is acting on an object with rest mass m_0, it will get accelerated (say from rest) to some speed (say v) and increase in kinetic energy from 0 to K

K as a function of v can be derived from first principle based on the definition of:

Force, $F = dp/dt$,

work done, $W = \int F\,dx$,

and conservation of mechanical energy, $\Delta K = W$

Derivation of relativistic kinetic energy

Force = rate change of momentum

$$W = \int_{x_1=0}^{x_2} F\,dx = \int_{x_1=0}^{x_2} \frac{dp}{dt}dx = \int_{x_1=0}^{x_2}\left(\frac{dp}{dx}\frac{dx}{dt}\right)dx$$

Chain rule in calculus

$$= \int_{x_1=0}^{x_2}\frac{dp}{dx}v\,dx = \int_{x_1=0}^{x_2}\left(\frac{dp}{dv}\frac{dv}{dx}\right)v\,dx = \int_0^v \frac{dp}{dv}v\,dv$$

where, by definition, $v = \dfrac{dx}{dt}$ is the velocity of the object

Explicitly, $p = \gamma m_0 v$,

Hence, $dp/dv = d/dv\,(\gamma m_0 v)$

$= m_0\,[v\,(d\gamma/dv) + \gamma]$

$= m_0\,[\gamma + (v^2/c^2)\,\gamma^3] = m_0\,(1-v^2/c^2)^{-3/2}$

in which we have inserted the relation

$$\frac{d\gamma}{dv} = \frac{d}{dv}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{v}{c^2}\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = \frac{v}{c^2}\gamma^3$$

integrate

$$W = m_0\int_0^v v\left(1-\frac{v^2}{c^2}\right)^{-3/2} dv$$

$$\Rightarrow K = W = m_0\gamma c^2 - m_0 c^2 = mc^2 - m_0 c^2$$

$$K = m_0\gamma c^2 - m_0 c^2 = mc^2 - m_0 c^2$$

- The relativistic kinetic energy of an object of rest mass m_0 traveling at speed v
- $E_0 = m_0 c^2$ is called the rest energy of the object. Its value is a constant for a given object
- Any object has non-zero rest mass contains energy $E_0 = m_0 c^2$
- One can imagine that masses are 'energies frozen in the form of masses' as per $E_0 = m_0 c^2$
- $E = mc^2$ is the total relativistic energy of an moving object

- Or in other words, the total relativistic energy of a moving object is the sum of its rest energy and its relativistic kinetic energy

$$E = mc^2 = m_0 c^2 + K$$

- The (relativistic) mass of an moving object m is larger than its rest mass m_0 due to the contribution from its relativistic kinetic energy – this is a pure relativistic effect not possible in classical mechanics
- $E = mc^2$ relates the relativistic mass of an object to the total energy released when the object is converted into pure energy

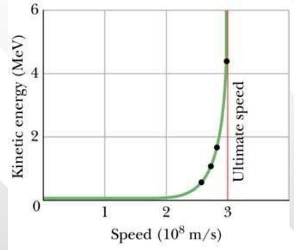
$$K = mc^2 - m_0 c^2$$

- When a given particle is not at rest in a given reference frame, its momentum p is not zero as measured in that frame.
- In this case the total particle energy (the total relativistic energy), E, must be greater than its rest value, E_0.
- The increase in energy due to its motion is the kinetic energy, $K = \Delta E = E - E_0$

$$K = mc^2 - m_0c^2 = \Delta mc^2$$

- One could also think that the kinetic energy gained by an moving object is 'stored' in the form of the mass increase of the object
- As a simile, imagine a man who is craze about cars.
- When his is poor, he owns a proton. When he earn an extra income, he uses them up to buy a new Toyota
- Here, the extra income is a simile to the kinetic energy gained, *K*
- The upgrade in car status (from Proton to Toyota) is a simile to the increase in mass, Δm

Relativistic Kinetic Energy of an electron



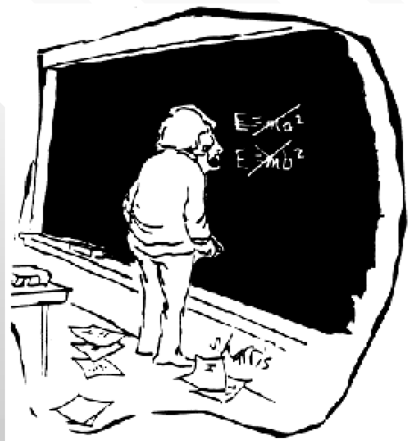
- The kinetic energy increases without limit as the particle speed *v* approaches the speed of light
- In principle we can add as much kinetic energy as we want to a moving particle in order to increase the kinetic energy of a particle without limit
- What is the kinetic energy required to accelerate an electron to the speed of light?

Example, 10 kg of mass, if converted into pure energy, it will be equivalent to $E = mc^2 = 10 \times (3 \times 10^8)^2$ J

$= 9 \times 10^{17}$ J

– equivalent to a few tons of TNT explosive

So, now you know how $E=mc^2$ comes about…



Example 38-6: Energy of Fast Particle

- A particle of rest mass m_0 moves so fast that its total (relativistic) energy is equal to 1.1 times its rest energy.
- (a) What is the speed v of the particle?
- (b) What is the kinetic energy of the particle?

Solution

(a)

- Rest energy $E_0 = m_0c^2$
- We are looking for a speed such that the energy is 1.1 times the rest energy.
- We know how the relativistic energy is related to the rest energy via
- $E = \gamma E_0 = 1.1E_0$
- $\Rightarrow 1/\gamma^2 = 1/1.1^2 = 1/1.21 = 0.8264$
- $1 - v^2/c^2 = 0.8264$
- $\Rightarrow v^2/c^2 = 1 - 0.8264 = 0.1736$
- $\Rightarrow v = 0.4166\ 2c$

(b) Kinetic energy is $K = E - E_0 = 1.1E_0 - E_0 = 0.1E_0 = 0.1\ m_0c^2$

Reduction of relativistic kinetic energy to the classical limit

- The expression of the relativistic kinetic energy

$$K = m_0\gamma c^2 - m_0c^2$$

must reduce to that of classical one in the limit $v/c \to 0$, i.e.

$$\lim_{v<<c} K_{relativistic} = \frac{p^2_{classical}}{2m_0} (= \frac{m_0v^2}{2})$$

Expand γ with binomial expansion

- For u << c, we can always expand γ in terms of $(u/c)^2$ as

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{v^2}{2c^2} + \text{terms of order } \frac{v^4}{c^4} \text{ and higher}$$

$$K = mc^2 - m_0c^2 = \gamma c^2(\gamma - 1)$$
$$= m_0c^2\left[\left(1 + \frac{1}{2}\frac{v^2}{c^2} + ...\right) - 1\right] \approx \frac{m_0v^2}{2}$$

i.e., the relativistic kinetic energy reduces to classical expression in the $v << c$ limit

Example

- A microscopic particle such as a proton can be accelerated to extremely high speed of $v = 0.85c$ in the Tevatron at Fermi National Accelerator Laboratory, US.
- Find its total energy and kinetic energy in eV.

Solution

Due to mass-energy equivalence, sometimes we express the mass of an object in unit of energy

- Electron has rest mass $m_p = 6.7 \times 10^{-27}$kg
- The rest mass of the proton can be expressed as energy equivalent, via
- $m_p c^2 = 1.67\times10^{-31}\text{kg} \times (3 \times 10^8\text{m/s})^2$
- $= 1.5\times10^{-10}$ J
- $= 1.5 \times 10^{-10} \times (1.6\text{x}10^{-19})^{-1}$ eV
- = 939,375,000 eV = 939 MeV

Solution

- First, find the Lorentz factor, $\gamma = 1.89$
- The rest mass of proton, m_0c^2, is 939 MeV
- Hence the total energy is

  $E = mc^2 = \gamma\,(m_0c^2) = 1.89 \times 939$ MeV = 1774 MeV
- Kinetic energy is the difference between the total relativistic energy and the rest mass,

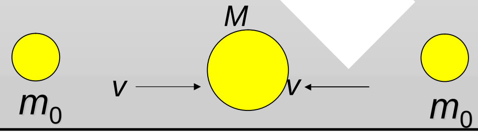
  $K = E - m_0c^2 = (1774 - 939)$ MeV = 835 MeV

Exercise

- Show that the rest mass of an electron is equivalent to 0.51 MeV

Conservation of Kinetic energy in relativistic collision

- Calculate (i) the kinetic energy of the system and (ii) mass increase for a completely inelastic head-on of two balls (with rest mass m_0 each) moving toward the other at speed $v/c = 1.5\times10^{-6}$ (the speed of a jet plane). M is the resultant mass after collision, assumed at rest.



Solution

- (i) $K = 2mc^2 - 2m_0c^2 = 2(\gamma-1)m_0c^2$
- (ii) $E_{before} = E_{after} \Rightarrow 2\gamma m_0c^2 = Mc^2 \Rightarrow M = 2\gamma m_0$
- Mass increase $\Delta M = M - 2m_0 = 2(\gamma-1)m_0$
- Approximation: $v/c = ...=1.5\text{x}10^{-6} \Rightarrow \gamma \approx 1 + ½\, v^2/c^2$ (binomail expansion) $\Rightarrow M \approx 2(1 + ½\, v^2/c^2)m_0$
- Mass increase $\Delta M = M - 2m_0$
- $\approx (v^2/c^2)m_0 = 1.5\text{x}10^{-6}m_0$
- Comparing K with ΔMc^2: the kinetic energy is not lost in relativistic inelastic collision but is converted into the mass of the final composite object, i.e. kinetic energy is conserved
- In contrast, in classical mechanics, momentum is conserved but kinetic energy is not in an inelastic collision

Relativistic momentum and relativistic Energy

In terms of relativistic momentum, the relativistic total energy can be expressed as followed

$$E^2 = \gamma^2 m_0^2 c^4;\, p^2 = \gamma^2 m_0^2 v^2 \Rightarrow \frac{v^2}{c^2} = \frac{c^2 p^2}{E^2}$$

$$\Rightarrow E^2 = \gamma^2 m_0^2 c^4 = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}} = m_0^2 c^2 \left(\frac{m_0^2 c^4 E^2}{E^2 - c^2 p^2} \right)$$

$$\boxed{E^2 = p^2 c^2 + m_0^2 c^4}$$

Conservation of energy-momentum

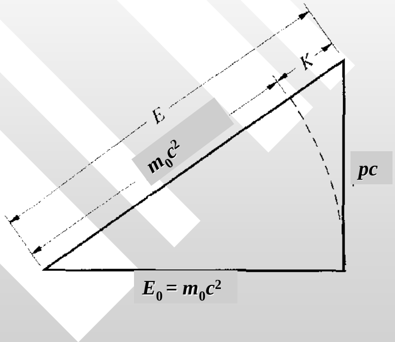
Invariance in relativistic dynamics

- Note that $E^2 - p^2c^2$ is an invariant, numerically equal to $m_0{}^2c^4$
- i.e., in any dynamical process, the difference between the total energy and total momentum of a given system must remain unchanged
- In additional, when observed in other frames of reference, the total relativistic energy and total relativistic momentum may have different values, but their difference, $E^2 - p^2c^2$, must remain invariant (i.e. to remain the same value in different frames)
- Such invariance greatly simplify the calculations in relativistic dynamics

A mnemonic energy-momentum invariance, $E^2 - p^2c^2 = m_0c^2$

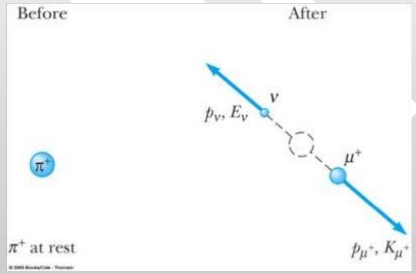
- A useful mnemonic device for recalling the relationships between E_0, p, K, and E. Note that to put all variables in energy units, the quantity pc must be used
- Note that when $m_0 = 0$, e.g. for the case of photon, the energy-momentum invariance reduces to

$E = pc = (h/\lambda)c$ (later topic)



Example: measuring pion mass using conservation of momentum-energy

- pi meson decays into a muon + massless neutrino
- If the mass of the muon is known to be 106 MeV/c^2, and the kinetik energy of the muon is measured to be 4.6 MeV, find the mass of the pion



Solution

Relationship between Kinetic energy and momentum:

$$E_\mu^2 = p_\mu^2c^2 + m_\mu^2c^4$$

Conservation of relativistic energy: $E_\pi = E_\mu + E_\nu$

$$\Rightarrow m_\pi c^2 = \sqrt{m_\mu^2c^4 + c^2p_\mu{}^2} + \sqrt{\not{m}_\nu^2c^4 + c^2p_\nu{}^2}$$

$$\Rightarrow m_\pi c = \sqrt{m_\mu^2c^2 + p_\mu{}^2} + p_\mu$$

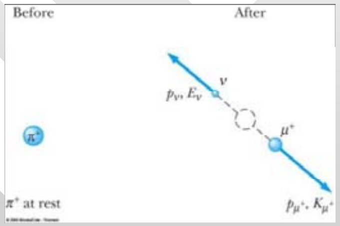
Momentum conservation: $p_\mu = p_\nu$

Also, total energy = K.E. + rest energy

$$E_\mu = K_\mu + m_\mu c^2$$

$$\Rightarrow p_\mu^2c^2 = (K_\mu + m_\mu c^2)^2 - m_\mu^2c^4;$$

$$p_\mu c = \sqrt{(K_\mu + m_\mu c^2)^2 - m_\mu^2c^4}$$



Plug $p_\mu^2c^2 = (K_\mu + m_\mu c^2)^2 - m_\mu^2c^4$ into

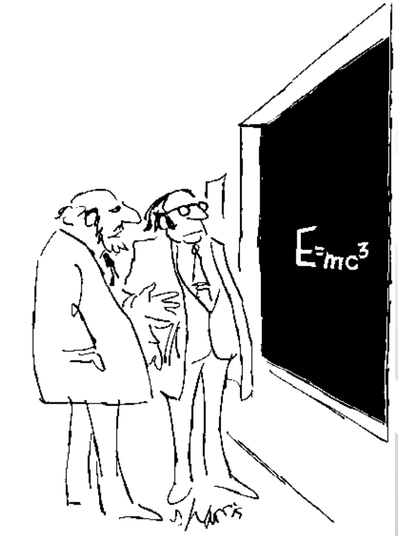
$$m_\pi c^2 = \sqrt{m_\mu^2c^4 + c^2p_\mu{}^2} + cp_\mu$$

$$= \sqrt{m_\mu{}^2c^4 + \left[(K_\mu + m_\mu c^2)^2 - m_\mu^2c^4\right]} + \sqrt{(K_\mu + m_\mu c^2)^2 - m_\mu^2c^4}$$

$$= (K_\mu + m_\mu c^2) + \sqrt{(K_\mu^2 + 2K_\mu m_\mu c^2)}$$

$$= \left(4.6\text{MeV} + \frac{106\text{MeV}}{c^2}c^2\right) + \sqrt{(4.6\text{MeV})^2 + 2(4.6\text{MeV})\left(\frac{106\text{MeV}}{c^2}\right)c^2}$$

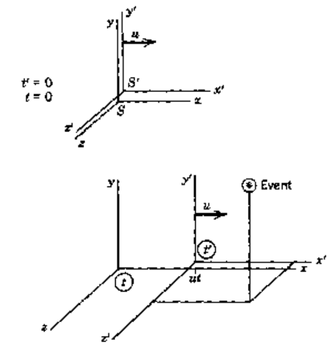
$$= 111\text{MeV} + \sqrt{996}\text{MeV} = 143\text{MeV}$$



"These days *everything* is higher."

Observing an event from lab frame and rocket frame

Figure 1-13
Reference systems S and S' in relative motion. An event occurs at (x, y, z, t) in S and (x', y', z', t') in S'. In this view, S' is moving through S.



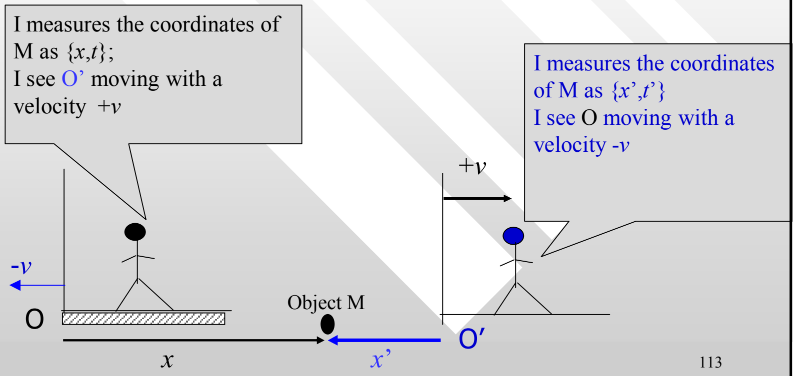
Lorentz Transformation

- All inertial frames are equivalent
- Hence all physical processes analysed in one frame can also be analysed in other inertial frame and yield consistent results
- Any event observed in two frames of reference must yield consistent results related by transformation laws
- Specifically such a transformation law is required to related the space and time coordinates of an event observed in one frame to that observed from the other

Different frame uses different notation for coordinates

- O' frame uses $\{x',y',z';t'\}$ to denote the coordinates of an event, whereas O frame uses $\{x,y,z;t\}$
- How to related $\{x',y',z',t'\}$ to $\{x,y,z;t\}$?
- In Newtonian mechanics, we use Galilean transformation

Two observers in two inertial frames with relative motion use different notation

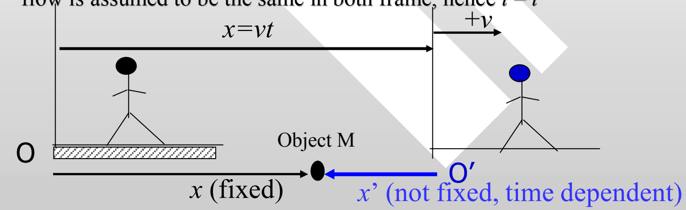


Galilean transformation (applicable only for $v \ll c$)

- For example, the spatial coordinate of the object M as observed in O is x and is being observed at a time t, whereas according to O', the coordinate for the space and time coordinates are x' and t'. At low speed $v \ll c$, the transformation that relates $\{x',t'\}$ to $\{x,t\}$ is given by Galilean transformation
- $\{x'=x-vt,\ t' = t\}$ (x' and t' in terms of x,t)
- $\{x = x' + vt,\ t = t'\}$ (x and t in terms of x',t')

Illustration on Galilean transformation of $\{x'=x-vt,\ t' = t\}$

- Assume object M is at rest in the O frame, hence the coordinate of the object M in O frame is fixed at x
- Initially, when $t = t' = 0$, O and O' overlap
- O' is moving away from O at velocity $+v$
- The distance of the origin of O' is increasing with time. At time t (in O frame), the origin of O' frame is at an instantaneous distance of $+vt$ away from O
- In the O' frame the object M is moving away with a velocity $-v$ (to the left)
- Obviously, in O' frame, the coordinate of the object M, denoted by x', is time-dependent, being $x' = x - vt$
- In addition, under current assumption (i.e. classical viewpoint) the rate of time flow is assumed to be the same in both frame, hence $t = t'$



However, GT contradicts the SR postulate when v approaches the speed of light, hence it has to be supplanted by a relativistic version of transformation law when near-to-light speeds are involved: Lorentz transformation

Derivation of Lorentz transformation

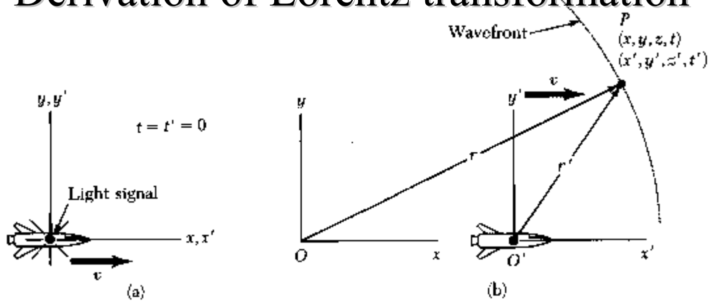
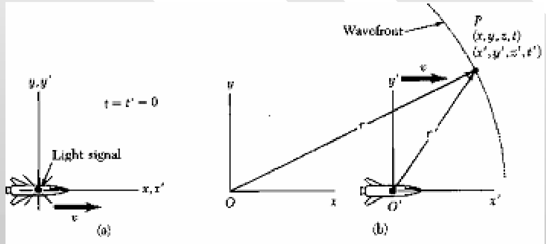


Figure 1.13 A rocket moves with a speed v along the xx' axes. (a) A pulse of light is sent out from the rocket at $t = t' = 0$ when the two systems coincide. (b) Coordinates of some point P on an expanding spherical wavefront as measured by observers in both inertial systems. (This figure is entirely schematic, and you should not be misled by the geometry.)

Our purpose is to find the transformation that relates $\{x,t\}$ to $\{x',t'\}$

Derivation of Lorentz transformation

- Consider a rocket moving with a speed u (O' frame) along the xx' direction wrp to the stationary O frame
- A light pulse is emitted at the instant $t' = t = 0$ when the two origins of the two reference frames coincide
- The light signal travels as a spherical wave at a constant speed c in both frames
- After in time interval of t, the origin of the wave centred at O has a radius $r = ct$, where $r^2 = x^2 + y^2 + z^2$



Arguments

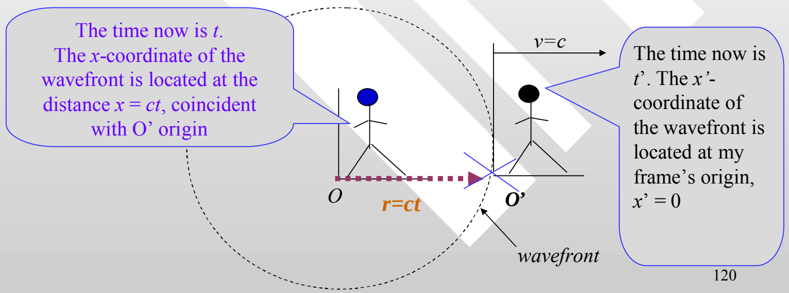
- From the view point of O', after an interval t' the origin of the wave, centred at O' has a radius:

$$r' = ct', \ (r')^2 = (x')^2 + (y')^2 + (z')^2$$

- $y'=y$, $z'=z$ (because the motion of O' is along the xx') axis – no change for y,z coordinates (*condition A*)
- The transformation from x to x' (and vice versa) must be linear, i.e. $x' \propto x$ (*condition B*)
- Boundary condition (1): If $v = c$, from the viewpoint of O, the origin of O′ is located on the wavefront (to the right of O)
- $\Rightarrow x' = 0$ must correspond to $x = ct$
- Boundary condition (2): In the same limit, from the viewpoint of O′, the origin of O is located on the wavefront (to the left of O′)
- $\Rightarrow x = 0$ corresponds to $x' = -ct'$
- Putting everything together we assume the transformation that relates x' to $\{x, t\}$ takes the form $x' = k(x - ct)$ as this will fulfill all the conditions (B) and boundary condition (1) ; (k some proportional constant to be determined)
- Likewise, we assume the form $x = k(x' + ct')$ to relate x to $\{x', t'\}$ as this is the form that fulfill all the conditions (B) and boundary condition (2)

Illustration of Boundary condition (1)

- $x = ct$ ($x' = ct'$) is defined as the x-coordinate (x'-coordinate) of the wavefront in the O (O') frame
- Now, we choose O as the rest frame, O' as the rocket frame. Furthermore, assume O' is moving away to the right from O with light speed, i.e. $v = +c$
- Since $u = c$, this means that the wavefront and the origin of O' coincides all the time
- For O, the x-coordinate of the wavefront is moving away from O at light speed; this is tantamount to the statement that $x = ct$
- From O' point of view, the x'-coordinate of the wavefront is at the origin of it's frame; this is tantamount to the statement that $x' = 0$
- Hence, in our yet-to-be-derived transformation, $x' = 0$ must correspond to $x = ct$

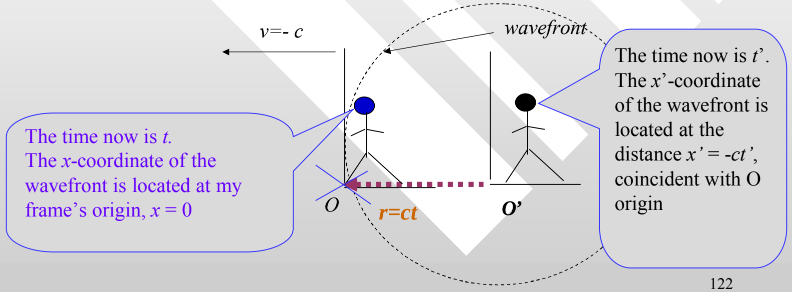


Permuting frames

- Since all frames are equivalent, physics analyzed in O' frame moving to the right with velocity $+v$ is equivalent to the physics analyzed in O frame moving to the left with velocity $-v$
- Previously we choose O frame as the lab frame and O' frame the rocket frame moving to the right (with velocity $+v$ wrp to O)
- Alternatively, we can also fix O' as the lab frame and let O frame becomes the rocket frame moving to the left (with velocity $-v$ wrp to O')

Illustration of Boundary condition (2)

- Now, we choose O' as the rest frame, O as the rocket frame. From O' point of view, O is moving to the left with a relative velocity $v = -c$
- From O' point of view, the wavefront and the origin of O coincides. The x'-coordinate of the wavefront is moving away from O' at light speed to the left; this is tantamount to the statement that $x' = -ct'$
- From O point of view, the x-coordinate of the wavefront is at the origin of it's frame; this is tantamount to the statement that $x = 0$
- Hence, in our yet-to-be-derived transformation, $x = 0$ must correspond to $x' = ct'$



Finally, the transformation obtained

- We now have
- $r = ct$, $r^2 = x^2 + y^2 + z^2$; $y' = y$, $z' = z$; $x = k(x' + ct')$;
- $r' = ct'$, $r'^2 = x'^2 + y'^2 + z'^2$; $x' = k(x - ct)$;
- With some algebra, we can solve for $\{x', t'\}$ in terms of $\{x, t\}$ to obtain the desired transformation law (do it as an exercise)
- The constant k turns out to be identified as the Lorentz factor, γ

$$x' = \frac{x - ut}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \gamma(x - ut) \qquad t' = \frac{t - \left(u/c^2\right)x}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \gamma\left[t - (u/c^2)x\right]$$

(x' and t' in terms of x, t)

Space and time now becomes state-of-motion dependent (via γ)

- Note that, now, the length and time interval measured become dependent of the state of motion (in terms of γ) – in contrast to Newton's classical viewpoint
- **Lorentz transformation reduces to Galilean transformation when $v \ll c$ (show this yourself)**
- **i.e. LT → GT in the limit $v \ll c$**

How to express $\{x, t\}$ in terms of $\{x', t'\}$?

- We have expressed $\{x',t'\}$ in terms of $\{x,t\}$ as per

$$x' = \frac{x - vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma(x - vt) \quad t' = \frac{t - (v/c^2)x}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\left[t - (v/c^2)x\right]$$

- Now, how do we express $\{x, t\}$ in terms of $\{x', t'\}$?

Simply permute the role of x and x' and reverse the sign of v

$$x \leftrightarrow x', v \rightarrow -v$$

$$x' = \gamma(x - vt) \rightarrow x = \gamma(x' + vt')$$

$$t' = \gamma\left[t - (v/c^2)x\right] \rightarrow t = \gamma\left[t' + (v/c^2)x'\right]$$

The two transformations above are equivalent; use which is appropriate in a given question

Length contraction

- Consider the rest length of a ruler as measured in frame O′ is $L' = \Delta x' = x'_2 - x'_1$ (proper length) measured at the same instance in that frame ($t'_2 = t'_1$)
- What is the length of the rule as measured by O?
- The length in O, according the LT is
  $L' = \Delta x' = x'_2 - x'_1 = \gamma[(x_2 - x_1) - v(t_2 - t_1)]$ (improper length)
- The length of the ruler in O is simply the distance btw x_2 and x_1 measured at the same instance in that frame ($t_2 = t_1$)
- As a consequence, we obtain the relation between the proper length measured by the observer at rest wrp to the ruler and that measured by an observer who is at a relative motion wrp to the ruler:

$$L' = \gamma L$$

Moving rulers appear shorter

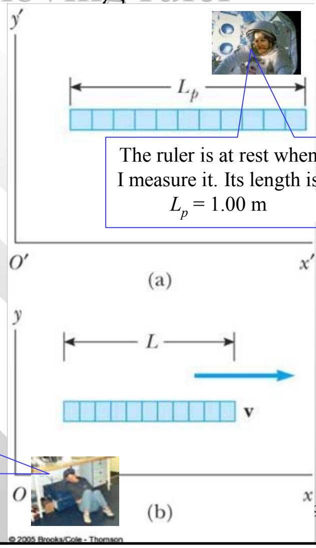
$$L' = \gamma L$$

- L' is defined as the proper length = length of and object measured in the frame in which the object is at rest
- L is the length measured in a frame which is moving wrp to the ruler
- If an observer at rest wrp to an object measures its length to be L', an observer moving with a relative speed u wrp to the object will find the object to be shorter than its rest length by a factor $1 / \gamma$
- i.e., the length of a moving object is measured to be shorter than the proper length – hence "length contraction"
- In other words, a moving rule will appear shorter!!

Example of a moving ruler

Consider a meter rule is carried on beard in a rocket (call the rocket frame O′)

- An astronaut in the rocket measure the length of the ruler. Since the ruler is at rest wrp to the astronaut in O′, the length measured by the astronaut is the proper length, L_p = 1.00 m, see (a)
- Now consider an observer on the lab frame on Earth. The ruler appears moving when viewed by the lab observer. If the lab observer attempts to measure the ruler



The ruler is at rest when I measure it. Its length is L_p = 1.00 m

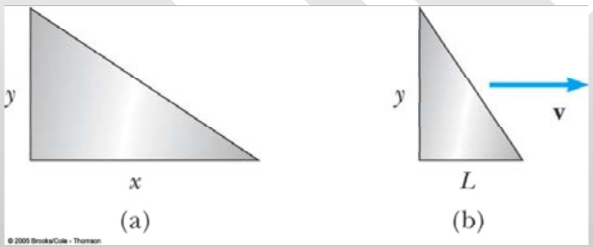
The ruler is at moving at a speed v when I measure it. Its length is L = 0.999 m

RE 38-11

- What is the speed v of a passing rocket in the case that we measure the length of the rocket to be half its length as measured in a frame in which the rocket is at rest?

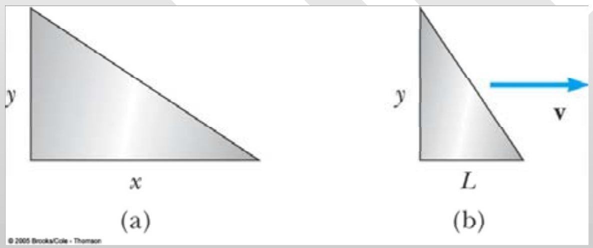
Length contraction only happens along the direction of motion

Example: A spaceship in the form of a triangle flies by an oberver at rest wrp to the ship (see fig (a)), the distance x and y are found to be 50.0 m and 25.0 m respectively. What is the shape of the ship as seen by an observer who sees the ship in motion along the direction shown in fig (b)?



Solution

- The observer sees the horizontal length of the ship to be contracted to a length of
- $L = L_p/\gamma$ = 50 m $\sqrt{(1 - 0.950^2)}$ = 15.6 m
- The 25 m vertical height is unchanged because it is perpendicular to the direction of relative motion between the observer and the spaceship.

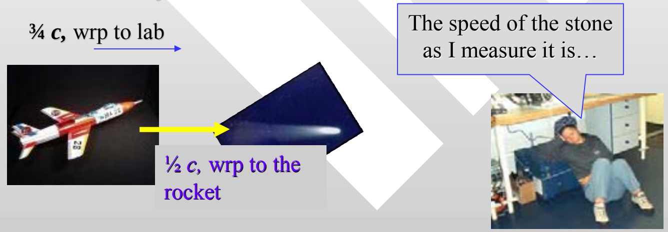


Similarly, one could also time dilation from the LT

Do it as homework

What is the velocities of the ejected stone?

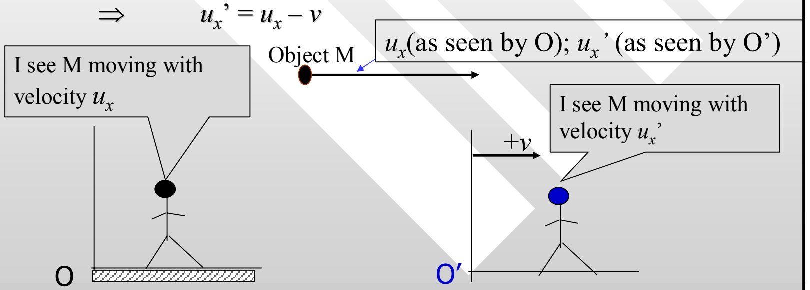
- Imagine you ride on a rocket moving ¾ c wrp to the lab. From your rocket you launch a stone forward at ½ c, as measured in your rocket frame. What is the speed of the stone observed by the lab observer?



Galilean transformation of velocity (applicable only for $u_x, v << c$)

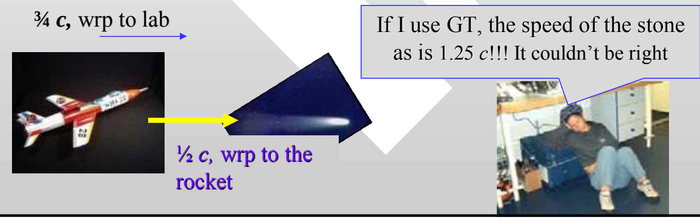
- Now, say object M is moving as a velocity of v wrp to the lab frame O
- What is the velocity of M as measured by O'?
- Differentiate $x' = x - vt$ wrp to t ($=t'$), we obtain
- $d(x')/dt' = d(x - vt)/dt = d(x)/dt - v$

$\Rightarrow \quad u_x' = u_x - v$



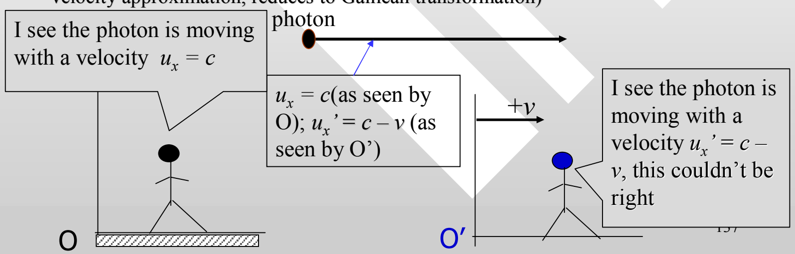
Adding relativistic velocities with Galilean

- According to GT of velocity (which is valid at low speed regime $u << c$),
- the lab observer would measure a velocity of $u_x = u_x' + v$ = ½ c + ¾ c
- = 1.25 c for the ejected stone.
- However, in SR, c is the ultimate speed and no object can ever exceed this ultimate speed limit
- So something is no right here…Galilean addition law is no more valid to handle addition of relativistic velocities (i.e. at speed near to c)



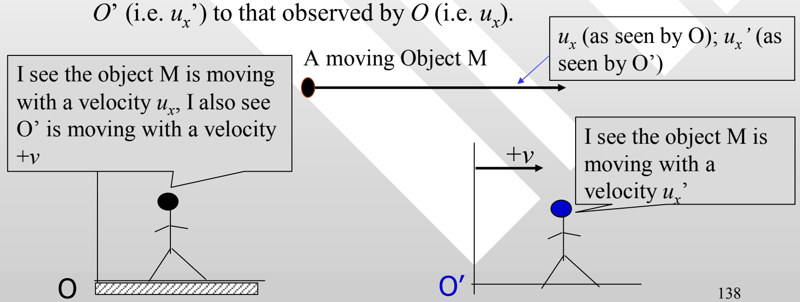
If applied to light Galilean transformation of velocity contradicts the SR Postulate

- Consider another case, now, a photon (particle of light) observed from different frames
- A photon. being a massless particle of light must move at a speed $u_x = c$ when observed in O frame
- However Galilean velocity addition law $u_x' = u_x - v$, if applied to the photon, says that in O' frame, the photon shall move at a lower speed of $u_x' = u_x - v = c - v$
- This is a contradiction to the constancy of light speed in SR
- Conclusion: GT cannot be applicable when dealing with object moving near or at the speed of light
- It has to be supplanted by a more general form of transformation (that must, as a low-velocity approximation, reduces to Galilean transformation)



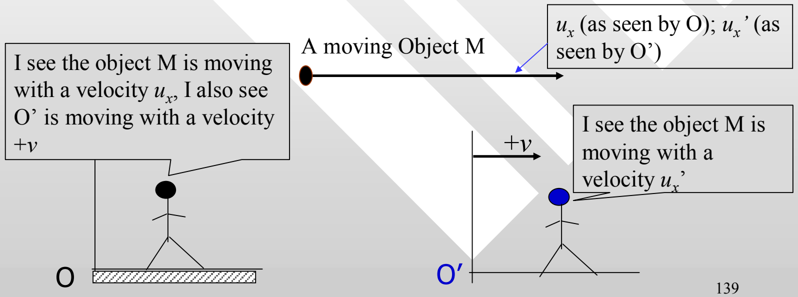
Relativity of velocities

- The generalised transformation law of velocity used for addition of relativistic velocities is called Lorentz transformation of velocities, derived from the Lorentz transformation of space-time
- Our task is to relate the velocity of the object M as observed by *O'* (i.e. u_x') to that observed by *O* (i.e. u_x).



Relativity of velocities

- Consider an moving object being observed by two observers, one in the lab frame and the other in the rocket frame
- We could derive the Lorentz transformation of velocities by taking time derivative wrp to the LT for space-time, see next slide



Derivation of Lorentz transformation of velocities

- By definition, $u_x = dx/dt$, $u'_x = dx'/dt'$
- The velocity in the O′ frame can be obtained by taking the differentials of the Lorentz transformation

$$x' = \gamma(x - vt) \quad t' = \gamma\left[t - (v/c^2)x\right]$$

$$dx' = \gamma(dx - vdt),\ dt' = \gamma(dt - \frac{v}{c^2}dx)$$

Combining

$$u'_x = \frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma(dt - \frac{v}{c^2}dx)} = \frac{dt\left(\frac{dx}{dt} - v\frac{dt}{dt}\right)}{dt\left(\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

$$= \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

where we have made used of the definition $u_x = dx/dt$

Comparing the LT of velocity with that of GT

Lorentz transformation of velocity:

$$u'_x = \frac{dx'}{dt'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

Galilean transformation of velocity:

$$u'_x = u_x - v$$

GT reduces to LT in the limit $u_x v << c^2$

- *Please try to be clear about the definition of u_x, u'_x, v so that you wont get confused*

LT is consistent with the constancy of speed of light

- In either O or O′ frame, the speed of light seen must be the same, *c*. LT is consistent with this requirement.
- Say object M is moving with speed of light as seen by O, i.e. $u_x = c$
- According to LT, the speed of M as seen by O′ is

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{c - u}{1 - \frac{v}{c}} = \frac{c - v}{\frac{1}{c}(c - v)} = c$$

- That is, in either frame, both observers agree that the speed of light they measure is the same, $c = 3 \times 10^8$m/s

How to express u_x in terms of u_x'?

- Simply permute v with $-v$ and change the role of u_x with that of u_x':

$$u_x \to u'_x, u'_x \to u_x, v \to -v$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \to u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

Recap: Lorentz transformation relates $\{x', t'\} \leftrightarrow \{x, t\}$; $u'_x \leftrightarrow u_x$

$$x' = \gamma(x - vt) \qquad t' = \gamma\left[t - (v/c^2)x\right]$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$x = \gamma(x' + vt') \qquad t = \gamma\left[t' + (v/c^2)x'\right]$$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

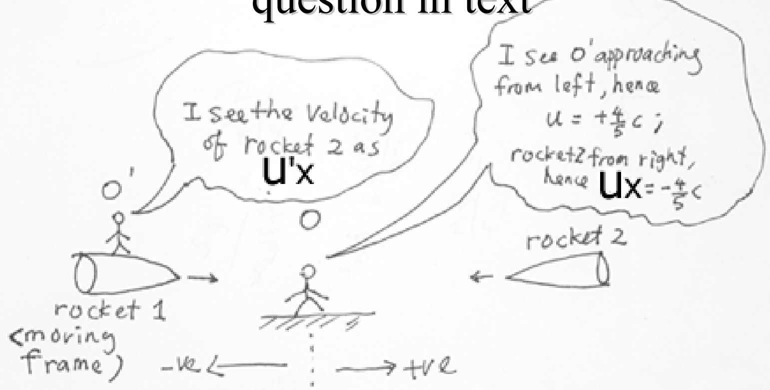
RE 38-12

- A rocket moves with speed 0.9c in our lab frame. A flash of light is sent toward from the front end of the rocket. Is the speed of that flash equal to 1.9 c as measured in our lab frame? If not, what is the speed of the light flash in our frame? Verify your answer using LT of velocity formula.

Example (relativistic velocity addition)

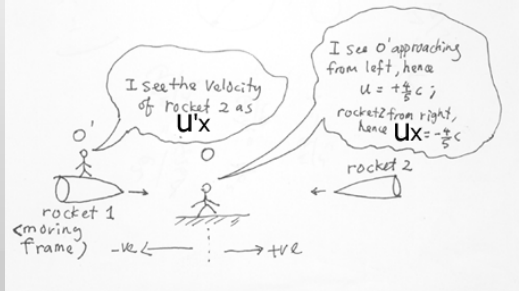
- Rocket 1 is approaching rocket 2 on a head-on collision course. Each is moving at velocity 4c/5 relative to an independent observer midway between the two. With what velocity does rocket 2 approaches rocket 1?

Diagramatical translation of the question in text

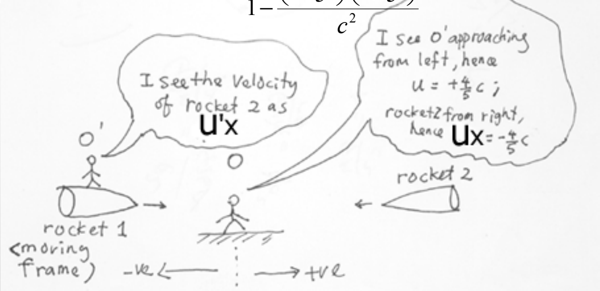


Note: c.f. in GT, their relative speed would just be 4c/5 + 4c/5 = 1.6 c – which violates constancy of speed of light postulate. See how LT handle this situation:

- Choose the observer in the middle as in the stationary frame, O
- Choose rocket 1 as the moving frame O‘
- Call the velocity of rocket 2 as seen from rocket 1 u'_x. This is the quantity we are interested in
- Frame O' is moving in the +ve direction as seen in O, so $v = +4c/5$
- The velocity of rocket 2 as seen from O is in the -ve direction, so $u_x = - 4c/5$
- Now, what is the velocity of rocket 2 as seen from frame O', $u'_x = ?$ (intuitively, u'_x must be in the negative direction)



Using LT: $$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{\left(-\frac{4c}{5}\right) - \left(+\frac{4c}{5}\right)}{1 - \frac{\left(-\frac{4c}{5}\right)\left(+\frac{4c}{5}\right)}{c^2}} = -\frac{40}{41}c$$



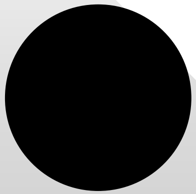
i.e. the velocity of rocket 2 as seen from rocket 1 (the moving frame, O') is $-40c/41$, which means that O' sees rocket 2 moving in the –ve direction (to the left in the picture), as expected.

Doppler Shift

- R.I.Y

Matter, energy and interactions

- One can think that our universe is like a stage existing in the form of space-time as a background
- All existence in our universe is in the form of either matter or energy (Recall that matter and energy are \`equivalent' as per the equation $E = mc^2$)



matter

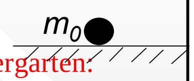
energy

Interactions

- Matter and energy exist in various forms, but they constantly transform from one to another according to the law of physics
- we call the process of transformation from one form of energy/matter to another energy/matter as 'interactions'
- Physics attempts to elucidate the interactions between them
- But before we can study the basic physics of the matter-energy interactions, we must first have some general idea to differentiate between the two different modes of physical existence:
- matter and wave
- This is the main purpose of this lecture

Matter (particles)

- **Consider a particles with mass:**



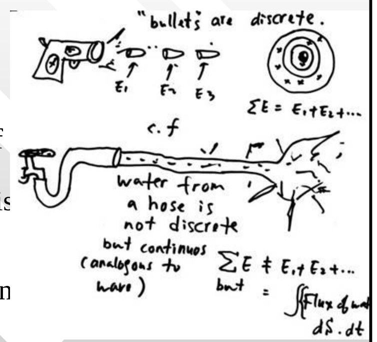
- you should know the following facts since kindergarten:
- A particle is discrete, or in another words, corpuscular, in nature.
- a particle can be localized completely, has mass and electric charge that can be determined with infinite precision (at least in principle)
- So is its momentum
- These are all implicitly assumed in Newtonian mechanics
- This is to be contrasted with energy exists in the forms of wave which is not corpuscular in nature (discuss later)

Energy in particle is corpuscular (discrete) i.e. not spread out all over the place like a continuum

- The energy carried by a particle is given by

$$E^2 = m_0^2c^4 + p^2c^2$$

- The energy of a particles is concentrated within the boundary of a particle (e.g. in the bullet)
- Hence we say "energy of a particle is corpuscular"
- This is in contrast to the energy carried by the water from the host, in which the energy is distributed spread all over the space in a continuous manner



Example of particles

- Example of `particles': bullet, billiard ball, you and me, stars, sands, etc...
- Atoms, electrons, molecules (or are they?)

What is not a `particle'?

- Waves - electromagnetic radiation (light is a form of electromagnetic radiation), mechanical waves and matter waves is classically thought to not have attributes of particles as mentioned

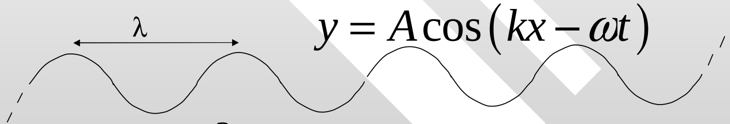
Analogy

- Imagine energy is like water
- A cup containing water is like a particle that carries some energy within it
- Water is contained within the cup as in energy is contained in a particle.
- The water is not to be found outside the cup because they are all retained inside it. Energy of a particle is corpuscular in the similar sense that they are all inside the carrier which size is a finite volume.
- In contrast, water that is not contained by any container will spill all over the place (such as water in the great ocean). This is the case of the energy carried by wave where energy is not concentrated within a finite volume but is spread throughout the space

Wave

- Three kinds of wave in Nature: mechanical, electromagnetical and matter waves
- The simplest type of wave is strictly sinusoidal and is characterised by a `sharp' frequency ν (= 1/*T*, *T* = the period of the wave), wavelength λ and its travelling speed *c*

$$y = A\cos(kx - \omega t)$$



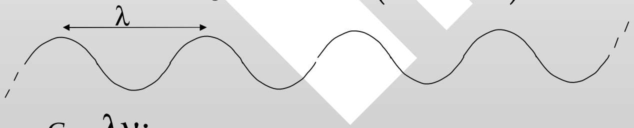
$$c = \lambda\nu; k = \frac{2\pi}{\lambda}$$

A `pure' (or 'plain') wave which has `sharp' wavelength and frequency

Quantities that haracterise a pure wave

- The quantities that quantify a pure (or called a plane) wave: λ, wave length, equivalent to $k = 2\pi / \lambda$, the wave number $\nu = 1/T$, frequency, equivalent angular frequency, $\omega = 2\pi\nu$
- c speed of wave, related to the above quantities via
- $c = \lambda\nu = \omega/k$

$$y = A\cos(kx - \omega t)$$



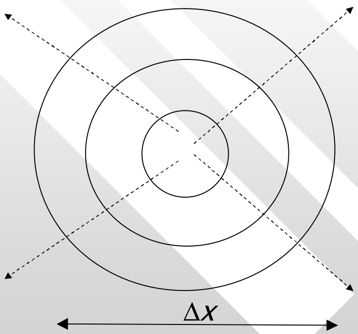
$C = \lambda\nu;$

Where is the wave?

- For the case of a particle we can locate its location and momentum precisely
- But how do we 'locate' a wave?
- Wave spreads out in a region of space and is not located in any specific point in space like the case of a particle
- To be more precise we says that a plain wave exists within some region in space, Δx
- For a particle, Δx is just the 'size' of its dimension, e.g. Δx for an apple is 5 cm, located exactly in the middle of a square table, x = 0.5 m from the edges. In principle, we can determine the position of x to infinity
- But for a wave, Δx could be infinity

In fact, for the `pure' (or 'plain') wave which has `sharp' wavelength and frequency mentioned in previous slide, the Δx is infinity

For example, a ripple



the ripple exists within the region Δx

A pure wave has Δx → infinity

- If we know the wavelength and frequency of a pure wave with infinite precision (= the statement that the wave number and frequency are 'sharp'), one can shows that :
- The wave cannot be confined to any restricted region of space but must have an infinite extension along the direction in which it is propagates
- In other words, the wave is 'everywhere' when its wavelength is 'sharp'
- This is what it means by the mathematical statement that "Δx is infinity"

More quantitatively,

$$\Delta x \Delta \lambda \geq \lambda^2$$

- This is the uncertainty relationships for classical waves

  $\Delta\lambda$ is the uncertainty in the wavelength.
- When the wavelength `sharp' (that we knows its value precisely), this would mean $\Delta\lambda = 0$.
- In other words, $\Delta\lambda \rightarrow$ infinity means we are totally ignorant of what the value of the wavelength of the wave is.

Δx is the uncertainty in the location of the wave (or equivalently, the region where the wave exists)

- $\Delta x = 0$ means that we know exactly where the wave is located, whereas $\Delta x \rightarrow$ infinity means the wave is spread to all the region and we cannot tell where is it's `location'

  $\Delta\lambda\ \Delta x \geqslant \lambda^2$ means the more we knows about x, the less we knows about λ as Δx is inversely proportional to $\Delta\lambda$

Other equivalent form

- $\Delta x \Delta \lambda \geq \lambda^2$ can also be expressed in an equivalence form

$$\Delta t \Delta \nu \geq 1$$

  via the relationship $c = \nu\lambda$ and $\Delta x = c\Delta t$
- Where Δt is the time required to measure the frequency of the wave
- The more we know about the value of the frequency of the wave, the longer the time taken to measure it
- If u want to know exactly the precise value of the frequency, the required time is Δt = infinity
- We will encounter more of this when we study the Heisenberg uncertainty relation in quantum physics

- The classical wave uncertain relationship

$$\Delta x \Delta \lambda \geq \lambda^2$$

- can also be expressed in an equivalence form

$$\Delta t \Delta \nu \geq 1$$

  via the relationship $c = \nu\lambda$ and $\Delta x = c\Delta t$
- Where Δt is the time required to measure the frequency of the wave
- The more we know about the value of the frequency of the wave, the longer the time taken to measure it
- If u want to know exactly the precise value of the frequency, the required time is Δt = infinity
- We will encounter more of this when we study the Heisenberg uncertainty relation in quantum physics

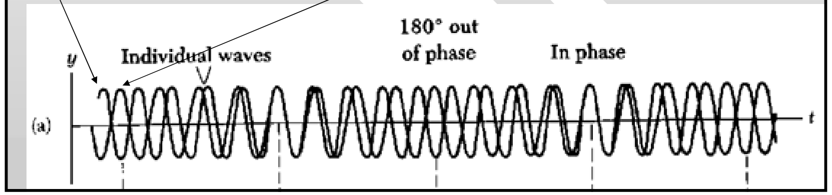
Wave can be made more ``localised''

- We have already shown that the 1-D plain wave is infinite in extent and can't be properly localised (because for this wave, $\Delta x \rightarrow$ infinity)
- However, we can construct a relatively localised wave (i.e., with smaller Δx) by :
- adding up two plain waves of slightly different wavelengths (or equivalently, frequencies)
- Consider the `beat phenomena'

Constructing wave groups

- Two pure waves with slight difference in frequency and wave number $\Delta\omega = \omega_1 - \omega_2$, $\Delta k = k_1 - k_2$, are superimposed

$$y_1 = A\cos(k_1 x - \omega_1 t); \quad y_2 = A\cos(k_2 x - \omega_2 t)$$

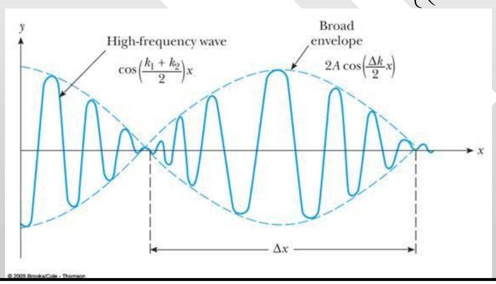


Envelop wave and phase wave

The resultant wave is a 'wave group' comprise of an `envelop' (or the group wave) and a phase waves

$$y = y_1 + y_2$$

$$= 2A\cos\frac{1}{2}\left(\{k_2 - k_1\}x - \{\omega_2 - \omega_1\}t\right)\cdot\cos\left\{\left(\frac{k_2 + k_1}{2}\right)x - \left(\frac{\omega_2 + \omega_1}{2}\right)t\right\}$$



- As a comparison to a plain waves, a group wave is more 'localised' (due to the existence of the wave envelop. In comparison, a plain wave has no `envelop' but only 'phase wave')
- It comprises of the slow envelop wave

$$2A\cos\frac{1}{2}\left(\{k_2 - k_1\}x - \{\omega_2 - \omega_1\}t\right) = 2A\cos\frac{1}{2}\left(\Delta kx - \Delta\omega t\right)$$

that moves at group velocity $v_g = \Delta\omega/\Delta k$

- and the phase waves (individual waves oscillating inside the envelop)

$$\cos\left\{\left(\frac{k_2 + k_1}{2}\right)x - \left(\frac{\omega_2 + \omega_1}{2}\right)t\right\} = \cos\{k_p x - \omega_p t\}$$

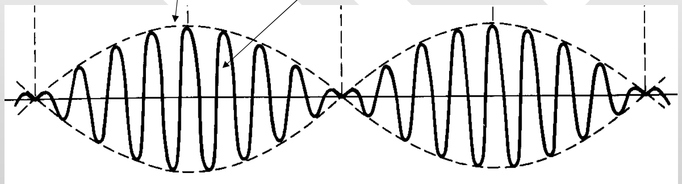
moving at phase velocity $v_p = \omega_p / k_p$

In general, $v_g = \Delta\omega/\Delta k << v_p = (\omega_1 + \omega_2)/(k_1 + k_2)$ because $\omega_2 \approx \omega_1$, $k_1 \approx k_2$

$$y = y_1 + y_2 = \left\{ 2A\cos\frac{1}{2}(\Delta kx - \Delta\omega t) \right\} \cdot \cos\{k_p x - \omega_p t\}$$

`envelop' (group waves). Sometimes it's called 'modulation'

Phase waves



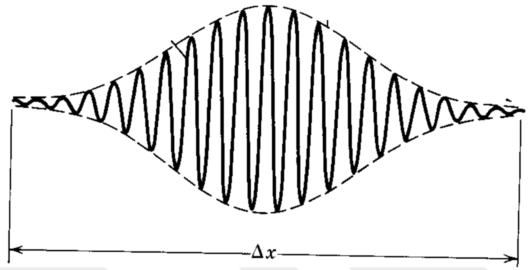
Energy is carried at the speed of the group wave

- The energy carried by the group wave is concentrated in regions in which the amplitude of the envelope is large
- The speed with which the waves' energy is transported through the medium is the speed with which the envelope advances, not the phase wave
- In this sense, the envelop wave is of more 'physical' relevance in comparison to the individual phase waves (as far as energy transportation is concerned)

Wave pulse – an even more `localised' wave

- In the previous example, we add up only two slightly different wave to form a train of wave group
- An even more `localised' group wave – what we call a "*wavepulse*" can be constructed by adding more sine waves of different numbers k_i *and* possibly different amplitudes so that they interfere constructively over a small region Δx and outside this region they interfere destructively so that the resultant field approach zero
- Mathematically,

$$y_{\text{wave pulse}} = \sum_i^{\infty} A_i \cos(k_i x - \omega_i t)$$



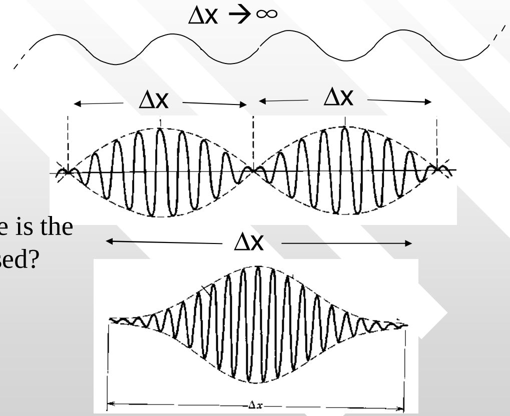
A wavepulse – the wave is well localised within Δx. This is done by adding a lot of waves with with their wave parameters $\{A_i, k_i, \omega_i\}$ slightly differ from each other (i = 1, 2, 3….as many as it can)

such a wavepulse will move with a velocity

$$v_g = \left.\frac{d\omega}{dk}\right]_{k_0}$$

(c.f the group velocity considered earlier $v_g = \Delta\omega/\Delta k$)

Comparing the three kinds of wave



Which wave is the most localised?

Why are waves and particles so important in physics?

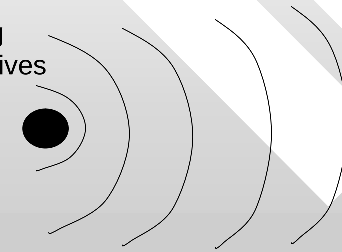
- Waves and particles are important in physics because they represent the only modes of energy transport (interaction) between two points.
- E.g we signal another person with a thrown rock (a particle), a shout (sound waves), a gesture (light waves), a telephone call (electric waves in conductors), or a radio message (electromagnetic waves in space).

Interactions take place between

(i) particles and particles (e.g. in particle-particle collision, a girl bangs into a guy) or

(ii) waves and particle, in which a particle gives up all or part of its energy to generate a wave, or when all or part of the energy carried by a wave is absorbed/dissipated by a nearby particle (e.g. a wood chip dropped into water, or an electric charge under acceleration, generates EM wave)

Oscillating electron gives off energy

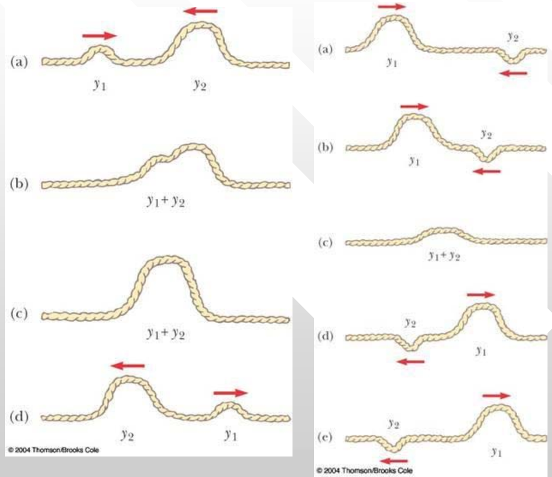


This is an example where particle is interacting with wave; energy transform from the electron's K.E. to the energy propagating in the form of EM wave wave

Waves superimpose, not collide

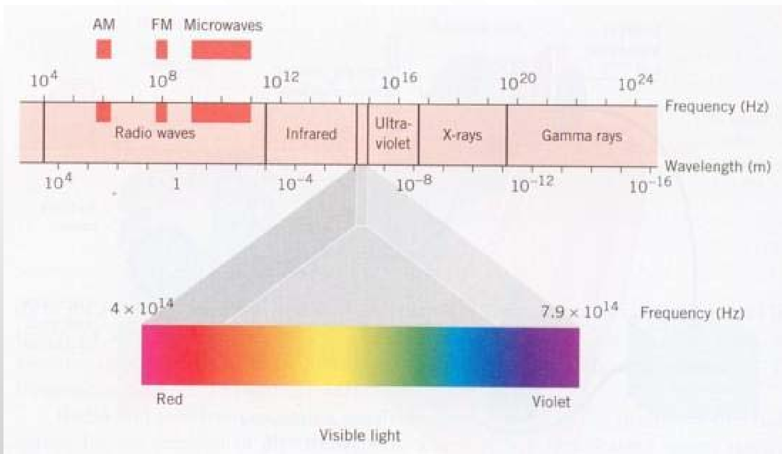
- In contrast, two waves do not interact in the manner as particle-particle or particle-wave do
- Wave and wave simply "**superimpose**": they pass through each other essentially unchanged, and their respective effects at every point in space simply add together according to the principle of superposition to form a resultant at that point -- a sharp contrast with that of two small, impenetrable particles

Superposition of waves

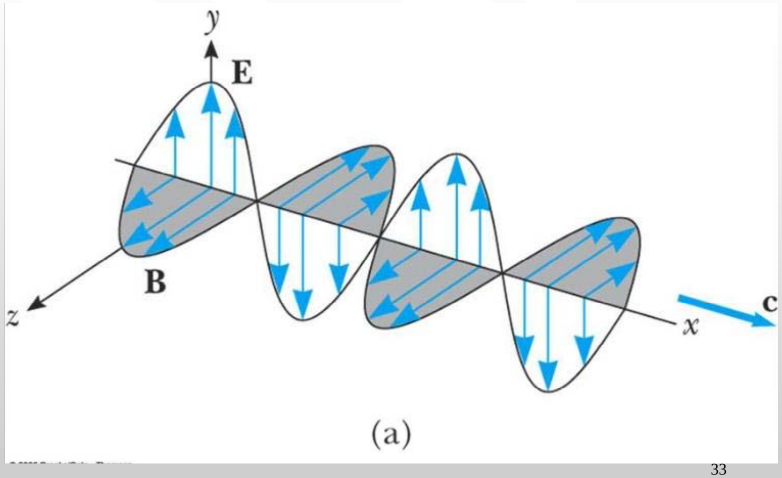


A pure EM wave

- According to Maxwell theory, light is a form of energy that propagates in the form of electromagnetic wave
- In Maxwell theory tight is synonym to electromagnetic radiation is synonym to electromagnetic wave
- Other forms of EM radiation include heat in the form of infra red radiation, visible light, gamma rays, radio waves, microwaves, x-rays



A pure EM wave

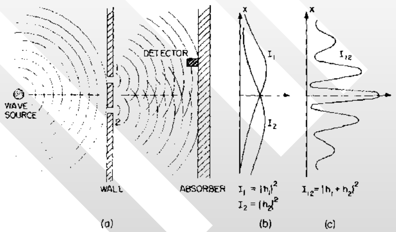


Heinrich Hertz (1857-1894), German, Established experimentally that light is EM wave

Interference experiment with water waves

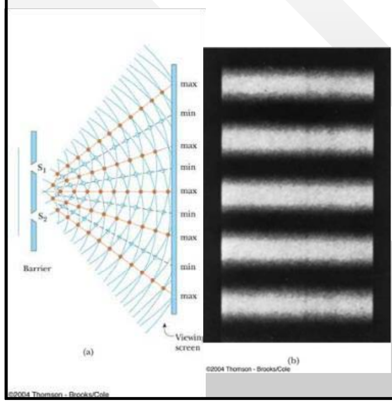
- If hole 1 (2) is block, intensity distribution of (I_1) I_2 is observed
- However, if both holes are opened, the intensity of I_{12} is such that $I_{12} \neq I_1 + I_2$
- Due to the wave nature, the intensities do not simply add
- In addition, and interference term exist,

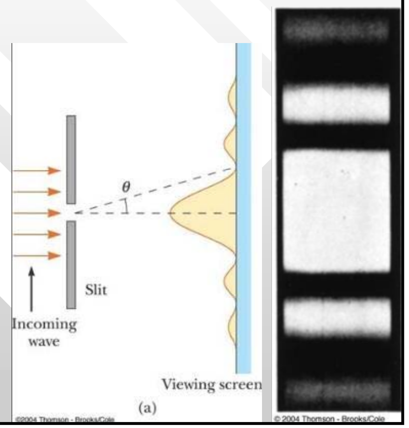
  $I_{12} = I_1 + I_2 + 2\cos\delta\,(I_1 + I_2)$
- “waves interfere”



Pictures of interference and diffraction pattern in waves

- Interference
- diffraction



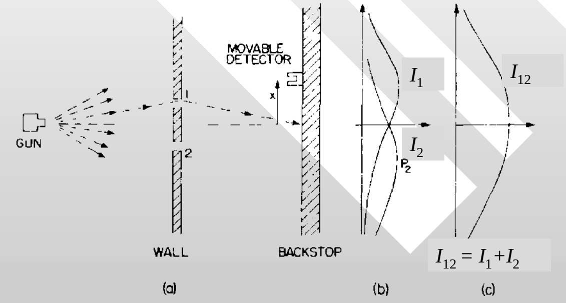


Since light display interference and diffraction pattern, it is wave

- Furthermore, Maxwell theory tell us what kind of wave light is
- It is electromagnetic wave
- (In other words it is not mechanical wave)

Interference experiment with bullets (particles)

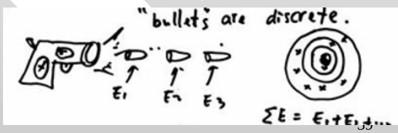
- I_2, I_1 are distribution of intensity of bullet detected with either one hole covered. I_{12} the distribution of bullets detected when both holes opened
- Experimentally, $I_{12} = I_1 + I_2$ (the individual intensity simply adds when both holes opened)
- Bullets always arrive in identical lump (corpuscular) and display no interference



EM radiation transports energy in flux, not in bundles of particles

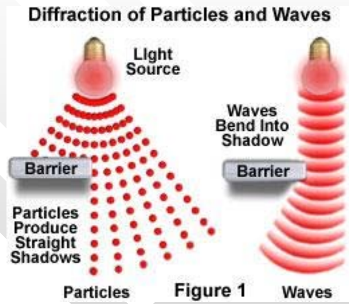
- The way how wave carries energy is described in terms of 'energy flux', in unit of energy per unit area per unit time
- Think of the continuous energy transported by a stream of water in a hose

This is in contrast to a stream of 'bullet' from a machine gun where the energy transported by such a steam is discrete in nature



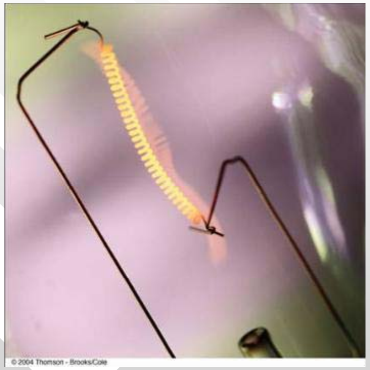
Essentially,

- Particles and wave are disparately distinct phenomena and are fundamentally different in their physical behaviour
- Free particles only travel in straight line and they don't bend when passing by a corner
- However, for light, it does
- Light, according to Maxwell's EM theory, is EM wave
- It display wave phenomena such as diffraction and interference that is not possible for particles
- Energy of the EM wave is transported in terms of energy flux



BLACK BODY RATDIATION

- Object that is HOT (anything > 0 K is considered “hot”) emit EM radiation
- For example, an incandescent lamp is red HOT because it emits a lot of IR radiation



41

Attempt to understand the origin of radiation from hot bodies from classical theories

- In the early years, around 1888 – 1900, light is understood to be EM radiation
- Since hot body radiate EM radiation, hence physicists at that time naturally attempted to understand the origin of hot body in terms of classical EM theory and thermodynamics (which has been well established at that time)

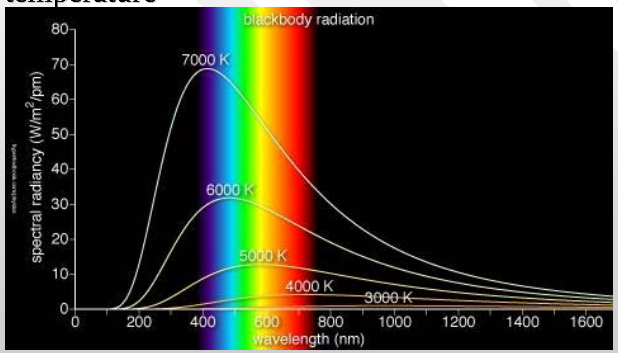
42

- All hot object radiate EM wave of all wavelengths
- However, the energy intensities of the wavelengths differ continuously from wavelength to wavelength (or equivalently, frequency)
- Hence the term: the spectral distribution of energy as a function of wavelength

43

Spectral distribution of energy in radiation depends only on temperature

- The distribution of intensity of the emitted radiation from a hot body at a given wavelength depends on the temperature



44

Introducing idealised black body

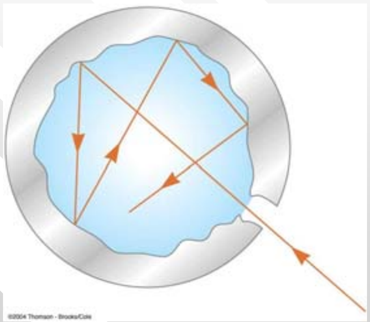
- In reality the spectral distribution of intensity of radiation of a given body could depend on the the type of the surface which may differ in absorption and radiation efficiency (i.e. frequency-dependent)
- This renders the study of the origin of radiation by hot bodies case-dependent (which means no good because the conclusions made based on one body cannot be applicable to other bodies that have different surface absorption characteristics)
- E.g. At the same temperature, the spectral distribution by the exhaust pipe from a Proton GEN2 and a Toyota Altis is different

Black body – a means of "strategy"

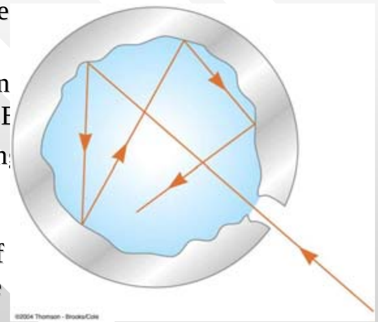
- As a strategy to overcome this non-generality, we introduce an idealised black body which, by definition, absorbs all radiation incident upon it, regardless of frequency
- Such idealised body is universal and allows one to disregard the precise nature of whatever is radiating, since all BB behave identically
- All real surfaces could be approximate to the behavior of a black body via a parameter EMMISSIVITY *e* (*e*=1 means ideally approximated, *e* << 1 means poorly approximated)

Blackbody Approximation

- A good approximation of a black body is a small hole leading to the inside of a hollow object
- The HOLE acts as a perfect absorber
- The Black Body is the HOLE



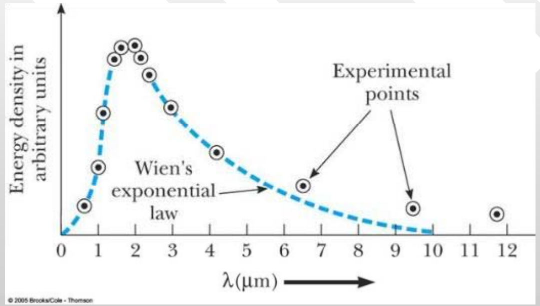
- Any radiation striking the HOLE enters the cavity, trapped by reflection until is absorbed by the inner walls
- The walls are constantly absorbin and emitting energy at thermal EI
- The nature of the radiation leavin the cavity through the hole depends only on the temperature of the cavity and not the detail of the surfaces nor frequency of the radiation



Essentially

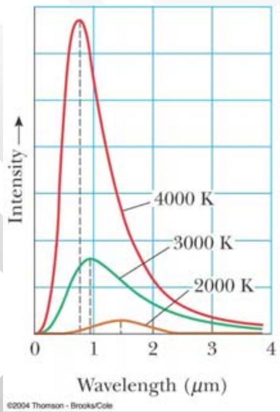
- A black body in thermal EB absorbs and emits radiation at the same rate
- The HOLE effectively behave like a Black Body because it effectively absorbs all radiation fall upon it
- And at the same time, it also emits all the absorbed radiations at the same rate as the radiations are absorbed
- MOST IMPORATANTLY: THE SPECTRAL DISTRIBUTION OF EMISSION DEPENDS SOLELY ON THE TEMPERATURE AND NOT OTHER DETAILS

Experimentally, the measured spectral distribution of black bodies is universal and depends only on temperature



Why does the spectral distribution of black bodies have the shape as measured?

- Lord Rayleigh and James Jeans at 1890's try to theoretically derive the distribution based on statistical mechanics (some kind of generalised thermodynamics) and classical Maxwell theory
- (Details omitted, u will learn this when u study statistical mechanics later)



For EM standing wave modes in the cavity, $\langle\varepsilon\rangle = kT$

- In RJ formula, the BB is modeled as the hole leading into a cavity supporting many modes of oscillation of the EM field caused by accelerated charges in the cavity walls, resulting in the emission of EM waves at all wavelength
- In the statistical mechanics used to derive the RJ formula, the average energy of each wavelength of the standing wave modes, $\langle\varepsilon\rangle$, is assumed to be proportional to kT, based on the theorem of equipartition of energy

Details not required

- What is essential here is that: in deriving the RJ formula the radiation in the cavity is treated as standing EM wave having classical average energy per standing wave as per

$$\langle \varepsilon \rangle = kT$$

- $\langle \varepsilon \rangle$ can take any value CONTINOUSLY and is NOT restricted to only discrete values

Rayleigh-Jeans Law

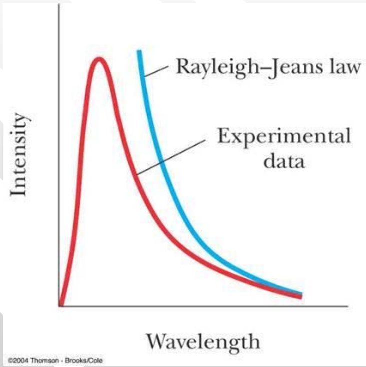
- **Rayleigh-Jeans law (based on classical physics):**

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4} \qquad k = 1.38\times10^{-23}\ \text{J/K, Boltzmann constant}$$

- At long wavelengths, the law matched experimental results fairly well

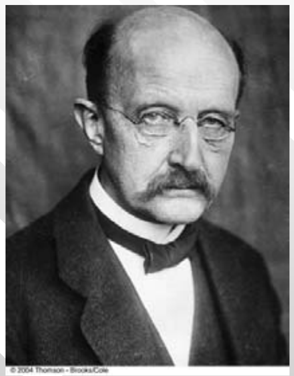
Rayleigh-Jeans Law, cont.

- At short wavelengths, there was a major disagreement between the Rayleigh-Jeans law and experiment
- This mismatch became known as the *ultraviolet catastrophe*
  - You would have infinite energy as the wavelength approaches zero



Max Planck

- Introduced the concept of "quantum of action"
- In 1918 he was awarded the Nobel Prize for the discovery of the quantized nature of energy



Planck's Theory of Blackbody Radiation

- In 1900 Planck developed a theory of blackbody radiation that leads to an equation for the intensity of the radiation
- This equation is in complete agreement with experimental observations

Planck's Wavelength Distribution Function

- Planck generated a theoretical expression for the wavelength distribution

$$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$$

  - h = 6.626 x 10^{-34} J·s
  - h is a fundamental constant of nature

Planck's Wavelength Distribution Function, cont.

- At long wavelengths, Planck's equation reduces to the Rayleigh-Jeans expression
- This can be shown by expanding the exponential term

$$e^{hc/\lambda k_B T} = 1 + \frac{hc}{\lambda k_B T} + \frac{1}{2!}\left(\frac{hc}{\lambda k_B T}\right)^2 + \ldots \approx 1 + \frac{hc}{\lambda k_B T}$$

in the long wavelength limit $hc << \lambda k_B T$

- At short wavelengths, it predicts an exponential decrease in intensity with decreasing wavelength
  - This is in agreement with experimental results

How Planck modelled the BB

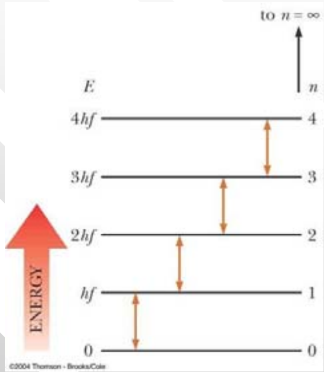
- He assumed the cavity radiation came from atomic oscillations in the cavity walls
- Planck made two assumptions about the nature of the oscillators in the cavity walls

Planck's Assumption, 1

- The energy of an oscillator can have only certain *discrete* values E_n
  - $E_n = nhf$
    - n is a positive integer called the quantum number
    - h is Planck's constant = 6.63×10^{-34} Js
    - f is the frequency of oscillation
  - This says the energy is **quantized**
  - Each discrete energy value corresponds to a different **quantum state**
  - This is in stark contrast to the case of RJ derivation according to classical theories, in which the energies of oscillators in the cavity must assume a continuous distribution

Energy-Level Diagram

- An **energy-level diagram** shows the quantized energy levels and allowed transitions
- Energy is on the vertical axis
- Horizontal lines represent the allowed energy levels
- The double-headed arrows indicate allowed transitions



Oscillator in Planck's theory is quantised in energies (taking only discrete values)

- The energy of an oscillator can have only certain *discrete* values $E_n = nhf$
- The average energy per standing wave in the Planck oscillator is

$$\langle \varepsilon \rangle = \frac{hf}{e^{hf/kT} - 1}$$ (instead of $\langle \varepsilon \rangle = kT$ in classical theories)

Planck's Assumption, 2

- The oscillators emit or absorb energy when making a transition from one quantum state to another
  - The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation
  - An oscillator emits or absorbs energy only when it changes quantum states

Example: quantised oscillator vs classical oscillator

- A 2.0 kg block is attached to a massless spring that has a force constant *k*=25 N/m. The spring is stretched 0.40 m from its EB position and released.
- (A) Find the total energy of the system and the frequency of oscillation according to classical mechanics.

Solution

- In classical mechanics, $E = \frac{1}{2}kA^2 = \ldots\ 2.0$ J
- The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \ldots = 0.56\ \text{Hz}$$

(B)

- (B) Assuming that the energy is quantised, find the quantum number *n* for the system oscillating with this amplitude

- Solution: This is a quantum analysis of the oscillator
- $E_n = nhf = n\,(6.63 \times 10^{-34}$ Js)(0.56 Hz) = 2.0 J
- $\Rightarrow n = 5.4 \times 10^{33}$ *!!!* A very large quantum number, typical for macroscopin system

To summarise

- Classical BB presents a “ultraviolet catastrophe”
- The spectral energy distribution of electromagnetic radiation in a black body CANNOT be explained in terms of classical Maxwell EM theory, in which the average energy in the cavity assumes continuous values of $\langle\varepsilon\rangle = kT$ (this is the result of the wave nature of radiation)
- To solve the BB catastrophe one has to assume that the energy of individual radiation oscillator in the cavity of a BB is quantised as per $E_n = nhf$
- This picture is in conflict with classical physics because in classical physics energy is in principle a continuous variable that can take any value between $0 \rightarrow \infty$
- One is then lead to the revolutionary concept that

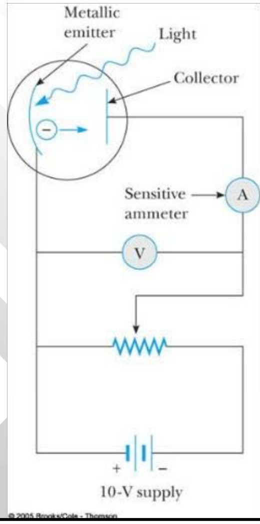
ENERGY OF AN OSCILLATOR IS QUANTISED

Photoelectricity

- Classically, light is treated as EM wave according to Maxwell equation
- However, in a few types of experiments, light behave in ways that is not consistent with the wave picture
- In these experiments, light behave like particle instead
- So, is light particle or wave? (recall that wave and particle are two mutually exclusive attributes of existence)
- This is a paradox that we will discuss in the rest of the course – wave particle duality

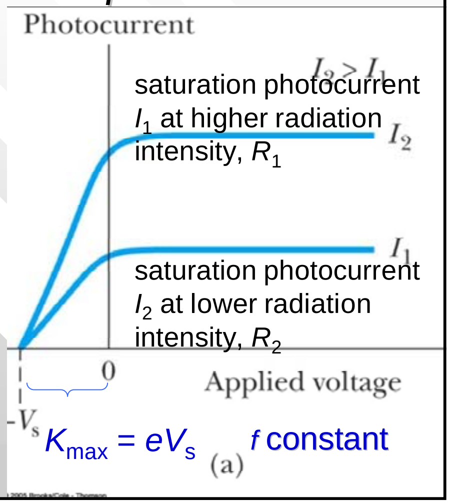
Photoelectric effect

- Photoelectrons are ejected from a metal surface when hit by radiation of sufficiently high frequency f (usually in the uv region)
- The photoelectrons are attracted to the collecting anode (positive) by potential difference applied on the anode and detected as electric current by the external circuits
- A negative voltage, relative to that of the emitter, can be applied to the collector.
- When this retarding voltage is sufficiently large the emitted electrons are repelled, and the current to the collector drops to zero (see later explanation).



Photocurrent I vs applied voltage at constant f

- No current flows for a retarding potential more negative than $-V_s$
- The photocurrent I saturates for potentials near or above zero
- Why does the I-V curve rises gradually from $-V_s$ towards more positive V before it flat off ?



Features of the experimental result

- When the external potential difference $V = 0$, the current is not zero because the photoelectrons carry some kinetic energy, K
- K range from 0 to a maximal value, K_{max}
- As V becomes more and more positive, there are more electrons attracted towards the anode within a given time interval. Hence the pthotocurrent, I, increases with V
- Saturation of I will be achieved when all of the ejected electron are immediately attracted towards the anode once they are kicked out from the metal plates (from the curve this happens approximately when $V \approx 0$ or larger

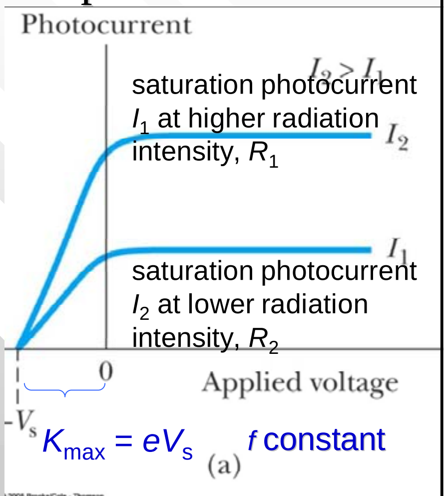
- On the other direction, when V becomes more negative, the photocurrent detected decreases in magnitude because the electrons are now moving against the potential
- K_{max} can be measured. It is given by eV_s, where V_s, is the value of $|V|$ when the current flowing in the external circuit = 0
- V_s is called the '**stopping potential**'
- When $V = -V_s$, e of the highest KE will be sufficiently retarded by the external electric potential such that they wont be able to reach the collector

$I_2 > I_1$ because more electrons are kicked out per unit time by radiation of larger intensity, R

- The photocurrent saturates at a larger value of I_2 when it is irradiated by higher radiation intensity R_2
- This is expected as larger R means energy are imparted at a higher rate on the metal surface

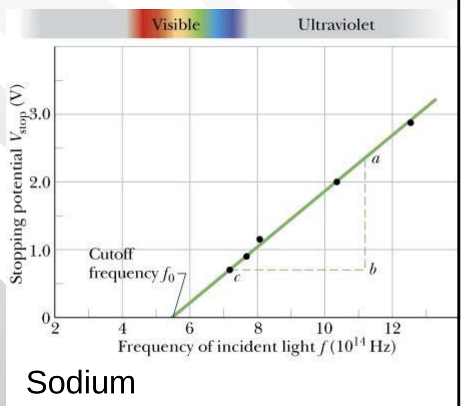
Stopping potential V_s is radiation intensity-independent

- Experimentalists observe that for a given type of surface:
- At constant frequency the maximal kinetic energy of the photoelectrons is measured to be a constant independent of the intensity of light.



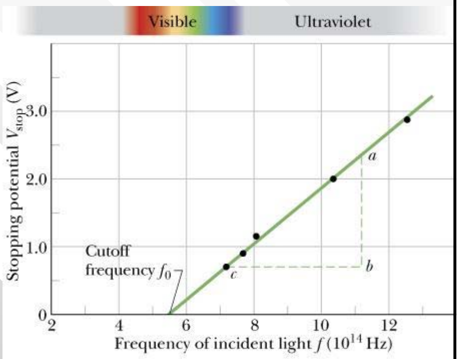
K_{max} of photoelectrons is frequency-dependent at constant radiation intensity

- One can also detect the stopping potential V_s for a given material at different frequency (at constant radiation intensity)
- K_{max} ($=eV_s$) is measured to linearly dependent on the radiation frequency $K_{max} \propto f$



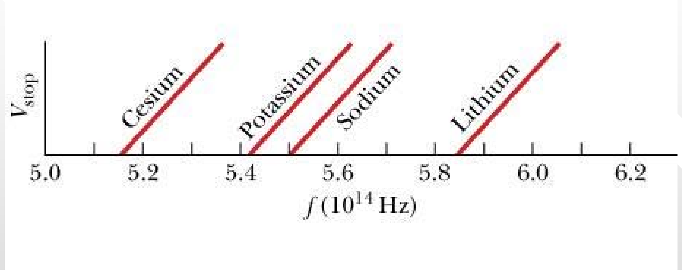
Cutoff frequency, f_0

- From the same graph one also found that there exist a **cut-off frequency**, f_0, below which no PE effect occurs no matter how intense is the radiation shined on the metal surface



Sodium

Different material have different cut-off frequency f_0



- For different material, the cut-off frequency is different

Classical physics can't explain PE

- The experimental results of PE pose difficulty to classical physicists as they cannot explain PE effect in terms of classical physics (Maxwell EM theory, thermodynamics, classical mechanics etc.)

Puzzle one

- If light were wave, the energy carried by the radiation will increases as the intensity of the monochromatic light increases
- Hence we would also expect K_{max} of the electron to increase as the intensity of radiation increases (because K.E. of the photoelectron must come from the energy of the radiation)
- YET THE OBSERVATION IS OTHERWISE.

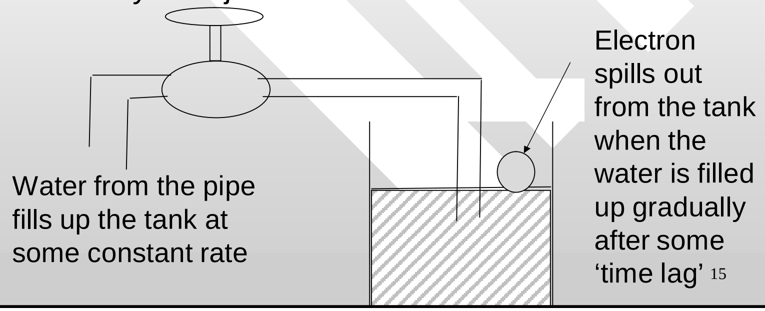
Puzzle two

- Existence of a characteristic cut-off frequency, ν_0. (previously I use f_0)
- Wave theory predicts that photoelectric effect should occur for any frequency as long as the light is intense enough to give the energy to eject the photoelectrons.
- No cut-off frequency is predicted in classical physics.

Puzzle three

- No detection time lag measured.
- Classical wave theory needs a time lag between the instance the light impinge on the surface with the instance the photoelectrons being ejected. Energy needs to be accumulated for the wave front, at a rate proportional to , $S = \frac{E_0}{2\mu_0 c}$ before it has enough energy to eject photoelectrons.
- But, in the PE experiments, PE is almost immediate

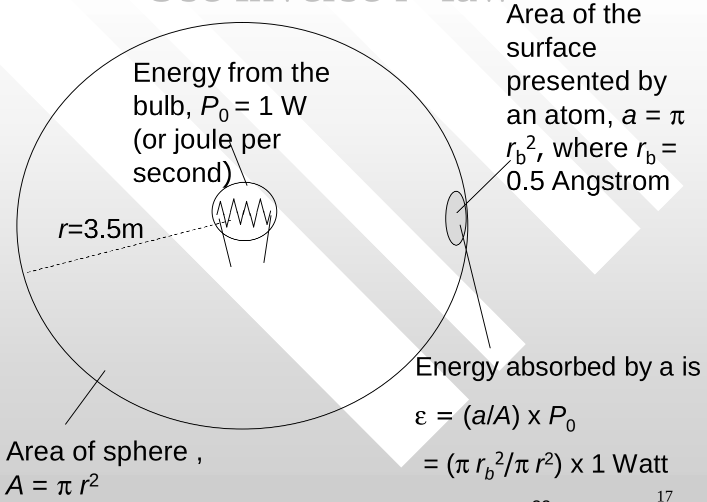
Cartoon analogy: in the wave picture, accumulating the energy required to eject an photoelectron from an atom is analogous to filling up a tank with water from a pipe until the tank is full. One must wait for certain length of time (time lag) before the tank can be filled up with water at a give rate. The total water filled is analogous to the total energy absorbed by electrons before they are ejected from the metal surface at



Wave theory and the time delay problem

- A potassium foil is placed at a distance r = 3.5 m from a light source whose output power P_0 is 1.0 W. How long would it take for the foil to soak up enough energy (=1.8 eV) from the beam to eject an electron? Assume that the ejected electron collected the energy from a circular area of the foil whose radius is 5.3 x 10^{-11} m

Use inverse r^2 law



Energy absorbed by a is

$\varepsilon = (a/A) \times P_0$

$= (\pi r_b^2/\pi r^2) \times 1$ Watt

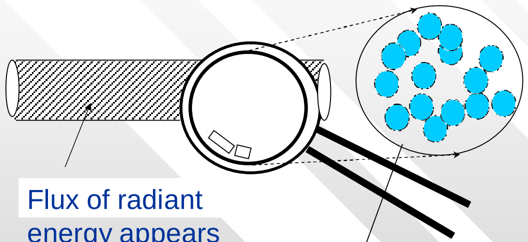
$= 2.0\times10^{-22}$ Watt

- Time taken for a to absorb 1.8 eV is simply 1.8 x 1.6 x 10^{-19} J / ε = 5000 s = 1.4 h!!!
- In PE, the photoelectrons are ejected almost immediately but not 1.4 hour later
- This shows that the wave model used to calculate the time lag in this example fails to account for the almost instantaneous ejection of photoelectron in the PE experiment

Einstein's quantum theory of the photoelectricity (1905)

- A Noble-prize winning theory (1905)
- To explain PE, Einstein postulates that the radiant energy of light is quantized into concentrated bundle. The discrete entity that carries the energy of the radiant energy is called photon
- Or, in quantum physics jargon, we say "photon is the quantum of light"
- Wave behaviour of light is a result of collective behaviour of very large numbers of photons

Photon is granular



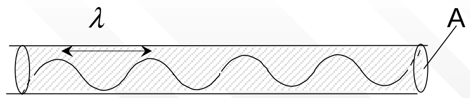
Flux of radiant energy appears like a continuum at macroscopic scale of intensity

Granularity of light (in terms of photon) becomes manifest when magnified

Wave and particle carries energy differently

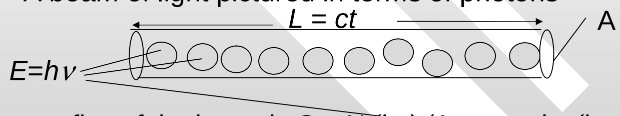
- The way how photon carries energy is in in contrast to the way wave carries energy.
- For wave the radiant energy is continuously distributed over a region in space and not in separate bundles
- (always recall the analogy of water in a hose and a stream of ping pong ball to help visualisation)

A beam of light if pictured as monochromatic wave (λ, ν)



Energy flux of the beam is $S = \frac{E_0}{2\mu_0 c}$ (in unit of joule per unit time per unit area), analogous to fluid in a host

A beam of light pictured in terms of photons



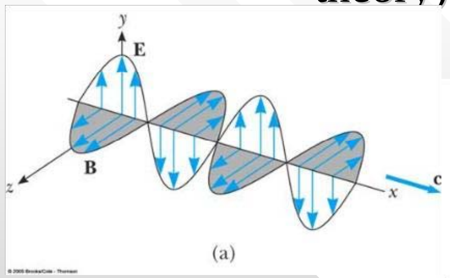
Energy flux of the beam is $S = N(h\nu)/At = n_0 ch\nu$ (in unit of joule per unit time per unit area). N is obtained by 'counting' the total number of photons in the beam volume, $N = n_0V = n_0$ x ($A\ ct$), where n_0 is the photon number density of the radiation (in unit of number per unit volume)

Einstein's 1st postulate

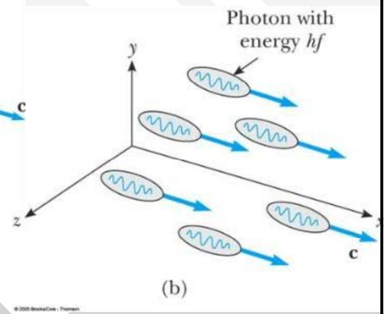
1. The energy of a single photon is $E = h\nu$. h is a proportional constant, called the Planck constant, that is to be determined experimentally.

- With this assumption, a photon will have a momentum given by $p = E/c = h/\lambda$.
- This relation is obtained from SR relationship $E^2 = p^2c^2 + (m_0c^2)^2$, for which the mass of a photon is zero.
- Note that in classical physics momentum is intrinsically a particle attribute not defined for wave. By picturing light as particle (photon), the definition of momentum for radiation now becomes feasible

Light as photon (in Einstein theory) instead of wave (in Classical EM theory)



(a)



(b)

$p=h/\lambda$, $E=h\nu=hc/\lambda$

Example

- (a) What are the energy and momentum of a photon of red light of wavelength 650nm?
- (b) What is the wavelength of a photon of energy 2.40 eV?
- In atomic scale we usually express energy in eV, momentum in unit of eV/*c*, length in nm; the combination of constants, *hc*, is conveniently expressed in
- 1 eV = 1.6×10^{-19} J
- $hc = (6.62 \times 10^{-34}\ \mathrm{Js}) \cdot (3 \times 10^{8}\ \mathrm{m/s})$
  $= [6.62 \times 10^{-34} \cdot (1.6 \times 10^{-19})^{-1} \mathrm{eV \cdot s}] \cdot (3 \times 10^{8}\ \mathrm{m/s})$
  $= 1.24 \mathrm{eV} \cdot 10^{-6} \mathrm{m} = 1240 \mathrm{eV \cdot nm}$
- $1\ \mathrm{eV}/c = (1.6 \times 10^{-19})\mathrm{J} / (3 \times 10^{8}\ \mathrm{m/s}) = 5.3 \times 10^{-28}$ Ns

solution

- (a) $E = hc/\lambda$
  $= 1240\ \mathrm{eV \cdot nm} / 650\ \mathrm{nm}$
  $= 1.91\ \mathrm{eV}\ (= 3.1 \times 10^{-19} \mathrm{J})$
- (b) $p = E/c = 1.91\ \mathrm{eV}/c\ (= 1 \times 10^{-27}\ \mathrm{Ns})$
- (c) $\lambda = hc/E$
  $= 1240 \mathrm{eV \cdot nm} / 2.40\ \mathrm{eV}$
  $= 517$ nm

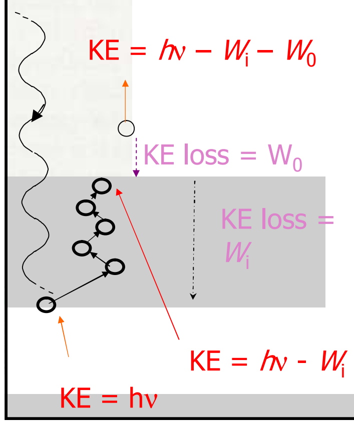
Einstein's 2nd postulate

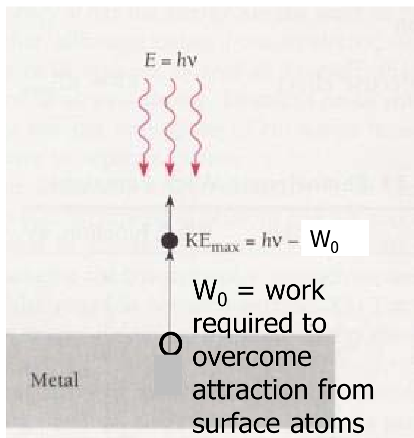
- In PE one photon is completely absorbed by one atom in the photocathode.
- Upon the absorption, one electron is 'kicked out' by the absorbent atom.
- The kinetic energy for the ejected electron is
  $K = h\nu - W$
- *W* is the worked required to
- (i) cater for losses of kinetic energy due to internal collision of the electrons (W_i),
- (ii) overcome the attraction from the atoms in the surface (W_0)
- When no internal kinetic energy loss (happens to electrons just below the surface which suffers minimal loss in internal collisions), *K* is maximum:
- $K_{max} = h\nu - W_0$

In general,

$K = h\nu - W$, where

$W = W_0 + W_i$





Einstein theory manage to solve the three unexplained features:

- First feature:
- In Einstein's theory of PE, $K_{max} = h\nu - W_0$
- Both $h\nu$ and W_0 do not depend on the radiation intensity
- Hence K_{max} is independent of irradiation intensity
- Doubling the intensity of light wont change K_{max} because only depend on the energy hν of individual photons and W_0
- W_0 is the intrinsic property of a given metal surface

Second feature explained

- **The cut-off frequency is explained**

- Recall that in Einstein assumption, a photon is completely absorbed by one atom to kick out one electron.
- Hence each absorption of photon by the atom transfers a discrete amount of energy by $h\nu$ only.
- If $h\nu$ is not enough to provide sufficient energy to overcome the required work function, W_0, no photoelectrons would be ejected from the metal surface and be detected as photocurrent

Cut-off frequency is related to work function of metal surface $W_0 = h\nu_0$

- A photon having the cut-off frequency ν_0 has just enough energy to eject the photoelectron and none extra to appear as kinetic energy.
- Photon of energy less than $h\nu_0$ has not sufficient energy to kick out any electron
- Approximately, electrons that are eject at the cut-off frequency will not leave the surface.
- This amount to saying that the have got zero kinetic energy: $K_{max} = 0$
- Hence, from $K_{max} = h\nu - W_0$, we find that the cut-off frequency and the work function is simply related by
  - $W_0 = h\nu_0$
- Measurement of the cut-off frequency tell us what the work function is for a given metal

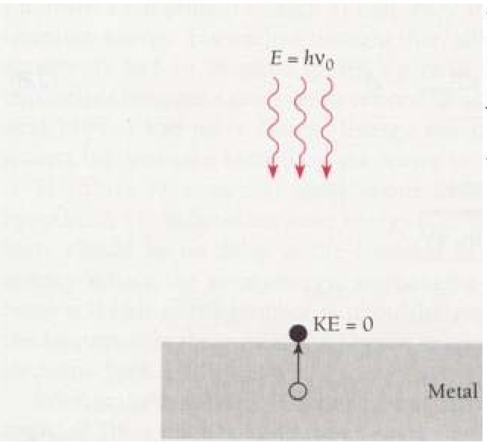


Table 3.1 Some Photoelectric Work Functions $W_0 = h\nu_0$

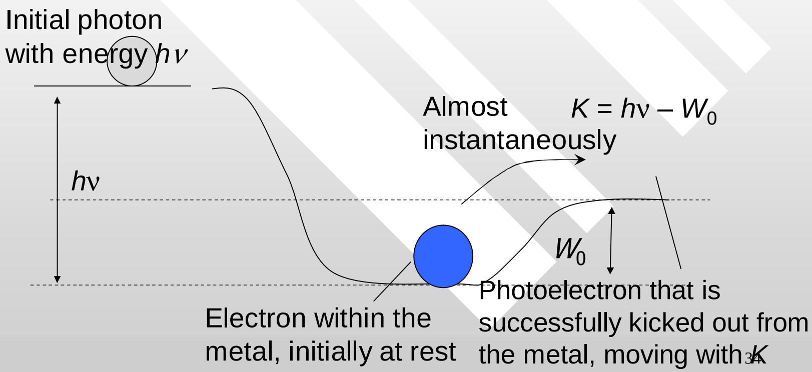
Material	W(eV
Na	2.28
Al	4.08
Co	3.90
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

Third feature explained

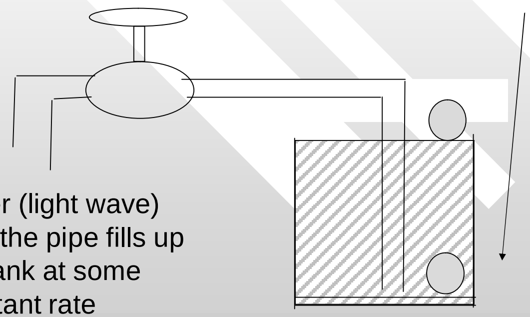
- The required energy to eject photoelectrons is supplied in concentrated bundles of photons, not spread uniformly over a large area in the wave front.
- Any photon absorbed by the atoms in the target shall eject photoelectron immediately.
- Absorption of photon is a discrete process at quantum time scale (almost 'instantaneously'): it either got absorbed by the atoms, or otherwise.
- Hence no time lag is expected in this picture

A simple way to picture photoelectricity in terms of particle-particle collision:

Energy of photon is transferred during the instantaneous collision with the electron. The electron will either get kicked up against the barrier threshold of W_0 almost instantaneously, or fall back to the bottom of the valley if $h\nu$ is less than W_0



Compare the particle-particle collision model with the water-filling-tank model:

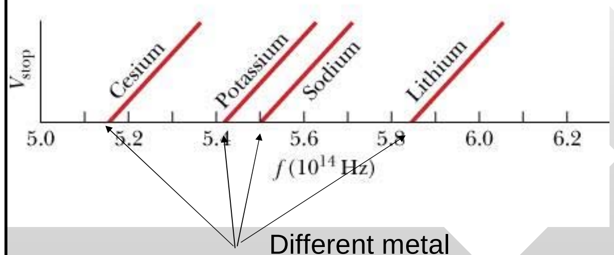


Experimental determination of Planck constant from PE

- Experiment can measure eV_s (= K_{max}) for a given metallic surface (e.g. sodium) at different frequency of impinging radiation
- We know that the work function and the stopping potential of a given metal is given by
- $eV_s = h\nu - W_0$

In experiment, we can measure the slope in the graph of V_s verses frequency ν for different metal surfaces. It gives a universal value of h/e = 4.1x10^{-15} Vs. Hence, h = 6.626 x 10^{-34} Js

$$V_s = (h/e)\nu - \nu_0$$



Different metal surfaces have different V_0

PYQ 2.16, Final Exam 2003/04

- Planck constant
- (i) is a universal constant
- (ii) is the same for all metals
- (iii) is different for different metals
- (iv) characterises the quantum scale
- **A.** I,IV **B.** I,II, IV **C.** I, III,IV
- **D.** I, III **E.** II,III
- **ANS: B, Machlup, Review question 8, pg. 496, modified**

PYQ 4(a,b) Final Exam 2003/04

- (a) Lithium, beryllium and mercury have work functions of 2.3 eV, 3.9 eV and 4.5 eV, respectively. If a 400-nm light is incident on each of these metals, determine
- (i) which metals exhibit the photoelectric effect, and
- (ii) the maximum kinetic energy for the photoelectron in each case (in eV)

Solution for Q3a

- The energy of a 400 nm photon is $E = hc/\lambda$ = **3.11 eV**
- The effect will occur only in **lithium***
- **Q3a(ii)**
- For lithium, $K_{max} = h\nu - W_0$
  = 3.11 eV – 2.30 eV
  = **0.81 eV**

*marks are deducted for calculating "K_{max}" for beryllium and mercury which is meaningless

PYQ 4(a,b) Final Exam 2003/04

- (b) Molybdenum has a work function of 4.2 eV.
- (i) Find the cut-off wavelength (in nm) and threshold frequency for the photoelectric effect.
- (ii) Calculate the stopping potential if the incident radiation has a wavelength of 180 nm.

Solution for Q4b

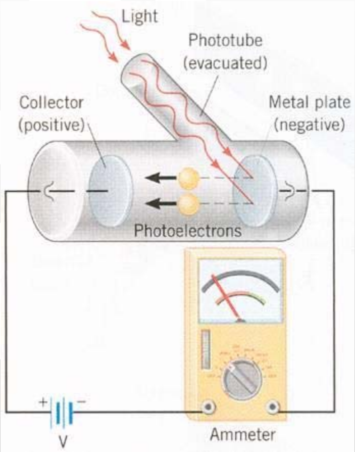
- **Q3a(ii)**
- Known $h\nu_{cutoff} = W_0$
- Cut-off wavelength = $\lambda_{cutoff} = c/\nu_{cutoff}$

  $= hc/W_0$ = 1240 nm eV / 4.2 eV = **295 nm**
- Cut-off frequency (or threshold frequency), ν_{cutoff}

  $= c / \lambda_{cutoff}$ = 1.01 x 1015 Hz
- **Q3b(ii)**
- Stopping potential $V_{stop} = (hc/\lambda - W_0) / e$ = (1240 nm·eV/180 nm – 4.2 eV)/e = **2. 7 V**

Example (read it yourself)

- Light of wavelength 400 nm is incident upon lithium (W_0 = 2.9 eV). Calculate
- (a) the photon energy and
- (b) the stopping potential, V_s
- (c) What frequency of light is needed to produce electrons of kinetic energy 3 eV from illumination of lithium?



Solution:

- (a) $E = h\nu = hc/\lambda$ = 1240eV·nm/400 nm = 3.1 eV
- (b) The stopping potential x e = Max Kinetic energy of the photon
- => $eV_s = K_{max} = h\nu - W_0$ = (3.1 - 2.9) eV
- Hence, V_s = 0.2 V
- i.e. a retarding potential of 0.2 V will stop all photoelectrons
- (c) $h\nu = K_{max} + W_0$ = 3 eV + 2.9 eV = 5.9 eV. Hence the frequency of the photon is

  $\nu = 5.9 \times (1.6 \times 10^{-19}\ \text{J}) / 6.63 \times 10^{-34}$ Js

  $= 1.42 \times 10^{15}$ Hz

PYQ, 1.12 KSCP 2003/04

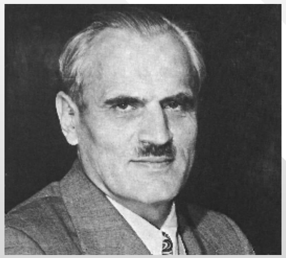
Which of the following statement(s) is (are) true?

- **I** The energy of the quantum of light is proportional to the frequency of the wave model of light
- **II** In photoelectricity, the photoelectrons has as much energy as the quantum of light which causes it to be ejected
- **III** In photoelectricity, no time delay in the emission of photoelectrons would be expected in the quantum theory
- **A. II, III** **B. I, III** **C. I, II, III** **D. I ONLY**
- **E.** Non of the above
- **Ans: B**
- Murugeshan, S. Chand & Company, New Delhi, pg. 136, Q28 (for I), Q29, Q30 (for II,III)

To summerise: In photoelectricity (PE), light behaves like particle rather than like wave.

Compton effect

- Another experiment revealing the particle nature of X-ray (radiation, with wavelength ~ 10^{-10} nm)

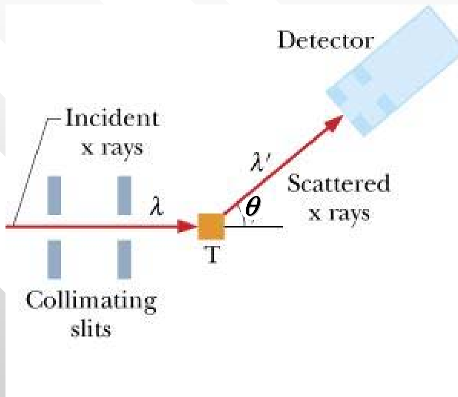


Compton, Arthur Holly (1892-1962), American physicist and Nobel laureate whose studies of X rays led to his discovery in 1922 of the so-called Compton effect.

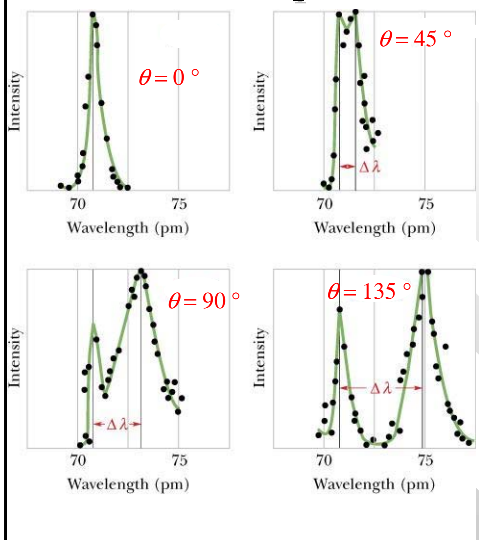
The Compton effect is the change in wavelength of high energy electromagnetic radiation when it scatters off electrons. The discovery of the Compton effect confirmed that electromagnetic radiation has both wave and particle properties, a central principle of quantum theory.

Compton's experimental setup

- A beam of x rays of wavelength 71.1 pm is directed onto a carbon target T. The x rays scattered from the target are observed at various angle θ to the direction of the incident beam. The detector measures both the intensity of the scattered x rays and their wavelength



Experimental data



Although initially the incident beam consists of only a single well-defined wavelength (λ) the scattered x-rays at a given angle θ have intensity peaks at two wavelength (λ' in addition), where $\lambda' > \lambda$

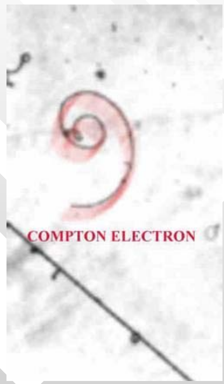
Compton shouldn't shift, according to classical wave theory of light

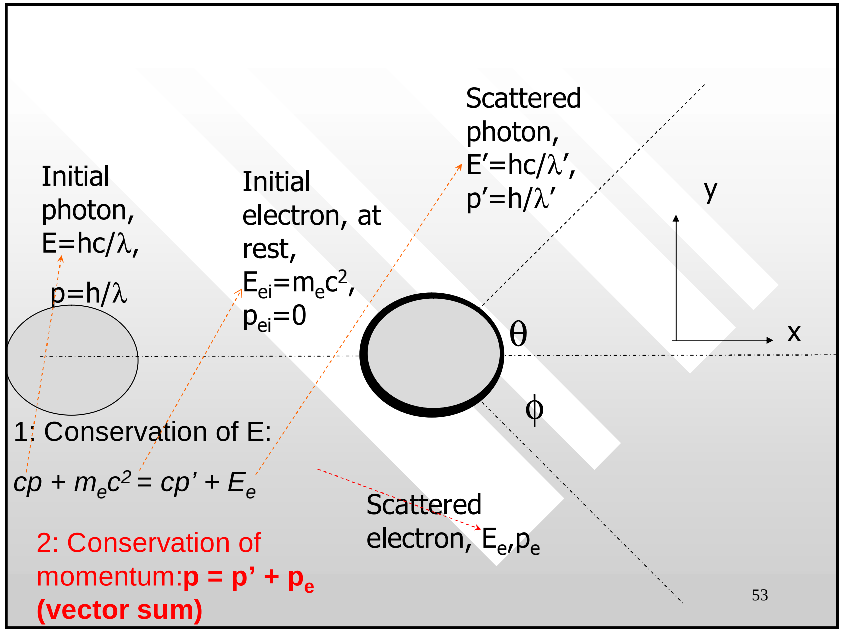
- **Unexplained by classical wave theory for radiation**
- **No shift of wavelength is predicted in wave theory of light**

Modelling Compton shift as "particle-particle" collision

- Compton (and independently by Debye) explain this in terms of collision between collections of (particle-like) photon, each with energy $E = h\nu$, with the *free* electrons in the target graphite (imagine billard balls collision)

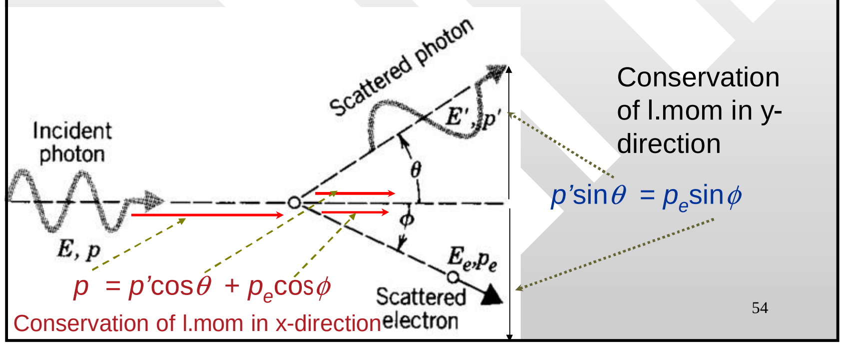
- ***Part of a bubble chamber picture (Fermilab'15 foot Bubble Chamber', found at the University of Birmingham). An electron was knocked out of an atom by a high energy photon.***





Conservation of momentum in 2-D

- $\mathbf{p} = \mathbf{p'} + \mathbf{p_e}$ (vector sum) actually comprised of two equation for both conservation of momentum in x- and y- directions



Some algebra…

Mom conservation in y : $p'\sin\theta = p_e \sin\phi$ (PY)

Mom conservation in x : $p - p'\cos\theta = p_e \cos\phi$ (PX)

Conservation of total relativistic energy:

$cp + m_e c^2 = cp' + E_e$ (RE)

$(PY)^2 + (PX)^2$, substitute into $(RE)^2$ to eliminate ϕ, p_e and E_e (and using $E_e^2 = c^2 p_e^2 + m_e^2 c^4$):

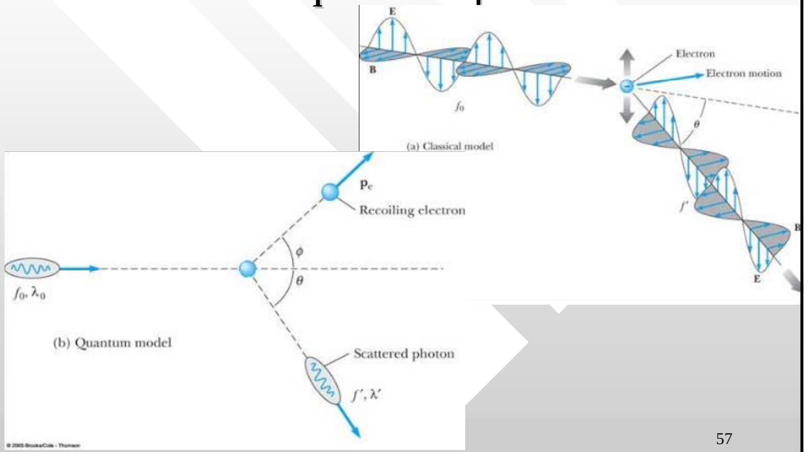
$$\Delta\lambda \equiv \lambda' - \lambda = (h/m_e c)(1 - \cos\theta)$$

Compton wavelength

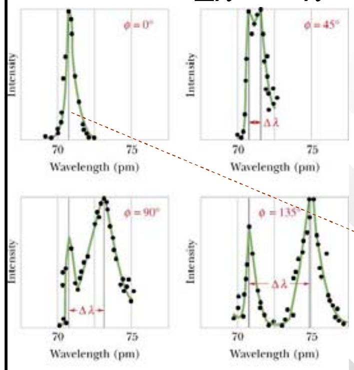
$\lambda_e = h/m_e c$ = 0.0243 Angstrom, is the Compton wavelength (for electron)

- Note that the wavelength of the x-ray used in the scattering is of the similar length scale to the Compton wavelength of electron
- The Compton scattering experiment can now be perfectly explained by the Compton shift relationship $\Delta\lambda \equiv \lambda' - \lambda = \lambda_e(1 - \cos\theta)$ as a function of the photon scattered angle
- Be reminded that the relationship is derived by assuming light behave like particle (photon)

X-ray scattering from an electron (Compton scattering): classical versus quantum picture



$$\Delta\lambda \equiv \lambda' - \lambda = (h/m_e c)(1 - \cos\theta)$$

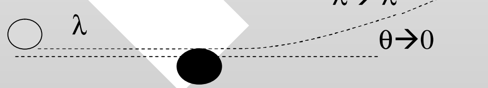


λ'=0.1795 nm

Notice that $\Delta\lambda$ depend on θ only, not on the incident wavelength, λ..

Consider some limiting behaviour of the Compton shift:

For $\theta = 0^0 \rightarrow$ "grazing" collision => $\Delta\lambda = 0$



For $\theta \rightarrow 180^0$ "head-on" collision => $\Delta\lambda = \Delta\lambda_{max}$

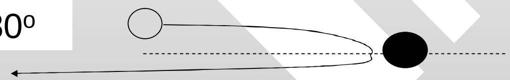
$\theta \rightarrow 180^0$ photon being reversed in direction

$\Delta\lambda_{max} = \lambda_{max}' - \lambda = (h/m_e c)(1 - \cos 180°)$

- $= 2\lambda_c = 2(0.00243\text{nm})$

initially λ

θ=180°



After collision

$\lambda'_{max} = \lambda + \Delta\lambda_{max}$

PYQ 2.2 Final Exam 2003/04

Suppose that a beam of 0.2-MeV photon is scattered by the electrons in a carbon target. What is the wavelength of those photon scattered through an angle of 90°?

A. 0.00620 nm

B. 0.00863 nm

C. 0.01106 nm

D. 0.00243 nm

E. Non of the above

Solution

First calculate the wavelength of a 0.2 MeV photon:

$E = hc/\lambda$= 1240 eV·nm/λ = 0.2 MeV

λ =1240 nm / 0.2 x 10^6 = 0.062 nm

From Compton scattering formula, the shift is

$\Delta\lambda = \lambda' - \lambda = \lambda_c (1 - \cos 90°) = \lambda_c$

Hence, the final wavelength is simply

$\lambda' = \Delta\lambda + \lambda = \lambda_c + \lambda =$ 0.00243nm + 0.062 nm = 0.00863 nm

ANS: B, Schaum's 3000 solved problems, Q38.31, pg. 712

Example

- X-rays of wavelength 0.2400 nm are Compton scattered and the scattered beam is observed at an angle of 60 degree relative to the incident beam.
- Find (a) the wave length of the scattered x-rays, (b) the energy of the scattered x-ray photons, (c) the kinetic energy of the scattered electrons, and (d) the direction of travel of the scattered electrons

solution

$\lambda' = \lambda + \lambda_c (1 - \cos\theta)$

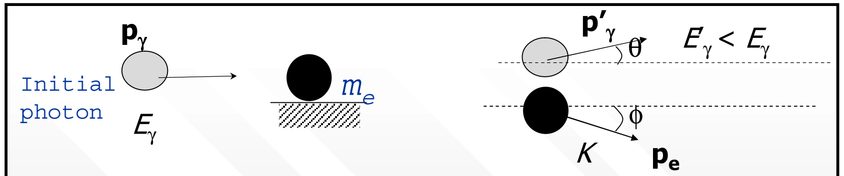
= 0.2400nm+0.00243nm(1–cos60°)

= 0.2412 nm

$E' = hc/\lambda'$

= 1240 eV·nm /0.2412 nm

= 5141 eV



kinetic energy gained by the scattered electron = energy transferred by the incident photon during the scattering:

$K = hc/\lambda - hc/\lambda'$ =(5167–5141)eV = 26 eV

Note that we ignore SR effect here because K << rest mass of electron, m_e = 0.5 MeV

By conservation of momentum in the x- and y- direction:

$p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$; $p'_\gamma \sin\theta = p_e \sin\phi$;

$\tan\phi = p_e \sin\phi / p_e \cos\phi = (p'_\gamma \sin\theta) / (p_\gamma - p'_\gamma \cos\theta)$

$= (E'_\gamma \sin\theta) / (E_\gamma - E'_\gamma \cos\theta)$

$= (5141 \sin 60^0 / [5167 – 5141 (\cos 60^0] = 0.43 = 1.71$

Hence, ϕ = 59.7 degree

PYQ 3(c), Final exam 2003/04

- (c) A 0.0016-nm photon scatters from a free electron. For what scattering angle of the photon do the recoiling electron and the scattered photon have the same kinetic energy?
- **Serway solution manual 2, Q35, pg. 358**

Solution

- The energy of the incoming photon is
  $E_i = hc/\lambda$ = **0.775 MeV**
- Since the outgoing photon and the electron each have half of this energy in kinetic form,
- $E_f = hc/\lambda'$ = 0.775 MeV / 2 = 0.388 MeV and
  $\lambda' = hc/E_f$ = 1240 eV· nm / 0.388 MeV = 0.0032 nm
- The Compton shift is
  $\Delta\lambda = \lambda' - \lambda$ = (0.0032 – 0.0016) nm = 0.0016 nm
- By $\Delta\lambda = \lambda_c (1 – \cos\theta)$
- $= (h/m_e c)\ (1 – \cos\theta)$ 0.0016 nm
- $= 0.00243$ nm $(1 – \cos\theta)$

  $\theta = 70^o$

PYQ 1.10 KSCP 2003/04

Which of the following statement(s) is (are) true?

- **I.** Photoelectric effect arises due to the absorption of electrons by photons
- **II.** Compton effect arises due to the scattering of photons by free electrons
- **III.** In the photoelectric effect, only part of the energy of the incident photon is lost in the process
- **IV**.*I*n the Compton effect, the photon completely disappears and all of its energy is given to the Compton electron
- **A. I,II** **B. II,III,IV** **C. I, II, III**
- **D. III,IV** **Ans: E**
- **[I = false; II = true; III = false; IV = false]**
- Murugeshan, S. Chand & Company, New Delhi, pg. 134, Q13,

X-ray:
The inverse of photoelectricity

- X-ray, discovered by Wilhelm Konrad Roentgen (1845-1923). He won the first Nobel prize in 1902. He refused to benefit financially from his work and died in poverty in the German inflation that followed the end of World War 1.

X-rays are simply EM radiation with very short wavelength, ~ 0.01 nm –10 nm

Some properties:

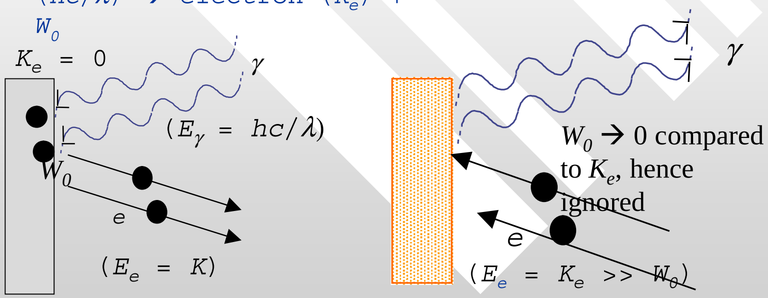
- energetic, according to $E = hc/\lambda \sim 0.1 - 100$ keV (c.f. $E \sim$ a few eV for visible light)
- travels in straight lines
- is unaffected by electric and magnetic fields
- passes readily through opaque materials – highly penetrative
- causes phosphorescent substances to glow
- exposes photographic plates

In photoelectricity, energy is transferred from photons to kinetic energy of electrons. The inverse of this process produces x-rays

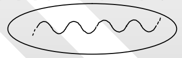
P.E:

electron (K_e=0) + photon (hc/λ) → electron (K_e) + W_0

x-ray: electron (K_e) → heat + photon (hc/λ)

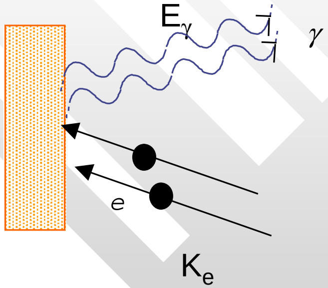


PE and x-rays production happen at different energy scale

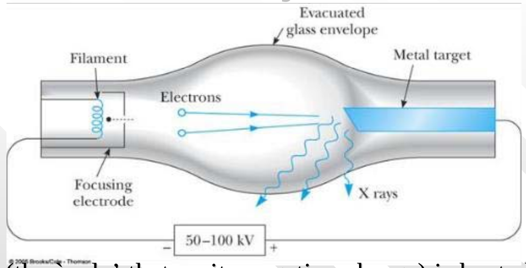
- However, both process occur at disparately different energy scale
- Roughly ,for PE, it occurs at eV scale with ultraviolet radiation 
- For x-ray production, the energy scale involved is much higher - at the order of 100 eV - 100 keV 

X-ray production

- X-rays is produced when electrons, accelerated by an electric field in a vacuum cathode-ray tube, are impacted on the glass end of the tube
- Part or all of the kinetic energy of a moving electron is converted into a x-ray photon

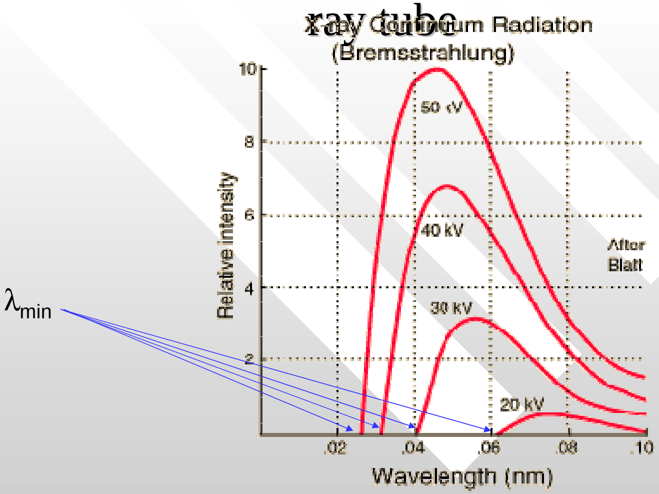


The x-ray tube



- A cathode (the `pole' that emits negative charge) is heated by means of electric current to produce thermionic emission of the electrons from the target
- A high potential difference *V* is maintained between the cathode and a metallic target
- The Thermionic electrons will get accelerated toward the latter
- The higher the accelerating potential *V*, the faster the electron and the shorter the wavelengths of the x-rays

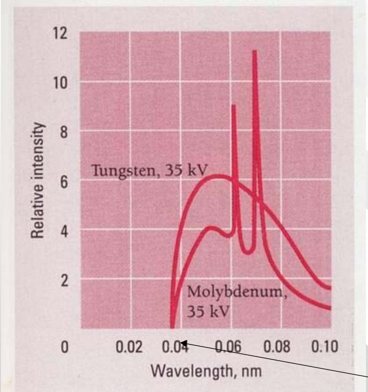
Typical x-ray spectrum from the x-ray tube



Important features of the x-ray spectrum

1. The spectrum is continuous
2. The existence of a minimum wavelength λ_{min} for a given *V*, below which no x-ray is observed
3. Increasing *V* decreases λ_{min} .

$\lambda_{min} \propto 1/V$, the same for all material surface



- At a particular V, λ_{min} is approximately

the same for different target materials. Experimentally one finds that lmin is inversely proportional to V,

$$\lambda_{min} = \left(\frac{1.24\times10^{-6}}{V}\right) \mathrm{m}\cdot\mathrm{V}$$

The peaks in the spectrum are due to the electronic transition occurring between the adjacent shells (orbit) in the atom. We would not discuss them further here.

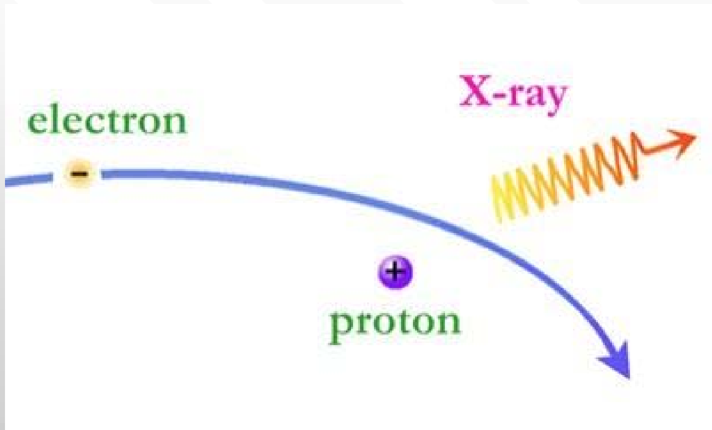
X-ray production heats up the target material

- Due to conversion of energy from the impacting electrons to x-ray photons is not efficient, the difference between input energy, K_e and the output x-ray energy E_γ becomes heat
- Hence the target materials have to be made from metal that can stand heat and must have high melting point (such as Tungsten and Molybdenum)

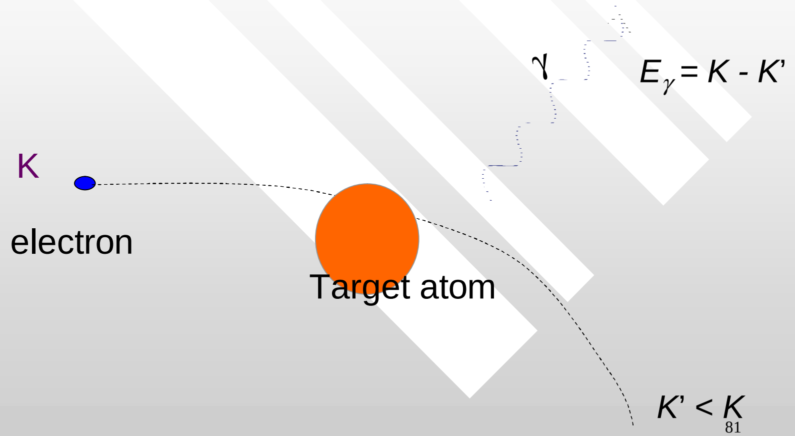
Classical explanation of continuous x-ray spectrum:

- The continuous X-ray spectrum is explained in terms of **Bremsstrahlung:** radiation emitted when a moving electron "tekan brake"
- According to classical EM theory, an accelerating (decelerating) electric charge will radiate EM radiation
- Electrons striking the target get slowed down and brought to eventual rest because of collisions with the atoms of the target material
- Within the target, many electrons collides with many atoms for many times before they are brought to rest
- Each collision causes some non-unique losses to the kinetic energy of the Bremsstrahlung electron
- As a net effect of the collective behavior by many individual collisions, the radiation emitted (a result due to the lost of KE of the electron) forms a continuous spectrum

Bremsstrahlung



Bremsstrahlung, simulation



Bremsstrahlung canot explain λ_{min}

- Notice that in the classical **Bremsstrahlung** process the x-ray radiated is continuous and there is no lower limit on the value of the wavelength emitted. Hence, the existence of λ_{min} is not explained with the classical **Bremsstrahlung** mechanism. All range of λ from 0 to a maximum should be possible in this classical picture.

λ_{min} can only be explained by assuming light as photons but not as EM wave

Energy of the x-ray photon in the quantum picture

- According to Einstein assumption on the energy of a photon, the energy of the photon emitted in the Bremsstrahlung is simply the difference between the initial and final kinetic energy of the electron:

$$h\nu = K - K'$$

- The shortest wavelength of the emitted photon gains its energy, $E = h\,\nu_{max} = hc/\lambda_{min}$ corresponds to the maximal loss of the K.E. of an electron in a single collision (happen when $K' = 0$ in a single collision)
- This (e.g. the maximal lose on KE) only happens to a small sample of collisions. Most of the other collisions loss their KE gradually in smaller amount in an almost continuous manner.

Theoretical explanation of the experimental Value of λ_{min}

- K (of the Bremsstrahlung electron) is converted into the photon with $E = hc/\lambda_{min}$
- Experimentally K is caused by the external potential V that accelerates the electron before it bombards with the target, hence

  $K = eV$
- Conservation of energy requires

$$K = eV = hc/\lambda_{min}$$

- or, $\lambda_{min} = hc/eV = (1240\text{ nm}\cdot\text{eV})/eV = (1240\text{V}/V)$ nm

which is the value measured in x-ray experiments

Why is λ_{min} the same for different material?

- The production of the x-ray can be considered as an inverse process of PE
- Hence, to be more rigorous, the conservation of energy should take into account the effects due to the work potential of the target material during the emission of x-ray process, W_0
- However, so far we have ignored the effect of W_0 when we were calculating the relationship between λ_{min} and K
- This approximation is justifiable because of the following reason:
- The accelerating potentials that is used to produce x-ray in a x-ray vacuum tube, V, is in the range of 10,000 V
- Whereas the work function W_0 is only of a few eV
- Hence, in comparison, W_0 is ignored wrp to eV
- This explains why is the same for different target materials

Example

- Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V
- Solution:

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1.24\times10^{-6}\,\mathrm{V\cdot m}}{5.00\times10^{4}\,\mathrm{V}} = 2.48\times10^{-11}\,\mathrm{m} = 0.0248\mathrm{nm}$$

This wavelength corresponds to the frequency

$$\nu_{\max} = \frac{c}{\lambda_{\min}} = \frac{3\times10^{8}\,\mathrm{m/s}}{2.48\times10^{-11}\,\mathrm{m}} = 1.21\times10^{19}\,\mathrm{Hz}$$

PYQ 1. 9 Final Exam 2003/04

- To produce an x-ray quantum energy of 10^{-15} J electrons must be accelerated through a potential difference of about
- A. 4 kV
- B. 6 kV
- C. 8 kV
- D. 9 kV
- E. 10 kV
- **ANS: B, OCR ADVANCED SUBSIDIARY GCE PHYSICS B (PDF), Q10, pg. 36**

Solution:

The energy of the x-rays photon comes from the external accelerating potential, V

$$E_\lambda = eV$$

$$V = E_\lambda / e = 1\times10^{-15}\,\mathrm{J}/e = \left(\frac{1\times10^{-15}}{1.6\times10^{-19}}\right)\mathrm{eV}/e = 6250\mathrm{V}$$

PYQ 1.9 KSCP 2003/04

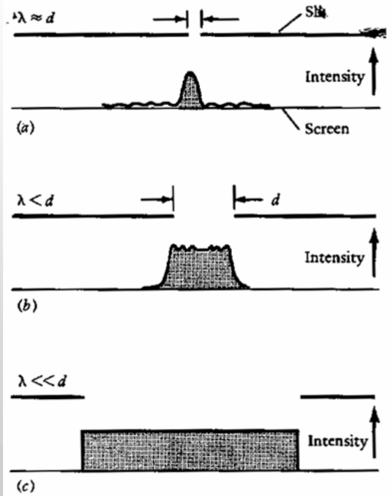
Which of the following statement(s) is (are) true?

- **I.** γ-rays have much shorter wavelength than *x*-rays
- **II.** The wavelength of *x*-rays in a *x*-ray tube can be controlled by varying the accelerating potential
- **III**. *x*-rays are electromagnetic waves
- **IV.** *x*-rays show diffraction pattern when passing through crystals
- **A. I,II** **B. I,II,III,IV** **C. I, II, III**
- **D. III.IV** **E.** Non of the above
- **Ans: B** Murugeshan, S. Chand & Company, New Delhi, pg. 132, Q1.(for I), pg. 132, Q3 (for II), pg. 132, Q4 (for III,IV)

X-ray diffraction

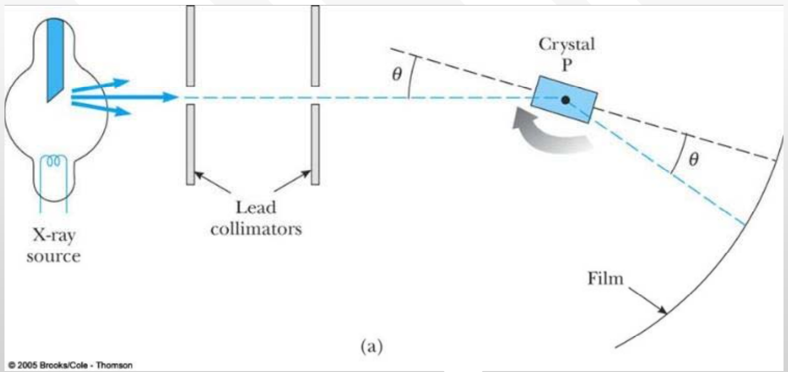
- X-ray wavelengths can be determined through diffraction in which the x-ray is diffracted by the crystal planes that are of the order of the wavelength of the x-ray, ~ 0.1 nm
- The diffraction of x-ray by crystal lattice is called 'Bragg's diffraction'
- It is also used to study crystal lattice structure (by analysing the diffraction pattern)

Condition for diffraction



- Note that as a general rule in wave optics, diffraction effect is prominent only when the wavelength and the 'hole/obstacle are comparable in their length scale

Experimental setup of Bragg's diffraction



X-ray diffraction pattern from crystal

The bright spots correspond to the directions where x-rays scattered from various layers (different Braggs planes) in the crystal interfere constructively.

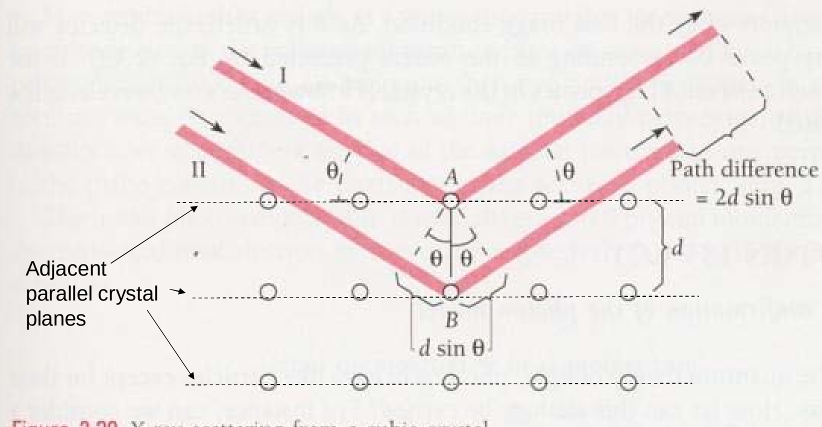


Figure 2.20 X-ray scattering from a cubic crystal.

Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly λ, 2 λ,3 λ and so on (beam I, II):

$2d \sin \theta = n\lambda$, n = 1, 2, 3 …**Bragg's law for x-ray diffraction**

Example

- **A single crystal of table salt (NaCl) is irradiated with a beam of x-rays of unknown wavelength. The first Bragg's reflection is observed at an angle of 26.3 degree. Given that the spacing between the interatomic planes in the NaCl crystal to be 0.282 nm, what is the wavelength of the x-ray?**

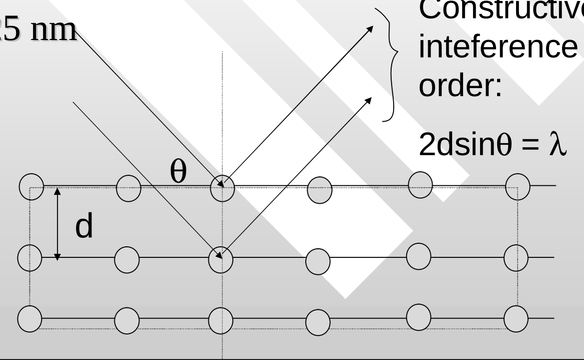
Solution

- Solving Bragg's law for the $n = 1$ order,

  $\lambda = 2d \sin \theta$ = 2 x 0.282 nm x sin (26.3º) = 0.25 nm

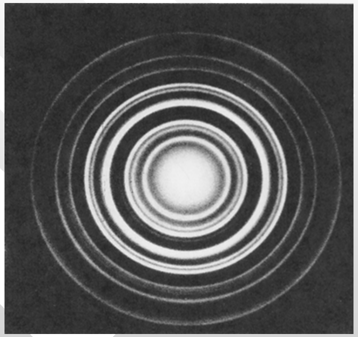
Constructive inteference of n=1 order:

2dsinθ = λ



If powder specimen is used (instead of single crystal)

- We get diffraction ring due to the large randomness in the orientation of the planes of scattering in the power specimen



PYQ 6 Test I, 2003/04

- X-ray of wavelength 1.2 Angstrom strikes a crystal of *d*-spacing 4.4 Angstrom. Where does the diffraction angle of the second order occur?
- **A.**16° **B.** 33° **C.**55 °
- **D.** 90° **E.** Non of the above
- Solution: $n\lambda = 2d \sin\theta$
- $\sin\theta = n\lambda/2d$ = 2 x 2.2 / (2 x 4.4) = 0. 5

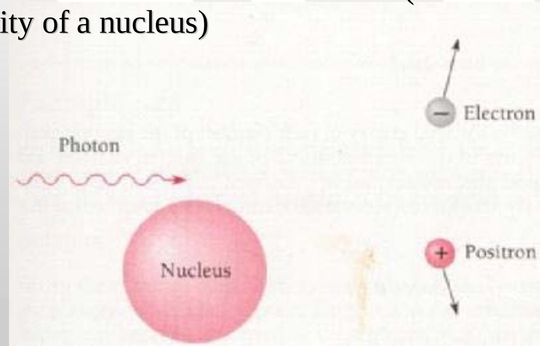
  $\theta = 30$ °
- ANS: B, Schaum's 3000 solved problems, Q38.46, pg. 715

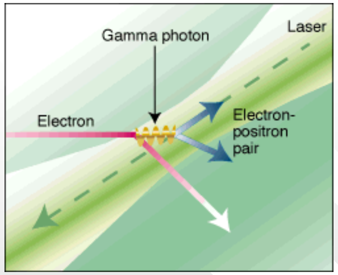
Pair Production: Energy into matter

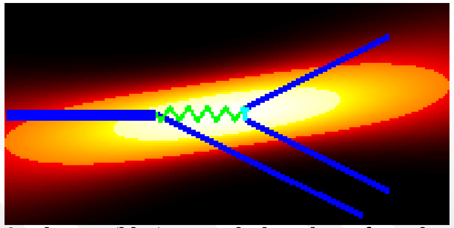
- In photoelectric effect, a photon gives an electron all of its energy. In Compton effect, a photon give parts of its energy to an electron
- A photon can also materialize into an electron and a positron
- Positron = anti-electron, positively charged electron with the exactly same physical characteristics with electron except opposite in charge and spin
- In this process, called pair production, electromagnetic energy is converted into matter
- Creation of something (electron-positron pair) out of nothing (pure EM energy) triggered by strong external EM field

Pictorial visualisation of pair production

- In the process of pair production, a photon of sufficient energy is converted into electron-positron pair. The conversion process must occur only in the presence of some external EM field (such as near the vicinity of a nucleus)







An electron (blue) enters the laser beam from the left, and collides with a laser photon to produce a high-energy gamma ray (wiggly yellow line). The electron is deflected downwards. The gamma ray then collides with four or more laser photons to produce an electron-positron pair

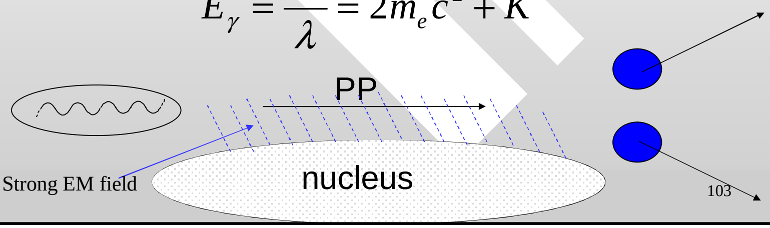
Conservational laws in pair-production

- The pair-production must not violate some very fundamental laws in physics:
- Charge conservation, total linear momentum, total relativistic energy are to be obeyed in the process
- Due to kinematical consideration (energy and linear momentum conservations) pair production cannot occur in empty space
- PP Must occur in the proximity of a nucleus where strong EM field is present to mediate interactions between the photon and the nucleus
- The presence of the nucleus is necessary to absorb part of the momentum transfer delivered by the incident photon so that momentum is conserved in the process of pair creation

Energy threshold

- Due to conservation of relativistic energy, pair production can only occur if E_γ is larger than 2 m_e = 2 x 0.51 MeV = 1.02 MeV
- Any additional photon energy becomes kinetic energy of the electron and positron, K

$$E_\gamma = \frac{hc}{\lambda} = 2m_e c^2 + K$$



Example

- What is the minimal wavelength of a EM radiation to pair-produce an electron-positron pair?
- Solutions: minimal photon energy occurs if the pair have no kinetic energy after being created, $K = 0$
- Hence,

$$\lambda_{\min} = \frac{hc}{2m_e c^2} = \frac{1240\text{nm}\cdot\text{eV}}{2\cdot 0.51\text{MeV}} = 1.21\times 10^{-12}\,\text{m}$$

These are very energetic EM radiation called gamma rays and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.

Electron-positron creation

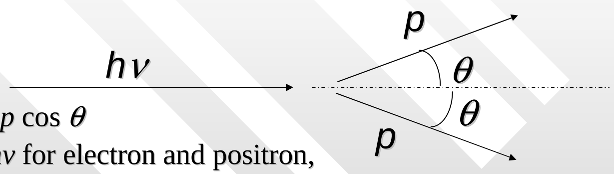
- ***Part of a bubble chamber picture (Fermilab'15 foot Bubble Chamber', found at the University of Birmingham). The curly line which turns to the left is an electron. Positron looks similar but turn to the right The magnetic field is perpendicular to the picture plan***

Solution

- Conservation of energy must me fulfilled,

$$h\nu = 2mc^2$$

- Conservation of linear momentum must be fulfilled:



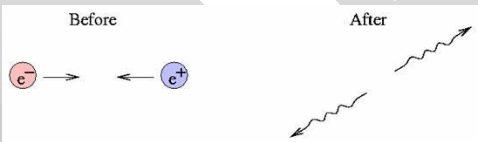
- $\Rightarrow h\nu/c = 2p\cos\theta$
- Since $p = mv$ for electron and positron,
- $\Rightarrow h\nu = 2c\,(mv)\cos\theta = 2mc^2\,(v/c)\cos\theta$
- Because $v/c < 1$ and $\cos\theta \leqslant 1$, $h\nu < 2mc^2$
- But conservation of energy requires $h\nu = 2mc^2$. Hence it is impossible for pair production to conserve both energy and momentum unless some other object (such as a nucleus) in involved in the process to carry away part of the initial of the photon momentum

Pair-annihilation

- The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges

$$e^+ + e^- \rightarrow \gamma + \gamma$$

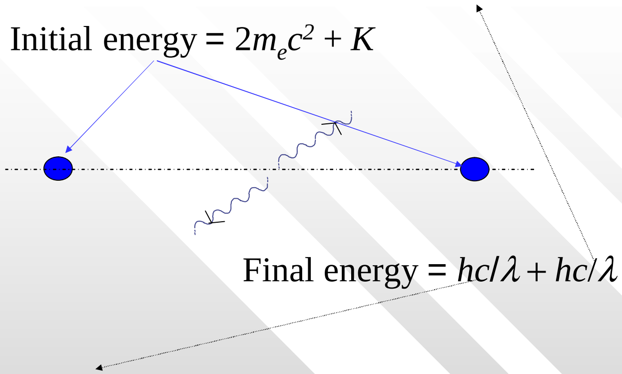
- Both particles vanish simultaneously, with the lost masses becoming energies in the form of two gamma-ray photons
- Positron and electron annihilate because they are anti particles to each other



Pair annihilation

- ***Part of a bubble-chamber picture from a neutrino experiment performed at the Fermilab (found at the University of Birmingham). A positron in flight annihilate with an electron. The photon that is produced materializes at a certain distance, along the line of flight, resulting a new electron-positron pair (marked with green)***

Initial energy = $2m_ec^2 + K$



Final energy = $hc/\lambda + hc/\lambda$

Conservation of relativistic energy:

$2m_ec^2 + K = 2\ hc/\lambda$

Energy and linear momentum are always conserved in pair annihilation

- The total relativistic energy of the e^--e+ pair is
  - $E = 2m_ec^2 + K = 1.02$ MeV + K
- where K the total kinetic energy of the electron-positron pair before annihilation
- Each resultant gamma ray photon has an energy

  $h\nu = 0.51$ MeV + $K/2$
- Both energy and linear momentum are automatically conserved in pair annihilation (else it wont occur at all)
- The gamma photons are always emitted in a back-to-back manner due to kinematical reasons (conservation of linear momentum)
- No nucleus or other particle is needed for pair annihilation to take place
- Pair annihilation always occurs whenever a matter comes into contact with its antimatter

As a tool to observe anti-world

- What is the characteristic energy of a gamma-ray that is produced in a pair-annihilation production process? What is its wavelength?
- Answer: 0.51 MeV, $\lambda_{annih} = hc/0.51$ MeV = 0.0243 nm
- The detection of such characteristic gamma ray in astrophysics indicates the annihilation of matter-antimatter in deep space

PYQ 4, Test I, 2003/04

- An electron and a positron collide and undergo pair-annihilation. If each particle is moving at a speed of 0.8*c* relative to the laboratory before the collision, determine the energy of each of the resultant photon.
- **A.** 0.85MeV **B.** 1.67 MeV
- **C.** 0.51 MeV **D.** 0.72MeV
- **E.** Non of the above

Solution

Total energy before and after anniliation must remain the same: *i.e.* the energy of each electron is converted into the energy of each photon. Hence the energy of each photon is simple equal to the total relativistic energy of each electron travelling at 0.8c :

$E_\gamma = E_e = \gamma m_e c^2$

where $\gamma = 1/\sqrt{1-(0.8)^2} = 1.678$

Hence $E_\gamma = 1.678 \times 0.51$ MeV = 0.85 MeV

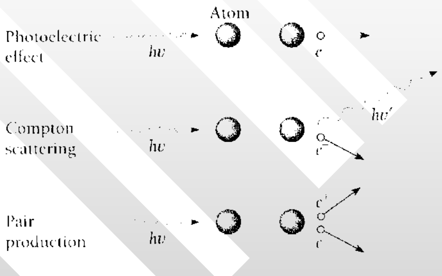
- ANS: A, Cutnell, Q17, pg. 878, modified

What is the wavelength of the gamma ray in a proton-antiproton annihilation?

$\lambda_{annih} = hc/M_p c^2$ = 1240 nm·eV/937 MeV = 1.3×10^{-15} m

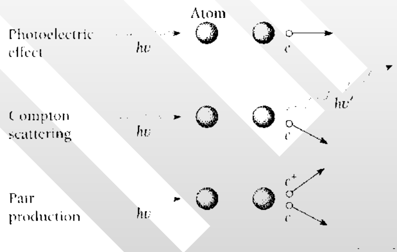
Photon absorption

- Three chief “channels” photons interact with matter are: photoelectric effect, Compton scattering and Pair-production
- In all of these process, photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material



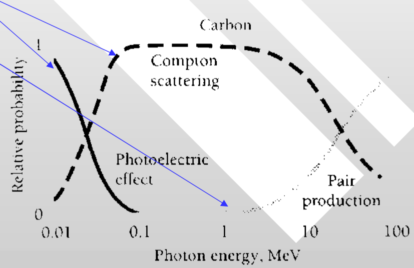
Photon absorption

- The probability of a photon undergoes a given channel of interaction with matter depends on
- (1) Photon energy, and
- (2) Atomic number of the absorbing material



Relative probabilities of photon absorption mechanisms

- For a fixed atomic number (say Carbon, A = 12)
- At low energy photoelectric effect dominates. It diminishes fast when E_γ approaches tens of keV
- At E_γ = a few tens of keV, Compton scattering start to take over
- Once E_γ exceeds the threshold of $2m_ec^2$ = 1.02 MeV, pair production becomes more likely. Compton scattering diminishes as energy increases from 1 MeV



Relative probabilities between different absorbers

- Compare with Lead absorber (much higher *A*):
- Photoelectric effect remains dominant up to a higher energy of a few hundreds of keV (c.f. Carbon of a few tens of keV)
- This is because the heavier the nucleus the better it is in absorbing the momentum transfer that occurs when the energetic photon impart its momentum to the atom
- Compton scattering starts to appears after a much higher energy of 1 MeV (c.f. a few tens of keV for Carbon).
- This is because a larger atomic number binds an electron stronger, rendering the electron less 'free'> In this case, to Compton scatter off an "free" electron the photon has to be more energetic
- (recall that in Compton scattering, only free electrons are scattered by photon).

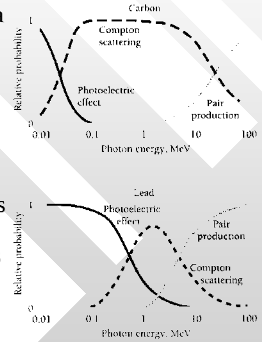


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

Relative probabilities of pair creation between different absorbers

- The energy at which pair production takes over as the principle mechanism of energy loss is called the crossover energy
- The crossover energy is 10 MeV for Carbon, 4 for Lead
- The greater atomic number, the lower the crossover energy
- This is because nuclear with larger atomic number has stronger electric field that is necessary to trigger pair-creation

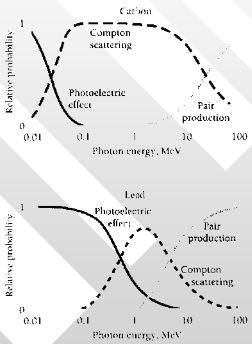


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

What is a photon

- Like an EM wave, photons move with speed of light *c*
- They have zero mass and rest energy
- The carry energy and momentum, which are related to the frequency and wavelength of the EM wave by $E=h\nu$ and $p=h/\lambda$
- They can be created or destroyed when radiation is emitted or absorbed
- They can have particle-like collisions with other particles such as electrons

Contradictory nature of photon

- In Photoelectric effect, Compton scatterings, inverse photoelectric effect, pair creation/annihilation, light behaves as particle. The energy of the EM radiation is confined to localised bundles production/annihilation
- In Young's Double slit interference, diffraction, light behave as waves. In the wave picture of EM radiation, the energy of wave is spread smoothly and continuously over the wavefronts
- Both the wave and particle explanations of EM radiation are obviously mutually exclusive

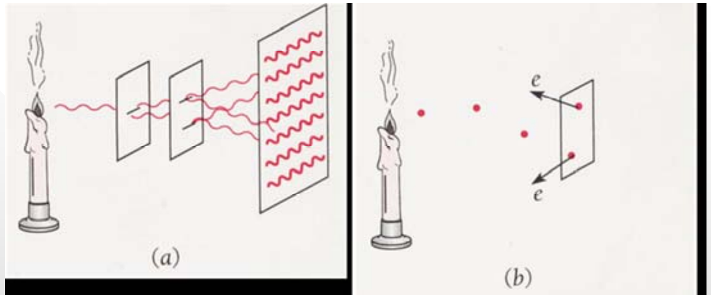
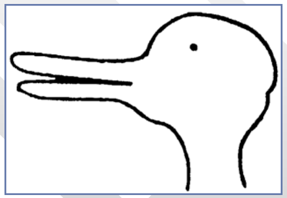


Figure 2.14 (a) The wave theory of light explains diffraction and interference, which the quantum theory cannot account for. (b) The quantum theory explains the photoelectric effect, which the wave theory cannot account

The identity of photon depends on how the experimenter looks at it

The face of a young or an old woman?



Is this a rabbit or a duck?

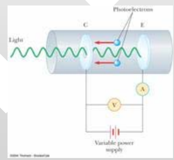
Gedanken experiment with remote light source

- The same remote light source is used to simultaneously go through two experimental set up
- In the left experiment, the EM radiation behaves as wave; the right one behave like particle
- This is weird: the "light source" from 100 M light years away seems to "know" in which direction to aim the waves and in which direction to aim the particles

Light source from 100 M light years away

Double slit experiment

Photoelectric experiment



Interference pattern observed

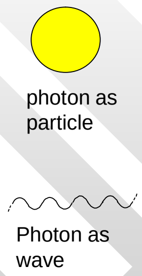
Photoelectron observed

So, is light wave of particle?

- So, it is not *either* particle or wave but *both* particles and waves
- However, both typed of nature cannot be simultaneously measured in a single experiment
- The light only shows one or the other aspect, depending on the kind of experiment we are doing
- Particle experiments show the particle nature, while a wave-type experiment shows the wave nature

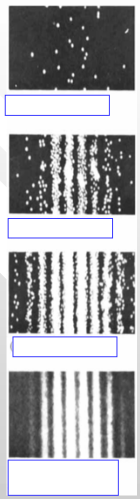
Coin a simile of wave-particle duality

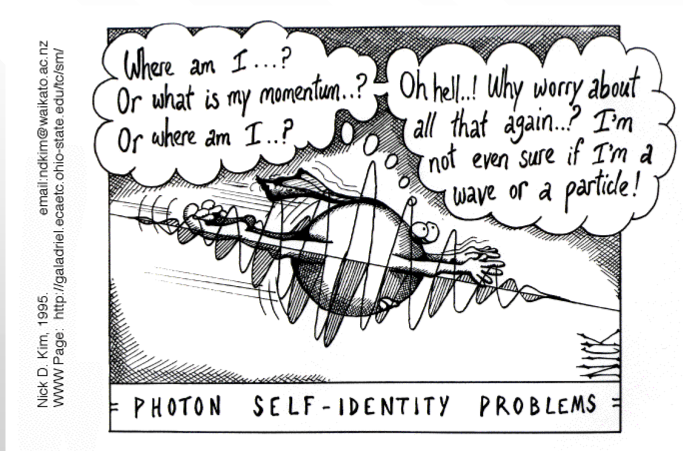
- It's like a coin with two faces. One can only sees one side of the coin but not the other at any instance
- This is the so-called wave-particle duality
- Neither the wave nor the particle picture is wholly correct all of the time, that both are needed for a complete description pf physical phenomena
- The two are *complementary* to another



Interference experiment with a single photon

- Consider an double slit experiment using an extremely weak source (say, a black body filament) that emits only one photon a time through the double slit and then detected on a photographic plate by darkening individual grains.
- When one follows the time evolution of the pattern created by these individual photons, interference pattern be observed
- At the source the light is being emitted as photon (radiated from a dark body) and is experimentally detected as a photon which is absorbed by an individual atom on the photographic plate to form a grain
- In between, we must interpret the light as electromagnetic energy that propagates smoothly and continuously as a wave
- However, the wave nature between the emission and detection is not directly detected. Only the particle nature are detected in this procedure.
- The correct explanation of the origin and appearance of the interference pattern comes from the wave picture, and the correct interpretation of the evolution of the pattern on the screen comes from the particle picture;
- Hence to completely explain the experiment, the two pictures must somehow be taken together – this is an example for which *both pictures are complimentary to each other*

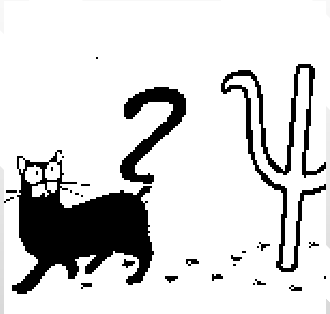




Both light and material particle display wave-particle duality

- Not only light manifest such wave-particle duality, but other microscopic material particles (e.g. electrons, atoms, muons, pions well).
- In other words:
- Light, as initially thought to be wave, turns out to have particle nature;
- Material particles, which are initially thought to be corpuscular, also turns out to have wave nature (next topic)

The wavelike properties of particles



Cat: "Am I a particle or wave?"

Wave particle duality

- "Quantum nature of light" refers to the particle attribute of light
- "Quantum nature of particle" refers to the wave attribute of a particle
- Light (classically EM waves) is said to display "wave-particle duality" – it behave like wave in one experiment but as particle in others (c.f. a person with schizophrenia)

- Not only light does have "schizophrenia", so are other microscopic ``particle'' such as electron, (see later chapters), i.e. particle" also manifest wave characteristics in some experiments
- Wave-particle duality is essentially the manifestation of the quantum nature of things
- This is an very weird picture quite contradicts to our conventional assumption with is deeply rooted on classical physics or intuitive notion on things

Planck constant as a measure of quantum effect

- When investigating physical systems involving its quantum nature, the theory usually involves the appearance of the constant h
- i.e. in Compton scattering, the Compton shift is proportional to h; So is photoelectricity involves h in its formula
- In general, when h appears, it means quantum effects arise
- In contrary, in classical mechanics or classical EM theory, h never appear both theories do not take into account quantum effects, hence the constant h never appear in such theories
- Roughly quantum effects arise in microscopic system (e.g. on the scale approximately of the order 10^{-10} m or smaller)

Wavelike properties of particle

- In 1923, while still a graduate student at the University of Paris, Louis de Broglie published a brief note in the journal *Comptes rendus* containing an idea that was to revolutionize our understanding of the physical world at the most fundamental level:
- *That particle has intrinsic wave properties*
- *For more interesting details:*
- *http://www.davis-inc.com/physics/index.shtml*

Prince de Broglie, 1892-1987

de Broglie's postulate (1924)

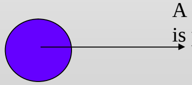
- The postulate: there should be a symmetry between matter and wave. The wave aspect of matter is related to its particle aspect in exactly the same quantitative manner that is in the case for radiation. The total (i.e. relativistic) energy E and momentum p of an entity, for both matter and wave alike, is related to the frequency ν of the wave associated with its motion via by Planck constant

$$E = hf, p = h/\lambda$$

A particle has wavelength!!!

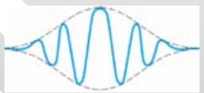
$$\lambda = h/p$$

- is the de Broglie relation predicting the wave length of the matter wave λ associated with the motion of a material particle with momentum p
- Note that classically the property of wavelength is only reserved for wave and particle was never associate with any wavelength
- But, following de Broglie's postulate, such distinction is removed



Particle with linear momentum p

A particle with momentum p is pictured as a wave (wavepulse)



Matter wave with de Broglie wavelength

$\lambda = p/h$

A physical entity possess both aspects of particle and wave in a complimentary manner

BUT <u>why</u> is the wave nature of material particle not observed?

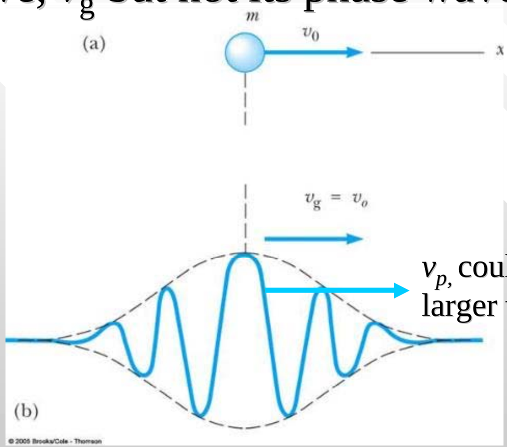
Because …

- Because…we are too large and quantum effects are too small
- Consider two extreme cases:
- (i) an electron with kinetic energy K = 54 eV, de Broglie wavelength, $\lambda = h/p = h / (2m_e K)^{1/2}$ = 1.65 Angstrom.
- Such a wavelength is comparable to the size of atomic lattice, and is experimentally detectable
- (ii) As a comparison, consider an macroscopic object, a billard ball of mass m = 100 g moving with momentum p
- $p = mv \approx 0.1$ kg $\times$ 10 m/s = 1 Ns (relativistic correction is negligible)
- It has de Broglie wavelength $\lambda = h/p \approx 10^{-34}$ m, too tiny to be observed in any experiments
- The total energy of the billard ball is
  - $E = K + m_0c^2 = mc^2 = 0.1\times (3 \times 10^8)^2$ J $= 9 \times 10^{15}$J
- The frequency of the de Broglie wave associated with the billard ball is
  $f = E/h = mc^2/h$= $(9\times10^{15}/6.63\times10^{34})$ Hz = $(9\times10^{15}/6.63\times10^{34})$ =10^{78} Hz, impossibly high for any experiment to detect

Matter wave is a quantum phenomena

- This also means that the wave properties of matter is difficult to observe for macroscopic system (unless with the aid of some specially designed apparatus)
- The smallness of h in the relation $\lambda = h/p$ makes wave characteristic of particles hard to be observed
- The statement that when $h \to 0$, λ becomes vanishingly small means that:
- the wave nature will become effectively “shut-off” and appear to loss its wave nature whenever the relevant p of the particle is too large in comparison with the quantum scale characterised by h
- More quantitatively, we could not detect the quantum effect if $h/p \sim 10^{-34}$ Js/p becomes too tiny in comparison to the length scale discernable by an experimental setup

The the particle’s velocity v_0 is identified with the de Broglie’ group wave, v_g but not its phase wave v_p



v_p, could be larger than c

Example

- An electron has a de Broglie wavelength of 2.00 pm. Find its kinetic energy and the phase and the group velocity of its de Broglie waves.
- Total energy $E^2 = c^2p^2 + m_0$i^2
- $K = E - m_0c^2 = (c^2p^2 + m_0^2c^4)^{½} - m_0c^2$ =
- $((hc/\lambda)^2 + m_0^2c^4)^{½} - m_0c^2$= 297 keV
- $v_g = v$; $1/\gamma^2 = 1 - (v/c)^2$;
- $(pc)^2 = (\gamma m_0 vc)^2 = (hc/\lambda)^2$ (from Relativity and de Broglie’s postulate)

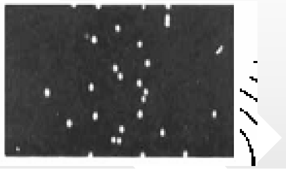
$\Rightarrow(\gamma v/c)^2 = (hc/\lambda)^2/(m_0c^2)^2$=(620 keV/510 keV)^2 = 1.4884;

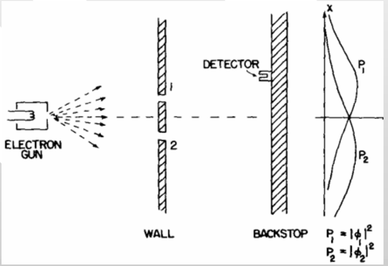
$(\gamma v/c)^2 = (v/c)^2 / 1 - (v/c)^2$

$\Rightarrow v_g/c = \sqrt{}$ (1.4884/(1+1.4884))=0.77

Interference experiment with a single electron, firing one in a time

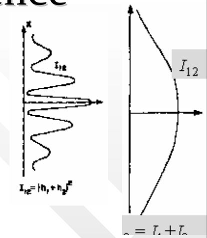
- Consider an double slit experiment using an extremely electron source that emits only one electron a time through the double slit and then detected on a fluorescent plate

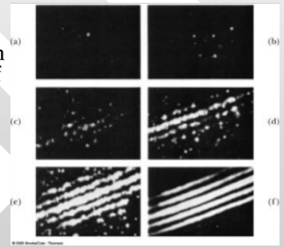




Electrons display interference pattern

- When one follows the time evolution of the pattern created by these individual photons, what sort of pattern do you think you will observed?
- It's the interference pattern that are in fact observed in experiments
- At the source the electron is being emitted as particle and is experimentally detected as a electron which is absorbed by an individual atom in the fluorescent plate
- In between, we must interpret the electron in the form of a wave. The double slits change the propagation of the electron wave so that it is 'processed' to forms diffraction pattern on the screen.
- Such process would be impossible if electrons are particle (because no one particle can go through both slits at the same time. Such a simultaneous penetration is only possible for wave.)
- Be reminded that the wave nature in the intermediate states is not measured. Only the particle nature are detected in this procedure.





Detection of electron as particle destroy the interference pattern

- If in the electron interference experiment one tries to place a detector on each hole to determine though which an electron passes, the wave nature of electron in the intermediate states are destroyed
- e.g. by the interference pattern on the screen shall be destroyed
- Why? It is the consistency of the wave-particle duality that demands such destruction must happen (think of the logics yourself or read up from the text)

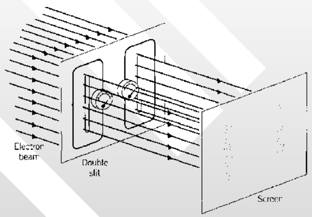
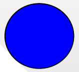


FIGURE 4.15 Apparatus to record passage of electrons through slits. Each slit is surrounded by a loop with a meter that signals the passage of an electron through the slit. No interference fringes are seen on the screen.

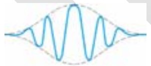
- The correct explanation of the origin and appearance of the interference pattern comes from the wave picture, and the correct interpretation of the evolution of the pattern on the screen comes from the particle picture;
- Hence to completely explain the experiment, the two pictures must somehow be taken together – this is an example for which *both pictures are complimentary to each other*
- Try to compare the last few slides with slide 119 from previous chapter for photon, which also displays wave-particle duality

So, is electron wave or particle?

- They are both…but not simultaneously
- In any experiment (or empirical observation) only one aspect of either wave or particle, but not both can be observed simultaneously.
- It's like a coin with two faces. But one can only see one side of the coin but not the other at any instance
- This is the so-called wave-particle duality



Electron as particle



Electron as wave

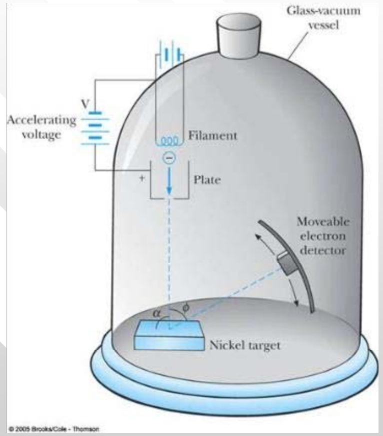
"Once and for all I want to know what I'm paying for. When the electric company tells me whether electron is a wave or a particle I'll write my check"

Extra readings

- Those quantum enthusiasts may like to read more about wave-particle duality in Section 5.7, page 179-185, Serway and Moses
- An even more recommended reading on wave-particle duality: the Feynman lectures on physics, vol. III, chapter 1 (Addison-Wesley Publishing)
- It's a very interesting and highly intellectual topic to investigate

Davisson and Gremer experiment

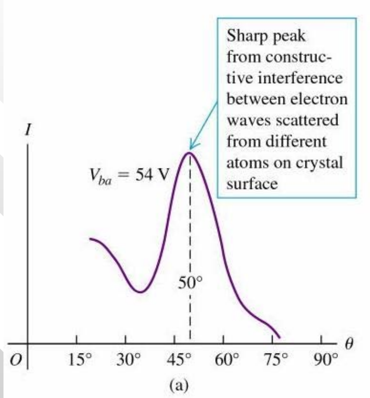
- DG confirms the wave nature of electron in which it undergoes Bragg's diffraction
- Thermionic electrons are produced by hot filament, accelerated and focused onto the target (all apparatus is in vacuum condition)
- Electrons are scattered at an angle ϕ into a movable detector



Pix of Davisson and Gremer

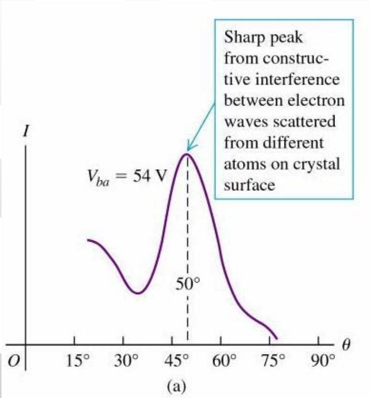
Result of the DG experiment

- Distribution of electrons is measured as a function of ϕ
- Strong scattered e-beam is detected at ϕ = 50 degree for V = 54 V



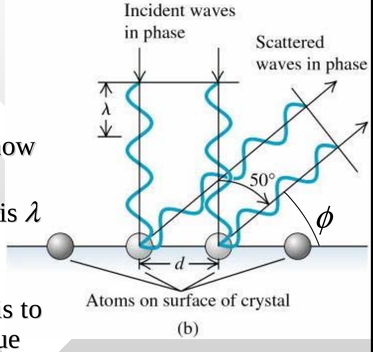
How to interpret the result of DG?

- Electrons get diffracted by the atoms on the surface (which acted as diffraction grating) of the metal as though the electron acting like they are WAVE
- Electron do behave like wave as postulated by de Broglie



Constructive Bragg's diffraction

- The peak of the diffraction pattern is the m=1st order constructive interference: $d\sin\phi = 1\lambda$
- where ϕ = 50 degree for V = 54 V
- From x-ray Bragg's diffraction experiment done independently we know d = 2.15 Amstrong
- Hence the wavelength of the electron is $\lambda = d\sin\theta$ = 1.65 Angstrom
- Here, 1.65 Angstrom is the experimentally inferred value, which is to be checked against the theoretical value predicted by de Broglie



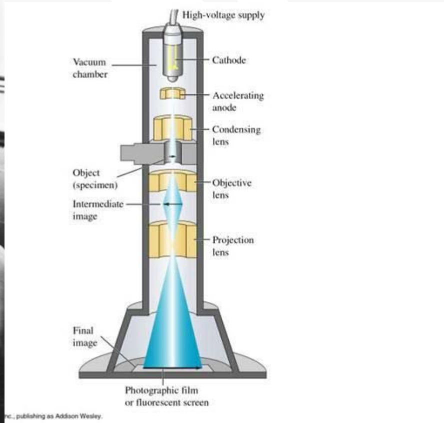
Theoretical value of λ of the electron

- An external potential V accelerates the electron via $eV=K$
- In the DG experiment the kinetic energy of the electron is accelerated to K = 54 eV (non-relativistic treatment is suffice because $K << m_e c^2$ = 0.51 MeV)
- According to de Broglie, the wavelength of an electron accelerated to kinetic energy of $K = p^2/2m_e$ = 54 eV has a equivalent matter wave wavelength
  $\lambda = h/p = h/(2Km_e)^{-1/2}$ = 1.67 Amstrong
- In terms of the external potential,
  $\lambda = h/(2eVm_e)^{-1/2}$

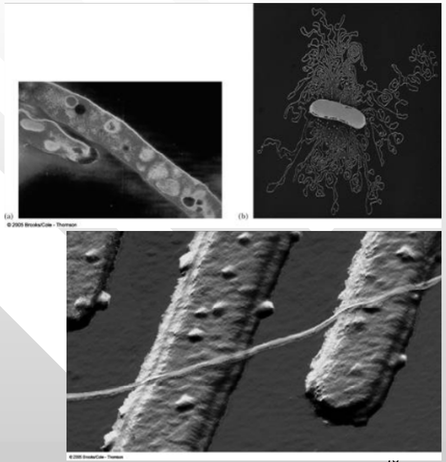
Theory's prediction matches measured value

- The result of DG measurement agrees almost perfectly with the de Broglie's prediction: 1.65 Angstrom measured by DG experiment against 1.67 Angstrom according to theoretical prediction
- Wave nature of electron is hence experimentally confirmed
- In fact, wave nature of microscopic particles are observed not only in e- but also in other particles (e.g. neutron, proton, molecules etc. – most strikingly Bose-Einstein condensate)

Application of electrons wave: electron microscope, Nobel Prize 1986 (Ernst Ruska)



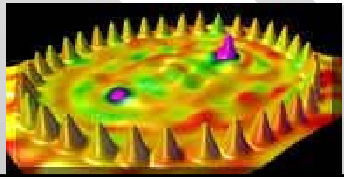
- Electron's de Broglie wavelength can be tuned via $\lambda = h/(2eVm_e)^{-1/2}$
- Hence electron microscope can magnify specimen (x4000 times) for biological specimen or 120,000 times of wire of about 10 atoms in width



Not only electron, other microscopic particles also behave like wave at the quantum scale

- The following atomic structural images provide insight into the threshold between prime radiant flow and the interference structures called matter.
- In the right foci of the ellipse a real cobalt atom has been inserted. In the left foci of the ellipse a phantom of the real atom has appeared. The appearance of the phantom atom was not expected.
- The ellipsoid coral was constructed by placing 36 cobalt atom on a copper surface. This image is provided here to provide a visual demonstration of the attributes of material matter arising from the harmonious interference of background radiation.

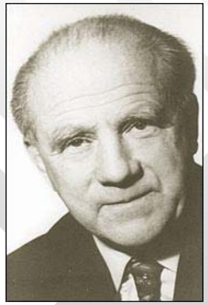
QUANTUM CORAL



http://home.netcom.com/~sbyers11/grav11E.htm

Heisenberg's uncertainty principle (Nobel Prize,1932)

- WERNER HEISENBERG (1901 - 1976)
- was one of the greatest physicists of the twentieth century. He is best known as a founder of quantum mechanics, the new physics of the atomic world, and especially for the uncertainty principle in quantum theory. He is also known for his controversial role as a leader of Germany's nuclear fission research during World War II. After the war he was active in elementary particle physics and West German science policy.
- *http://www.aip.org/history/heisenberg/p01.htm*



A particle is represented by a wave packet/pulse

- Since we experimentally confirmed that particles are wave in nature at the quantum scale *h* (matter wave) we now have to describe particles in term of waves (relevant only at the quantum scale)
- Since a real particle is localised in space (not extending over an infinite extent in space), the wave representation of a particle has to be in the form of wave packet/wave pulse

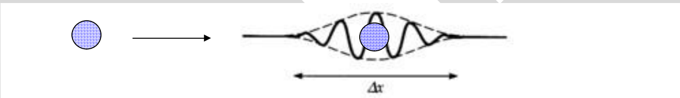
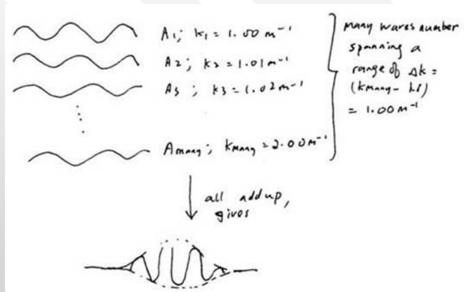


FIGURE 6.14 An idealized wave packet localized in space over a region Δx is the perposition of many waves of different amplitudes and frequencies.

- As mentioned before, wavepulse/wave packet is formed by adding many waves of different amplitudes and with the wave numbers spanning a range of Δk (or equivalently, $\Delta\lambda$)



Δx

Recall that $k = 2\pi/\lambda$, hence

$\Delta k/k = \Delta\lambda/\lambda$

Still remember the uncertainty relationships for classical waves?

- As discussed earlier, due to its nature, a wave packet must obey the uncertainty relationships for classical waves (which are derived mathematically with some approximations)

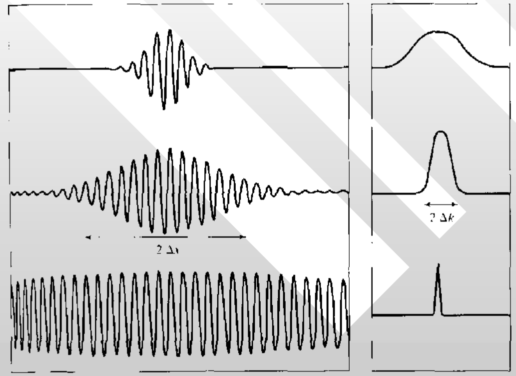
$$\Delta\lambda\Delta x \gtrsim \lambda^2 \equiv \Delta k \Delta x \gtrsim 2\pi \qquad \Delta t \Delta \nu \geq 1$$

- However a more rigorous mathematical treatment (without the approximation) gives the exact relations

$$\Delta\lambda\Delta x \geq \frac{\lambda^2}{4\pi} \equiv \Delta k \Delta x \geq 1/2 \qquad \Delta \nu \Delta t \geq \frac{1}{4\pi}$$

- To describe a particle with wave packet that is localised over a small region Δx requires a large range of wave number; that is, Δk is large. Conversely, a small range of wave number cannot produce a wave packet localised within a small distance.

- A narrow wave packet (small Δx) corresponds to a large spread of wavelengths (large Δk).
- A wide wave packet (large Δx) corresponds to a small spread of wavelengths (small Δk).



Matter wave representing a particle must also obey similar wave uncertainty relation

- For matter waves, for which their momentum and wavelength are related by $p = h/\lambda$, the uncertainty relationship of the classical wave
- $\Delta\lambda\Delta x \geq \frac{\lambda^2}{4\pi} \equiv \Delta k \Delta x \geq 1/2$ is translated into

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

- where $\hbar = h/2\pi$
- Prove this relation yourselves (hint: from $p = h/\lambda$, $\Delta p/p = \Delta\lambda/\lambda$)

Time-energy uncertainty

- Just as $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ implies position-momentum uncertainty = relation, the classical wave uncertainty relation $\Delta \nu \Delta t \geq \frac{1}{4\pi}$ also implies a corresponding relation of time and energy

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- This uncertainty relation can be easily obtained:

$$h\Delta \nu \Delta t \geq \frac{h}{4\pi} = \frac{\hbar}{2};$$

$$\because E = h\nu, \Delta E = h\Delta \nu \Rightarrow h\Delta \nu \Delta t = \Delta E \Delta t = \frac{\hbar}{2}$$

Heisenberg uncertainty relations

$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

- The product of the uncertainty in momentum (energy) and in position (time) is at least as large as Planck's constant

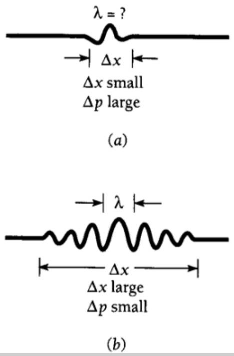
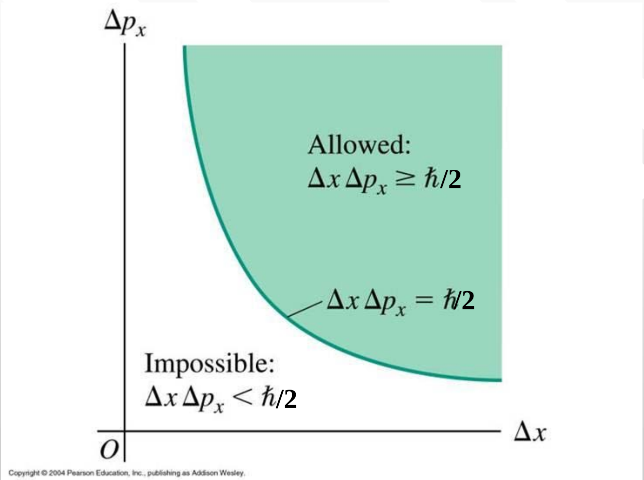


Figure 3.12 (*a*) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (*b*) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

What $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ means

- It sets the intrinsic lowest possible limits on the uncertainties in knowing the values of p_x and x, no matter how good an experiments is made
- It is impossible to specify simultaneously and with infinite precision the linear momentum and the corresponding position of a particle

It is impossible for the product $\Delta x \Delta p_x$ to be less than $h/4\pi$

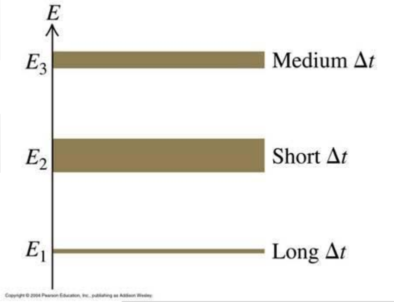


What $\Delta E \Delta t \geq \frac{\hbar}{2}$ means

- Uncertainty principle for energy.
- The energy of a system also has inherent uncertainty, ΔE
- ΔE *is* dependent on the *time interval* Δt during which the system remains in the given states.
- If a system is known to exist in a state of energy E over a limited period Δt, then this energy is uncertain by at least an amount $h/(4\pi\Delta t)$. This corresponds to the 'spread' in energy of that state (see next page)
- Therefore, the energy of an object or system can be measured with infinite precision (ΔE=0) only if the object of system exists for an infinite time ($\Delta t \rightarrow \infty$)

What $\Delta E \Delta t \geq \frac{\hbar}{2}$ means

- A system that remains in a metastable state for a very long time (large Δt) can have a very well-defined energy (small ΔE), but if remain in a state for only a short time (small Δt), the uncertainty in energy must be correspondingly greater (large ΔE).



Conjugate variables (Conjugate observables)

- $\{p_x, x\}$, $\{E, t\}$ are called *conjugate variables*
- The conjugate variables cannot in principle be measured (or known) to infinite precision simultaneously

Heisenberg's Gedanken experiment

- The U.P. can also be understood from the following gedanken experiment that tries to measure the position and momentum of an object, say, an electron at a certain moment
- In order to measure the momentum and position of an electron it is necessary to "interfere" it with some "probe" that will then carries the required information back to us – such as shining it with a photon of say a wavelength of λ

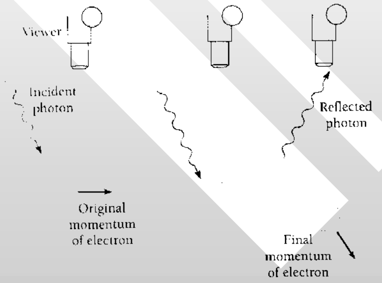
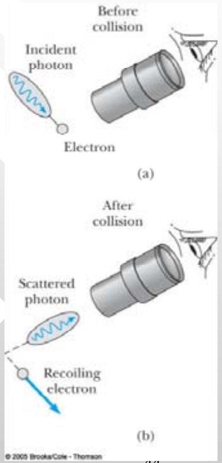


Figure 3.17 An electron cannot be observed without changing its momentum.

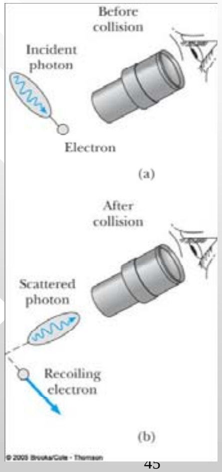
Heisenberg's Gedanken experiment

- Let's say the "unperturbed" electron was initially located at a "definite" location x and with a "definite" momentum p
- When the photon 'probes' the electron it will be bounced off, associated with a changed in its momentum by some uncertain amount, Δp.
- Δp cannot be predicted but must be of the similar order of magnitude as the photon's momentum h/λ
- Hence $\Delta p \approx h/\lambda$
- The longer λ (i.e. less energetic) the smaller the uncertainty in the measurement of the electron's momentum
- In other words, electron cannot be observed without changing its momentum



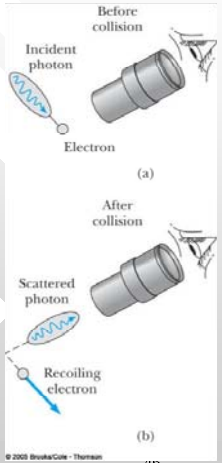
Heisenberg's Gedanken experiment

- How much is the uncertainty in the position of the electron?
- By using a photon of wavelength λ we cannot determine the location of the electron better than an accuracy of $\Delta x = \lambda$
- Hence $\Delta x \geq \lambda$
- Such is a fundamental constraint coming from optics (Rayleigh's criteria).
- The shorter the wavelength λ (i.e. more energetic) the smaller the uncertainty in the electron's position



Heisenberg's Gedanken experiment

- However, if shorter wavelength is employed (so that the accuracy in position is increased), there will be a corresponding decrease in the accuracy of the momentum, measurement (recall $\Delta p \approx h/\lambda$)
- A higher photon momentum will disturb the electron's motion to a greater extent
- Hence there is a 'zero sum game' here
- Combining the expression for Δx and Δp, we then have $\Delta p \, \Delta\lambda \geq h$, a result consistent with $\Delta p \, \Delta\lambda \geq h/2$



Heisenberg's kiosk

Example

- A typical atomic nucleus is about 5.0×10^{-15} m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus

Solution

- Letting $\Delta x = 5.0\times10^{-15}$ m, we have
- $\Delta p \geq h/(4\pi\Delta x)=\ldots=1.1\times10^{-20}$ kg·m/s

If this is the uncertainty in a nuclear electron's momentum, the momentum p must be at lest comparable in magnitude. An electron of such a momentum has a

- KE = $pc \geq 3.3\times10^{-12}$ J

  $= 20.6$ MeV >> $m_e c^2 = 0.5$ MeV
- i.e., if electrons were contained within the nucleus, they must have an energy of at least 20.6 MeV
- However such an high energy electron from radioactive nuclei never observed
- Hence, by virtue of the uncertainty principle, we conclude that electrons emitted from an unstable nucleus cannot comes from within the nucleus

Example

- A measurement established the position of a proton with an accuracy of $\pm1.00\times10^{-11}$ m. Find the uncertainty in the proton's position 1.00 s later. Assume $v << c$.

Solution

- Let us call the uncertainty in the proton's position Δx_0 at the time $t = 0$.
- The uncertainty in its momentum at $t = 0$ is

$$\Delta p \geq h/(4\pi\,\Delta x_0)$$

- Since $v << c$, the momentum uncertainty is

$$\Delta p = m\Delta v$$

- The uncertainty in the proton's velocity is

$$\Delta v = \Delta p/m \geq h/(4\pi\,\Delta x_0)$$

- The distance x of the proton covers in the time t cannot be known more accurately than

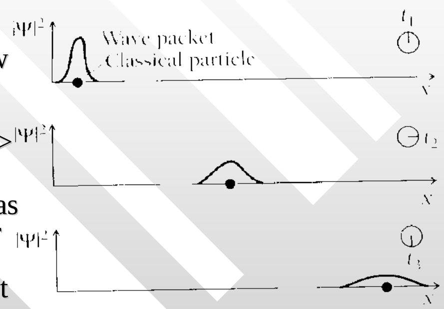
$$\Delta x = t\Delta v \geq ht/(4\pi\,\Delta x_0)$$

- The value of at $t = 1.00$ s is 3.15 km.

A moving wave packet spreads out in space

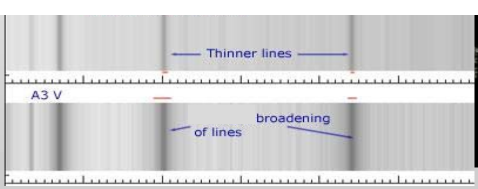
$$\Delta x = t\Delta v \geq ht/(4\pi\,\Delta x_0)$$

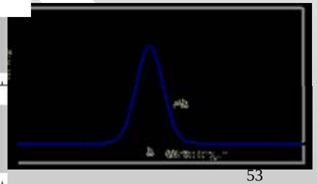
- Note that Δx is inversely proportional to Δx_0
- It means the more we know about the proton's position at $t = 0$ the less we know about its later position at $t > 0$.
- The original wave group has spread out to a much wider one because the phase velocities of the component wave vary with wave number and a large range of wave numbers must have been present to produce the narrow original wave group



Broadening of spectral lines due to uncertainty principle

- An excited atom gives up it excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time is radiates is 1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon.





Solution

- The photon energy is uncertain by the amount
- $\Delta E \geq hc/(4c\pi\Delta t) = 5.3\times10^{-27}$ J $= 3.3\times10^{-8}$ eV
- The corresponding uncertainty in the frequency of light is $\Delta\nu = \Delta E/h \geq 8\times10^{6}$ Hz
- This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom.
- As a result, the radiation from a group of excited atoms does not appear with the precise frequency ν.
- For a photon whose frequency is, say, 5.0×10^{14} Hz,
- $\Delta\nu/\nu = 1.6\times10^{-8}$

Example

Estimating quantum effect of a macroscopic particle

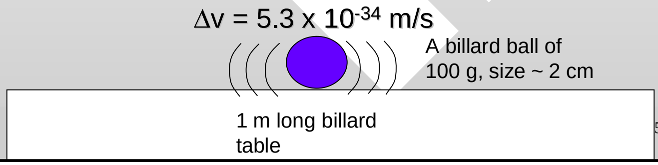
- Estimate the minimum uncertainty velocity of a billard ball (m ~ 100 g) confined to a billard table of dimension 1 m

Solution

For Δx ~ 1 m, we have

$\Delta p \geq h/4\pi\Delta x = 5.3\text{x}10^{-35}$ Ns,

- So $\Delta v = (\Delta p)/m \geq 5.3\text{x}10^{-34}$ m/s
- One can consider $\Delta v = 5.3\text{x}10^{-34}$ m/s (extremely tiny) is the speed of the billard ball at anytime caused by quantum effects
- In quantum theory, no particle is absolutely at rest due to the Uncertainty Principle

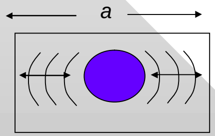


A particle contained within a finite region must has some minimal KE

- One of the most dramatic consequence of the uncertainty principle is that a particle confined in a small region of finite width cannot be exactly at rest (as already seen in the previous example)
- Why? Because…
- ...if it were, its momentum would be precisely zero, (meaning $\Delta p = 0$) which would in turn violate the uncertainty principle

What is the K_{ave} of a particle in a box due to Uncertainty Principle?

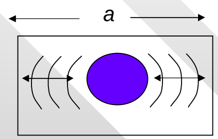
- We can estimate the minimal KE of a particle confined in a box of size *a* by making use of the U.P.
- If a particle is confined to a box, its location is uncertain by $\Delta x = a$
- Uncertainty principle requires that $\Delta p \geq (h/2\pi)a$
- (don't worry about the factor 2 in the uncertainty relation since we only perform an estimation)



Zero-point energy

$$K_{ave} = \left(\frac{p^2}{2m}\right)_{av} \gtrsim \frac{(\Delta p)^2}{2m} \gtrsim \frac{\hbar^2}{2ma^2}$$

This is the zero-point energy, the minimal possible kinetic energy for a quantum particle confined in a region of width *a*



Particle in a box of size *a* can never be at rest (e.g. has zero K.E) but has a minimal KE K_{ave} (its zero-point energy)

We will formally re-derived this result again when solving for the Schrodinger equation of this system (see later).

PYQ 2.11 Final Exam 2003/04

- Assume that the uncertainty in the position of a particle is equal to its de Broglie wavelength. What is the minimal uncertainty in its velocity, v_x?
- A. v_x/4p B. v_x/2p C. v_x/8p
- D. v_x E. v_x/p
- ANS: A, Schaum's 3000 solved problems, Q38.66, pg. 718

Recap

- Measurement necessarily involves interactions between observer and the observed system
- Matter and radiation are the entities available to us for such measurements
- The relations $p = h/\lambda$ and $E = h\nu$ are applicable to both matter and to radiation because of the intrinsic nature of wave-particle duality
- When combining these relations with the universal waves properties, we obtain the Heisenberg uncertainty relations
- In other words, the uncertainty principle is a necessary consequence of particle-wave duality


OH ALICE.. YOU'RE THE ONE FOR ME
BUT BOB.. IN A QUANTUM WORLD HOW CAN WE BE SURE?
ψ+ or ψ- ?
Physics World

Atomic Models

- Much of the luminous matter in the Universe is hydrogen. In fact hydrogen is the most abundance atom in the Universe. The colours of this Orion Nebula come from the transition between the quantized states in hydrogen atoms.

© 2004 Thomson - Brooks/Cole

INTRODUCTION

- The purpose of this chapter is to build a simplest atomic model that will help us to understand the structure of atoms
- This is attained by referring to some basic experimental facts that have been gathered since 1900's (e.g. Rutherford scattering experiment, atomic spectral lines etc.)
- In order to build a model that well describes the atoms which are consistent with the experimental facts, we need to take into account the wave nature of electron
- This is one of the purpose we explore the wave nature of particles in previous chapters

Basic properties of atoms

- 1) Atoms are of microscopic size, ~ 10^{-10} m. Visible light is not enough to resolve (see) the detail structure of an atom as its size is only of the order of 100 nm.
- 2) Atoms are stable
- 3) Atoms contain negatively charges, electrons, but are electrically neutral. An atom with Z electrons must also contain a net positive charge of $+Ze$.
- 4) Atoms emit and absorb EM radiation (in other words, atoms interact with light quite readily)

Because atoms interacts with EM radiation quite strongly, it is usually used to probe the structure of an atom. The typical of such EM probe can be found in the atomic spectrum as we will see now

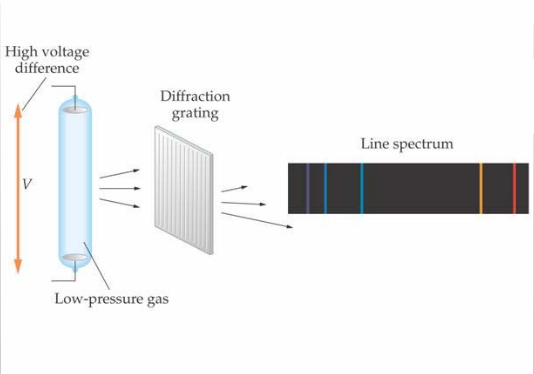
Emission spectral lines

- Experimental fact: A single atom or molecule in a very diluted sample of gas emits radiation characteristic of the particular atom/molecule species
- The emission is due to the de-excitation of the atoms from their excited states
- e.g. if heating or passing electric current through the gas sample, the atoms get excited into higher energy states
- When a excited electron in the atom falls back to the lower energy states (de-excites), EM wave is emitted

- The spectral lines are analysed with ***spectrometer***, which give important physical information of the atom/molecules by analysing the wavelengths composition and pattern of these lines.

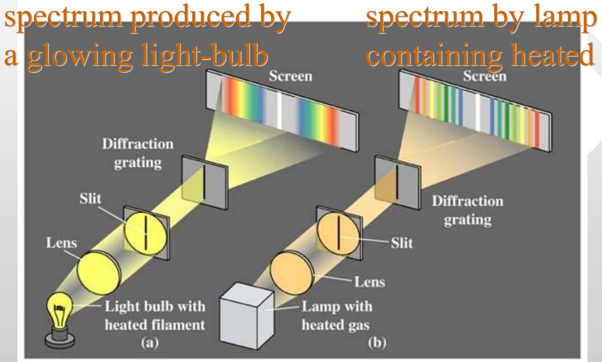
Line spectrum of an atom

- The light given off by individual atoms, as in a low-pressure gas, consist of a series of discrete wavelengths corresponding to different colour.



Comparing continuous and line spectrum

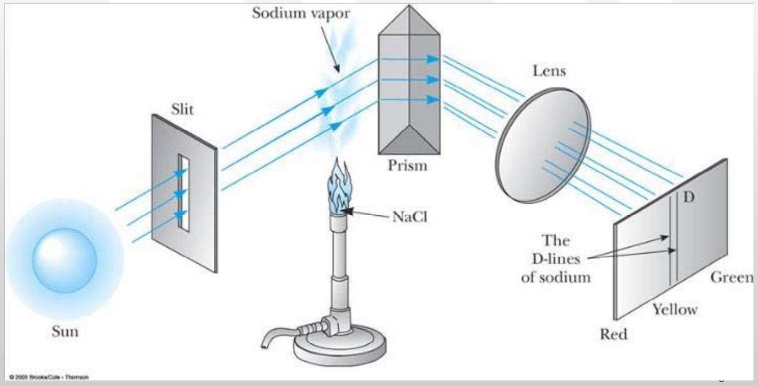
- (a) continuous spectrum produced by a glowing light-bulb
- (b) Emission line spectrum by lamp containing heated gas



Absorption line spectrum

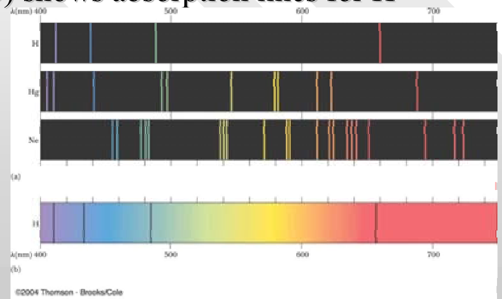
- We also have absorption spectral line, in which white light is passed through a gas. The absorption line spectrum consists of a bright background crossed by dark lines that correspond to the absorbed wavelengths by the gas atom/molecules.

Experimental arrangement for the observation of the absorptions lines of a sodium vapour



Comparing emission and absorption spectrum

The emitted and absorption radiation displays characteristic discrete sets of spectrum which contains certain discrete wavelengths only
(a) shows 'finger print' emission spectral lines of H, Hg and Ne. (b) shows absorption lines for H

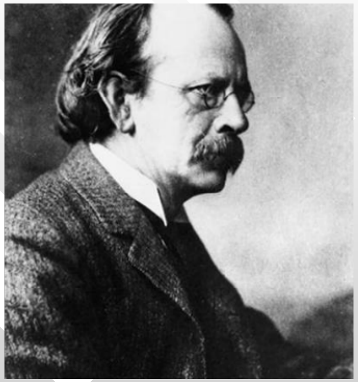


A successful atomic model must be able to explain the observed discrete atomic spectrum

We are going to study two attempts to built model that describes the atoms: the Thompson Plum-pudding model (which fails) and the Rutherford-Bohr model (which succeeds)

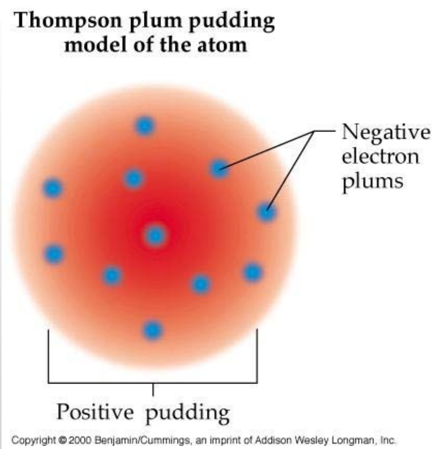
The Thompson model – Plum-pudding model

Sir J. J. Thompson (1856-1940) is the Cavandish professor in Cambridge who discovered electron in cathode rays. He was awarded Nobel prize in 1906 for his research on the conduction of electricity by bases at low pressure.
He is the first person to establish the particle nature of electron. Ironically his son, another renown physicist proves experimentally electron behaves like wave…

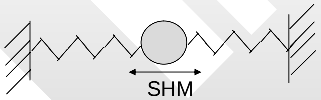


Plum-pudding model

- An atom consists of *Z* electrons is embedded in a cloud of positive charges that exactly neutralise that of the electrons'
- The positive cloud is heavy and comprising most of the atom's mass
- Inside a stable atom, the electrons sit at their respective equilibrium position where the attraction of the positive cloud on the electrons balances the electron's mutual repulsion



One can treat the electron in the pudding like a point mass stressed by two springs



13

The "electron plum" stuck on the pudding vibrates and executes SHM

- The electron at the EQ position shall vibrate like a simple harmonic oscillator with a frequency

$$\nu = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$$

- Where $k = \frac{Ze^2}{4\pi\varepsilon_o R^3}$, R radius of the atom, m mass of the electron
- From classical EM theory, we know that an oscillating charge will emit radiation with frequency identical to the oscillation frequency ν as given above

14

The plum-pudding model predicts unique oscillation frequency

- Radiation with frequency identical to the oscillation frequency.
- Hence light emitted from the atom in the plum-pudding model is predicted to have exactly one unique frequency as given in the previous slide.
- This prediction has been falsified because observationally, light spectra from all atoms (such as the simplest atom, hydrogen,) have sets of discrete spectral lines correspond to many different frequencies (already discussed earlier).

15

Experimental verdict on the plum pudding model

- Theoretically one expect the deviation angle of a scattered particle by the plum-pudding atom to be small: $\Theta = \sqrt{N}\theta_{ave} \sim 1^\circ$
- This is a prediction of the model that can be checked experimentally
- Rutherford was the first one to carry out such experiment

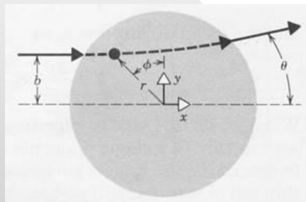


FIGURE 6.2 A positively charged alpha particle is deflected by an angle θ as it passes through a Thomson-model atom. The coordinates r and ϕ locate the alpha particle while it is inside the atom.

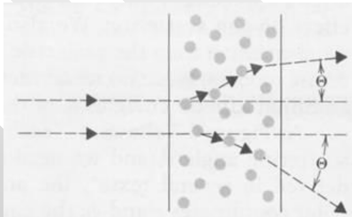
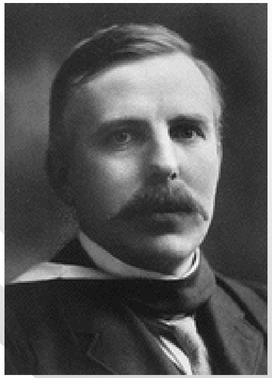


FIGURE 6.6 A microscopic representation of the scattering. Some individual scatterings tend to increase Θ, while others tend to decrease Θ.

16

Ernest Rutherford

British physicist Ernest Rutherford, winner of the 1908 Nobel Prize in chemistry, pioneered the field of nuclear physics with his research and development of the nuclear theory of atomic structure



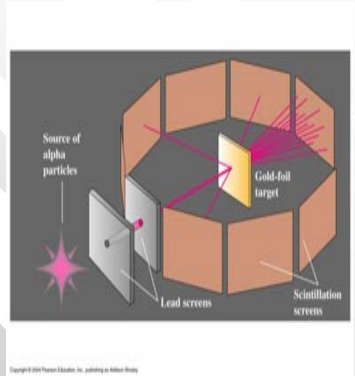
Born in New Zealand, teachers to many physicists who later become Nobel prize laureates

Rutherford stated that an atom consists largely of empty space, with an electrically positive nucleus in the center and electrically negative electrons orbiting the nucleus. By bombarding nitrogen gas with *alpha particles* (nuclear particles emitted through radioactivity), Rutherford engineered the transformation of an atom of nitrogen into both an atom of oxygen and an atom of hydrogen.

This experiment was an early stimulus to the development of nuclear energy, a form of energy in which nuclear transformation and disintegration release extraordinary power.

Rutherford experimental setup

- Alpha particles from source is used to be scattered by atoms from the thin foil made of gold
- The scattered alpha particles are detected by the background screen



"…fire a 15 inch artillery shell at a tissue paper and it came back and hit you"

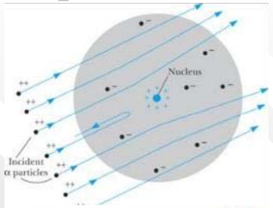
- In the scattering experiment Rutherford saw some electrons being bounced back at 180 degree.
- He said this is like firing "a 15-inch shell at a piece of a tissue paper and it came back and hit you"
- Hence Thompson plum-pudding model fails in the light of these experimental result

So, is the plum pudding model utterly useless?

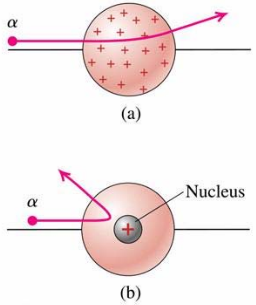
- So the plum pudding model does not work as its predictions fail to fit the experimental data as well as other observations
- Nevertheless it's a perfectly sensible scientific theory because:
- It is a mathematical model built on sound and rigorous physical arguments
- It predicts some physical phenomenon with definiteness
- It can be verified or falsified with experiments
- It also serves as a prototype to the next model which is built on the experience gained from the failure of this model

How to interpret the Rutherford scattering experiment?

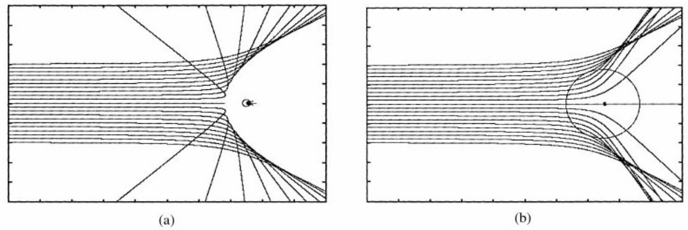
- The large deflection of alpha particle as seen in the scattering experiment with a thin gold foil must be produced by a close encounter between the alpha particle and a very small but massive kernel inside the atom



- In contrast, a diffused distribution of the positive charge as assumed in plum-pudding model cannot do the job



Comparing model with nucleus concentrated at a point-like nucleus and model with nucleus that has large size

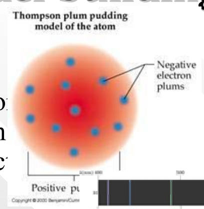


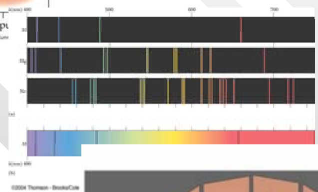
Recap
the atomic model building story

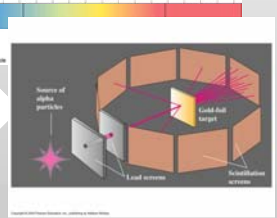
- Plum-pudding model by Thompson
- It fails to explain the emission and absorption line spectrum from atoms because it predicts only a single emission frequency

$$\nu = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$$

- Most importantly it fails to explain the back-scattering of alpha particle seen in Rutherford's scattering experiment because the model predicts only $\Theta = \sqrt{N}\theta_{ave} \sim 1^{\circ}$







The Rutherford model (planetary model)

- Rutherford put forward an model to explain the result of the scattering experiment: the Rutherford model
- An atom consists of a very small nucleus of charge $+Ze$ containing almost all of the mass of the atom; this nucleus is surrounded by a swarm of Z electrons
- The atom is largely comprised of empty space
- $R_{atom} \sim 10^{-10}$m
- $R_{nucleus} \sim 10^{-13} - 10^{-15}$ m

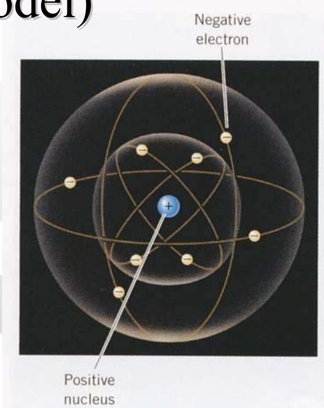
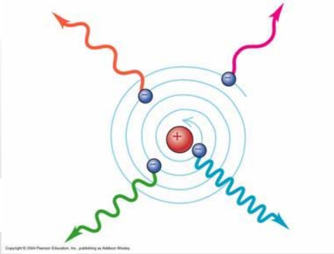
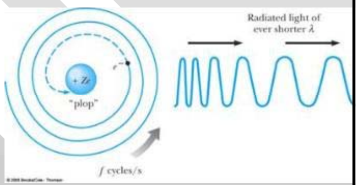


Figure 30.1 In the nuclear atom a small positively charged nucleus is surrounded at relatively large distances by a number of electrons.

Infrared catastrophe: insufficiency of the Rutherford model

- According to classical EM, the Rutherford model for atom (a classical model) has a fatal flaw: it predicts the collapse of the atom within 10^{-10} s
- A accelerated electron will radiate EM radiation, hence causing the orbiting electron to loss energy and consequently spiral inward and impact on the nucleus





Rutherford model also can't explain the discrete spectrum

- The Rutherford model also cannot explain the pattern of discrete spectral lines as the radiation predicted by Rutherford model is a continuous burst.

So how to fix up the problem?

NEILS BOHR COMES TO THE RESCUE

- **Niels Bohr** (1885 to 1962) is best known for the investigations of atomic structure and also for work on radiation, which won him the 1922 Nobel Prize for physics
- He was sometimes dubbed "the God Father" in the physicist community
- http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Bohr_Niels.html

To fix up the infrared catastrophe …

Neils Bohr put forward a model which is a hybrid of the Rutherford model with the wave nature of electron taken into account

Bohr's model of hydrogen-like atom

- We shall consider a simple atom consists of a nucleus with charge Ze and mass of $M_{nucleus} << m_e$
- The nucleus is urrounded by only a single electron
- We will assume the centre of the circular motion of the electron coincides with the centre of the nucleus
- We term such type of simple system: hydrogen-like atoms
- For example, hydrogen atom corresponds to $Z = 1$; a singly ionised Helium atom He^+ corresponds to $Z = 2$ etc

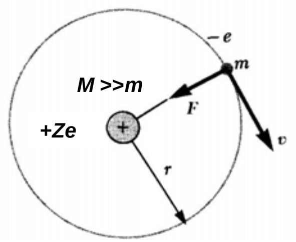
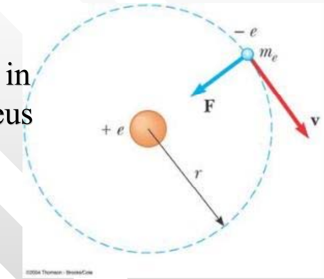


Diagram representing the model of a hydrogen-like atom

Bohr's postulate, 1913

- Postulate No.1: Mechanical stability (classical mechanics)
- An electron in an atom moves in a circular orbit about the nucleus under Coulomb attraction obeying the law of classical mechanics



Coulomb's attraction = centripetal force

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{m_e v^2}{r}$$

Assumption: the mass of the nucleus is infinitely heavy compared to the electron's

Postulate 2: condition for orbit stability

- Instead of the infinite orbit which could be possible in classical mechanics (c.f the orbits of satellites), it is only possible for an electron to move in an orbit that contains an integral number of de Broglie wavelengths,
- $n\lambda_n = 2\pi r_n$, $n = 1,2,3...$

Bohr's 2nd postulate means that *n* de Broglie wavelengths must fit into the circumference of an orbit

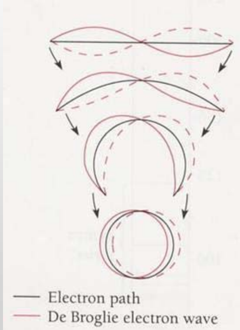
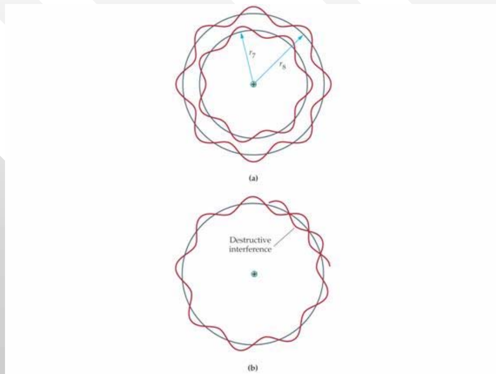


Figure 4.12 The orbit of the electron in a hydrogen atom corresponds to a complete electron de Broglie wave joined on itself.



Electron that don't form standing wave

- Since the electron must form standing waves in the orbits, the the orbits of the electron for each *n* is quantised
- Orbits with the perimeter that do not conform to the quantisation condition cannot persist
- All this simply means: all orbits of the electron in the atom must be quantised, and orbit that is not quantised is not allowed (hence can't exist)

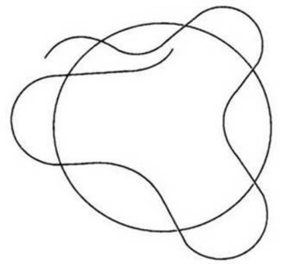
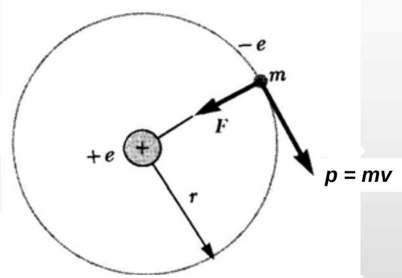


Figure 4.14 A fractional number of wavelengths cannot persist because destructive interference will occur.

Quantisation of angular momentum

- As a result of the orbit quantisation, the angular momentum of the orbiting electron is also quantised:
- $L = (m_e v)\, r = pr$ (definition)
- $n\lambda = 2\pi r$ (orbit quantisation)
- Combining both:
- $p = h/\lambda = nh/2\pi r$
- $L = m_e vr = p\, r = nh/2\pi$



Angular momentum of the electron,

L* = *p* x *r*. It is a vector quantity with its direction pointing to the direction perpendicular to the plane defined by *p* and *r

Third postulate

- Despite the fact that it is constantly accelerating, an electron moving in such an allowed orbit does not radiate EM energy (hence total energy remains constant)
- As far as the stability of atoms is concerned, classical physics is invalid here
- My Comment: At the quantum scale (inside the atoms) some of the classical EM predictions fail (e.g. an accelerating charge radiates EM wave)

Quantisation of velocity and radius

- Combining the quantisation of angular momentum and the equation of mechanical stability we arrive at the result that:
- the allowed radius and velocity at a given orbit are also quantised:

$$r_n = 4\pi\varepsilon_0 \frac{n^2\hbar^2}{m_e Z e^2} \qquad v_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{n\hbar}$$

Some mathematical steps leading to quantisation of orbits, $r_n = 4\pi\varepsilon_0 \frac{n^2\hbar^2}{m_e Z e^2}$

$$m_e v r = \frac{nh}{2\pi} \qquad \text{(Eq.1)}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{m_e v^2}{r} \Rightarrow v^2 = \frac{Ze^2}{4\pi\varepsilon_0 m_e}\frac{1}{r} \qquad \text{(Eq. 2)}$$

- (Eq.2) → (Eq.1)^2,
- $(m_e v r)^2 = (nh/2\pi)^2$
- LHS: $m_e^2 r^2 v^2 = m_e^2 r^2 (Ze^2 / 4\pi\varepsilon_0 m_e r)$

  $= m_e r Ze^2 / 4\pi\varepsilon_0 =$ RHS $= (nh/2\pi)^2$

  $r = n^2 (h/2\pi)^2\, 4\pi\varepsilon_0 / Ze^2 m_e \equiv r_n$,

  $n = 1,2,3\ldots$

Prove it yourself the quantisation of the electron velocity

$$v_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{n\hbar}$$

using Eq.(1) and Eq.(2)

Important comments

- The smallest orbit charaterised by
- $Z = 1$, $n=1$ is the ground state orbit of the hydrogen

$$r_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = 0.5\ \overset{0}{A}$$

- It's called the Bohr's radius = the typical size of an atom
- In general, the radius of an hydrogen-like ion/atom with charge Ze in the nucleus is expressed in terms of the Bohr's radius as

$$r_n = n^2 \frac{r_0}{Z}$$

- Note also that the ground state velocity of the electron in the hydrogen atom is $v_0 = 2.2\times 10^6$ m/s << c
- non-relativistic

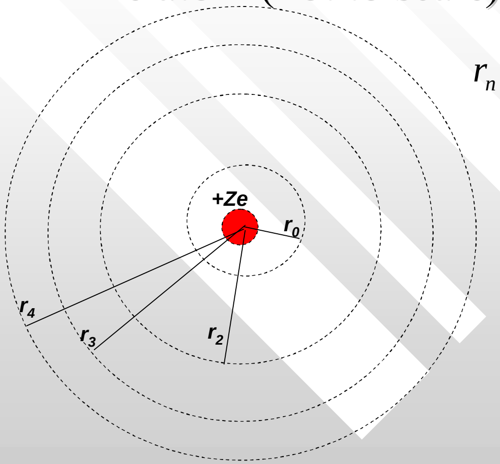
PYQ 7 Test II 2003/04

- In Bohr's model for hydrogen-like atoms, an electron (mass m) revolves in a circle around a nucleus with positive charges Ze. How is the electron's velocity related to the radius r of its orbit?
- **A.** $v = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr}$ **B.** $v = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr^2}$ **C.** $v = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{mr^2}$
- **D.** $v^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr}$ **E.** Non of the above

- Solution: I expect your to be able to derive it from scratch without memorisation
- ANS: D, Schaum's series 3000 solved problems, Q39.13, pg 722 modified

The quantised orbits of hydrogen-like atom (not to scale)

$$r_n = n^2 \frac{r_0}{Z}$$



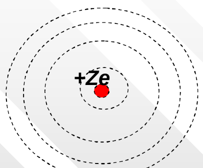
Strongly recommending the Physics 2000 interactive physics webpage by the University of Colorado

For example the page
http://www.colorado.edu/physics/2000/quantumzone/bohr.html
provides a very interesting explanation and simulation on atom and Bohr model in particular.
Please visit this page if you go online

Recap

- The hydrogen-like atom's radii are quantised according to:

$$r_n = n^2 \frac{r_0}{Z}$$



- The quantisation is a direct consequence of the postulate that electron wave forms stationary states (standing waves) at the allowed orbits
- The smallest orbit or hydrogen, the Bohr's radius

$$r_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 0.5 \overset{0}{A}$$

Postulate 4

- Similar to Einstein's postulate of the energy of a photon

EM radiation is emitted if an electron initially moving in an orbit of total energy E_i, discontinuously changes it motion so that it moves in an orbit of total energy E_f. The frequency of the emitted radiation,

$\nu = (E_f - E_i)/h$

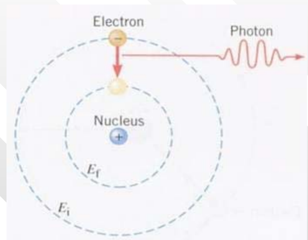


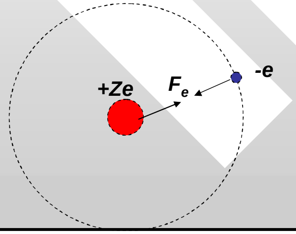
Figure 30.5 In the Bohr model, a photon is emitted when the electron drops from a larger, higher-energy orbit (energy = E_i) to a smaller, lower-energy orbit (energy = E_f).

Energies in the hydrogen-like atom

- Potential energy of the electron at a distance *r* from the nucleus is, as we learned from standard electrostatics, ZCT 102, form 6, matriculation etc. is simply

$$V = -\int_r^{\infty} \frac{Ze^2}{4\pi\varepsilon_0 r^2} dr = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

- -ve means that the EM force is attractive



Kinetic energy in the hydrogen-like atom

- According to definition, the KE of the electron is $K = \frac{m_e v^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$

The last step follows from the equation $\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

- Adding up KE + V, we obtain the total mechanical energy of the atom:

$$E = K + V = \frac{Ze^2}{8\pi\varepsilon_0 r} + \left(-\frac{Ze^2}{4\pi\varepsilon_0 r}\right) = -\frac{Ze^2}{8\pi\varepsilon_0}\left(\frac{1}{r}\right) = -\frac{Ze^2}{8\pi\varepsilon_0}\left[\frac{m_e Ze^2}{4\pi\varepsilon_0 n^2 \hbar^2}\right]$$

$$= -\frac{m_e Z^2 e^4}{(4\pi\varepsilon_0)^2 2\hbar^2} \frac{1}{n^2} \equiv E_n$$

The ground state energy

- For the hydrogen atom (*Z* = 1), the ground state energy (which is characterised by *n* =1)

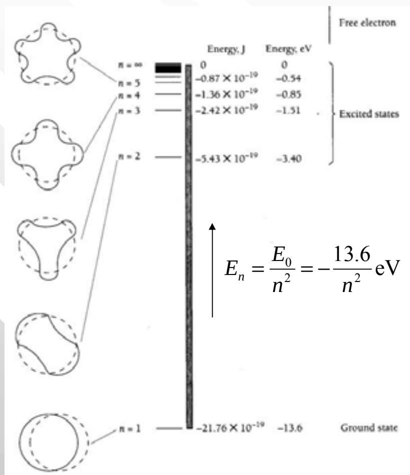
$$E_0 \equiv E_n(n=1) = -\frac{m_e e^4}{(4\pi\varepsilon_0)^2 2\hbar^2} = -13.6\text{eV}$$

In general the energy level of a hydrogen like atom with *Ze* nucleus charges can be expressed in terms of

$$E_n = \frac{Z^2 E_0}{n^2} = -\frac{13.6 Z^2}{n^2}\text{eV}$$

Quantisation of energy levels

- The energy level of the electrons in the atomic orbit is quantised
- The quantum number, *n*, that characterises the electronic states is called principle quantum number
- Note that the energy state is –ve (because it's a bounded system)

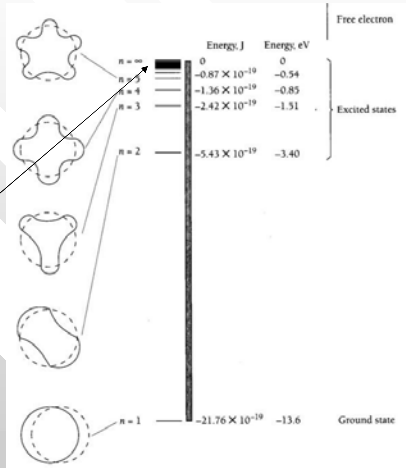


Energy of the electron at very large *n*

- An electron occupying an orbit with very large *n* is "almost free" because its energy approaches zero:

$$E_n(n \to \infty) = 0$$

- *E* = 0 means the electron is free from the bondage of the nucleus' potential field
- Electron at high *n* is not tightly bounded to the nucleus by the EM force
- Energy levels at high *n* approaches to that of a continuum, as the energy gap between adjacent energy levels become infinitesimal in the large *n* limit

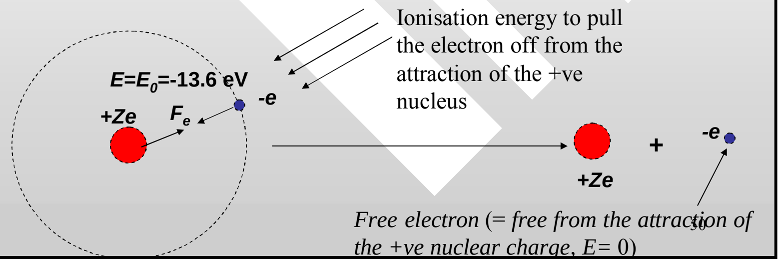


Ionisation energy of the hydrogen atom

- The energy input required to remove the electron from its ground state to infinity (ie. to totally remove the electron from the bound of the nucleus) is simply

$$E_{ionisation} = E_\infty - E_0 = -E_0 = 13.6\text{eV}$$

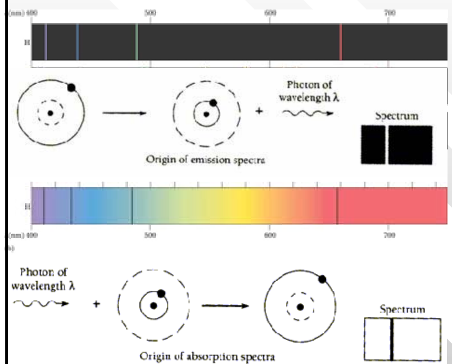
- this is the ionisation energy of hydrogen



Two important quantities to remember

- As a practical rule, it is strongly advisable to remember the two very important values
- (i) the Bohr radius, $r_0 = 0.53$A and
- (ii) the ground state energy of the hydrogen atom, $E_0 = -13.6$ eV

Bohr's 4th postulate explains the line spectrum

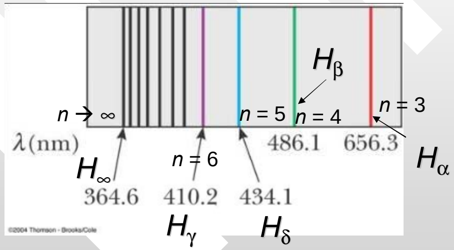


- When atoms are excited to an energy state above its ground state, they shall radiate out energy (in forms of photon) within at the time scale of ~10^{-8} s upon their de-excitations to lower energy states –emission spectrum explained
- When a beam of light with a range of wavelength from sees an atom, the few particular wavelengths that matches the allowed energy gaps of the atom will be absorbed, leaving behind other unabsorbed wavelengthsto become the bright background in the absorption spectrum. Hence absorption spectrum explained

Balmer series and the empirical emission spectrum equation

- Since 1860 – 1898 Balmer have found an empirical formula that correctly predicted the wavelength of four visible lines of hydrogen:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$



where n = 3,4,5,….R_H is called the Rydberg constant, experimentally measured to be R_H = 1.0973732 x 10^7 m^{-1}

Example

- For example, for the H_β (486.1 nm) line, $n = 4$ in the empirical formula

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

- According to the empirical formula the wavelength of the hydrogen beta line is

$$\frac{1}{\lambda_\beta} = R_H\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R_H\left(\frac{3}{16}\right) = \frac{3(1.0973732\times10^7\,\mathrm{m}^{-1})}{16}$$

$$\Rightarrow \lambda_\beta = 486\mathrm{nm}$$

- which is consistent with the observed value

Other spectra series

- Apart from the Balmer series others spectral series are also discovered: Lyman, Paschen and Brackett series
- The wavelengths of these spectral lines are also given by the similar empirical equation as

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1} - \frac{1}{n^2}\right), \quad n = 2,3,4,...$$

Lyman series, ultraviolet region

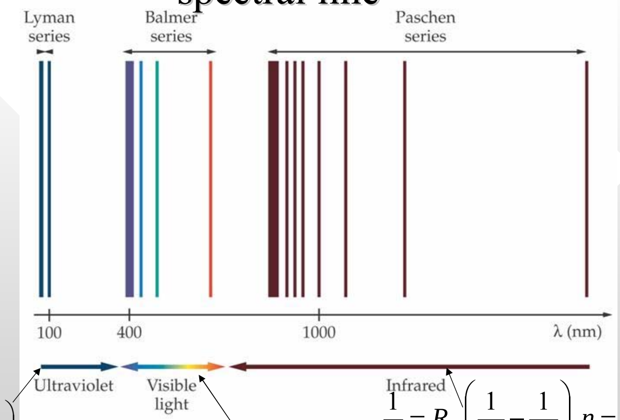
$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \quad n = 4,5,6,...$$

Paschen series, infrared region

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \quad n = 5,6,7,...$$

Brackett series, infrared region

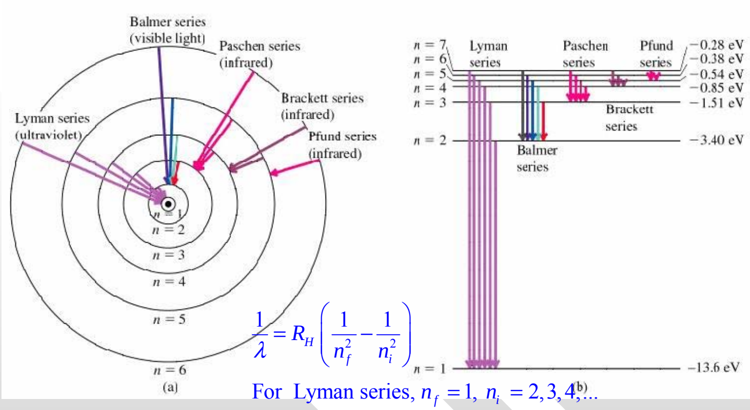
These are experimentally measured spectral line



$$\frac{1}{\lambda} = R_H\left(\frac{1}{1} - \frac{1}{n^2}\right), n = 2,3,4...$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 3,4,5,6,$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4,5,6...$$



$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For Lyman series, $n_f = 1,\ n_i = 2, 3, 4...$

For Balmer series, $n_f = 2,\ n_i = 3, 4, 5...$

For Paschen series, $n_f = 3,\ n_i = 4, 5, 6...$

For Brackett series, $n_f = 4,\ n_i = 5, 6, 7...$

For Pfund series, $n_f = 5,\ n_i = 6, 7, 8...$

The empirical formula needs a theoretical explanation

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

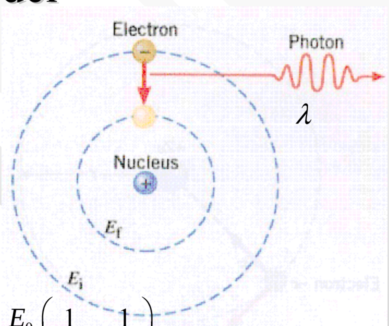
is an empirical formula with R_H measured to be R_H = 1.0973732 x 10^7 m^{-1}.

Can the Bohr model provide a sound theoretical explanation to the form of this formula and the numerical value of R_H in terms of known physical constants?

The answer is: YES

Theoretical derivation of the empirical formula from Bohr's model

- According to the 4th postulate, $\Delta E = E_i - E_f = h\nu = hc/\lambda$, and
- $E_k = E_0 / n_k^2$
- $= -13.6$ eV $/ n_k^2$
- where $k = i$ or j
- Hence we can easily obtain the theoretical expression for the emission line spectrum of hydrogen-like atom



$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{E_i - E_f}{ch} = \frac{E_0}{ch}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$= \frac{m_e e^4}{4c\pi\hbar^3(4\pi\varepsilon_0)^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \equiv R_\infty\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The theoretical Rydberg constant

$$R_\infty \equiv \frac{m_e e^4}{4c\pi\hbar^3(4\pi\varepsilon_0)^2} = 1.0984119\times10^7\,\mathrm{m}^{-1}$$

- The theoretical Rydberg constant, R_∞, agrees with the experimental one up a precision of less than 1%

$$R_H = 1.0973732\times10^7\,\mathrm{m}^{-1}$$

This is a remarkable experimental verification of the correctness of the Bohr model

Real life example of atomic emission

- AURORA are caused by streams of fast photons and electrons from the sun that excite atoms in the upper atmosphere. The green hues of an auroral display come from oxygen

Example

- Suppose that, as a result of a collision, the electron in a hydrogen atom is raised to the second excited state ($n = 3$).
- What is (i) the energy and (ii) wavelength of the photon emitted if the electron makes a direct transition to the ground state?
- What are the energies and the wavelengths of the two photons emitted if, instead, the electron makes a transition to the first excited state (n=2) and from there a subsequent transition to the ground state?

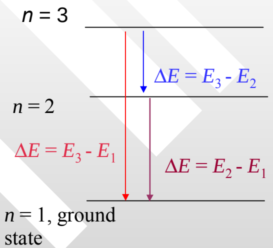
Make use of $E_k = E_0 / n_k^2 = -13.6 \text{ eV} / n_k^2$

The energy of the proton emitted in the transition from the $n = 3$ to the $n = 1$ state is

$$\Delta E = E_3 - E_1 = -13.6\left(\frac{1}{3^2} - \frac{1}{1^2}\right)\text{eV} = 12.1\text{eV}$$

the wavelength of this photon is

$$\lambda = \frac{c}{\nu} = \frac{ch}{\Delta E} = \frac{1242eV \cdot nm}{12.1eV} = 102\text{nm}$$



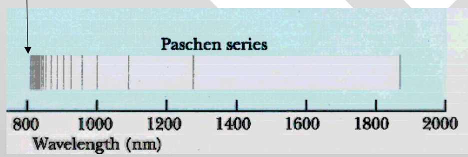
Likewise the energies of the two photons emitted in the transitions from n = 3 → n = 2 and n = 2 → n = 1 are, respectively,

$$\Delta E = E_3 - E_2 = -13.6\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.89\text{eV}$$ with wavelength $$\lambda = \frac{ch}{\Delta E} = \frac{1242eV \cdot nm}{1.89eV} = 657\text{nm}$$

$$\Delta E = E_2 - E_1 = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2\text{eV}$$ with wavelength $$\lambda = \frac{ch}{\Delta E} = \frac{1242eV \cdot nm}{10.2eV} = 121\text{nm}$$

Example

- The series limit of the Paschen ($n_f = 3$) is 820.1 nm (The series limit of a spectral series is the wavelength corresponds to $n_i \to \infty$).
- What are two longest wavelengths of the Paschen series?



Solution

- Note that the Rydberg constant is not provided
- But by definition the series limit and the Rydberg constant is closely related
- We got to make use of the series limit to solve that problem
- By referring to the definition of the series limit,

$$\frac{1}{\lambda}=R_H\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)\xrightarrow{n_i\to\infty}\frac{1}{\lambda_\infty}=\frac{R_H}{n_f^2}$$

- Hence we can substitute $R_H = n_f^2 / \lambda_\infty$ into

$$\frac{1}{\lambda}=R_H\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

- and express it in terms of the series limit as

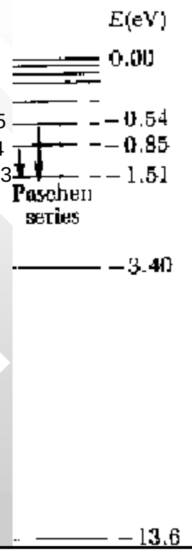
$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left(1-\frac{n_f^2}{n_i^2}\right)$$

- n = 4,5,6…

- For Paschen series, $n_f = 3, \lambda_\infty = 820.1$ nm

$$\frac{1}{\lambda}=\frac{1}{820.1\,\text{nm}}\left(1-\frac{3^2}{n_i^2}\right)$$

- The two longest wavelengths correspond to transitions of the two smallest energy gaps from the energy levels closest to n = 3 state (i.e the n = 4, n = 5 states) to the n = 3 state



$$n_i=4:\lambda=820.1\,\text{nm}\left(\frac{n_i^2}{n_i^2-9}\right)=820.1\,\text{nm}\left(\frac{4^2}{4^2-9}\right)=1875\,\text{nm}$$

$$n_i=5:\lambda=820.1\,\text{nm}\left(\frac{n_i^2}{n_i^2-9}\right)=820.1\,\text{nm}\left(\frac{5^2}{5^2-9}\right)=1281\,\text{nm}$$

Example

- Given the ground state energy of hydrogen atom -13.6 eV, what is the longest wavelength in the hydrogen's Balmer series?
- Solution:

  $\Delta E = E_i - E_f = -13.6\text{ eV }(1/n_i^2 - 1/n_f^2) = hc/\lambda$
- Balmer series: $n_f = 2$. Hence, in terms of 13.6 eV the wavelengths in Balmer series is given by

$$\lambda_{Balmer}=\frac{hc}{13.6\text{eV}\left(\frac{1}{4}-\frac{1}{n_i^2}\right)}=\frac{1240\text{eV}\cdot\text{nm}}{13.6\text{eV}\left(\frac{1}{4}-\frac{1}{n_i^2}\right)}=\frac{91\text{nm}}{\left(\frac{1}{4}-\frac{1}{n_i^2}\right)},\quad n_i=3,4,5...$$

$$\lambda_{Balmer}=\frac{91\text{nm}}{\left(\frac{1}{4}-\frac{1}{n_i^2}\right)},\quad n_i=3,4,5...$$

- longest wavelength corresponds to the transition from the n_i = 3 states to the n_f = 2 states
- Hence

$$\lambda_{Balmer,\max}=\frac{91\text{nm}}{\left(\frac{1}{4}-\frac{1}{3^2}\right)}=655.2\text{nm}$$

- This is the red H_α line in the hydrogen's Balmer series
- Can you calculate the shortest wavelength (the series limit) for the Balmer series? Ans = 364 nm

PYQ 2.18 Final Exam 2003/04

- Which of the following statements are true?
- **I.**. the ground states are states with lowest energy
- **II.** ionisation energy is the energy required to raise an electron from ground state to free state
- **III**. Balmer series is the lines in the spectrum of atomic hydrogen that corresponds to the transitions to the $n = 1$ state from higher energy states

- **A.** I,IV **B.** I,II, IV **C.** I, III,IV
- **D.** I, II **E.** II,III

- **ANS: D, My own question**
- **(note: this is an obvious typo error with the statement IV missing. In any case, only statement I, II are true.)**

PYQ 1.5 KSCP 2003/04

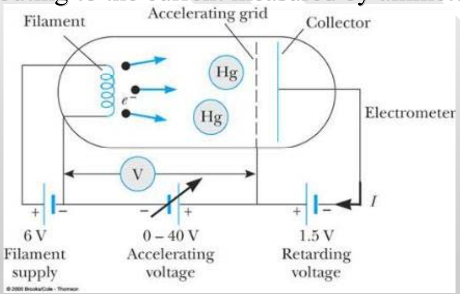
- An electron collides with a hydrogen atom in its ground state and excites it to a state of n =3. How much energy was given to the hydrogen atom in this collision?
- **A.** -12.1 eV **B.** 12.1 eV **C.** -13.6 eV
- **D.** 13.6 eV **E.** Non of the above
- **Solution:**

$$\Delta E = E_3 - E_0 = \frac{E_0}{3^2} - E_0 = \frac{(-13.6\text{eV})}{3^2} - (-13.6\text{eV}) = 12.1\text{eV}$$

- **ANS: B**, Modern Technical Physics, Beiser, Example 25.6, pg. 786

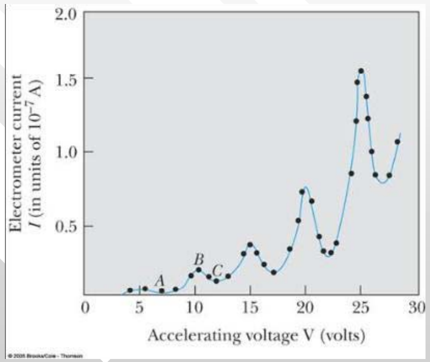
Frank-Hertz experiment

- The famous experiment that shows the excitation of atoms to discrete energy levels and is consistent with the results suggested by line spectra
- Mercury vapour is bombarded with electron accelerated under the potential V (between the grid and the filament)
- A small potential V_0 between the grid and collecting plate prevents electrons having energies less than a certain minimum from contributing to the current measured by ammeter



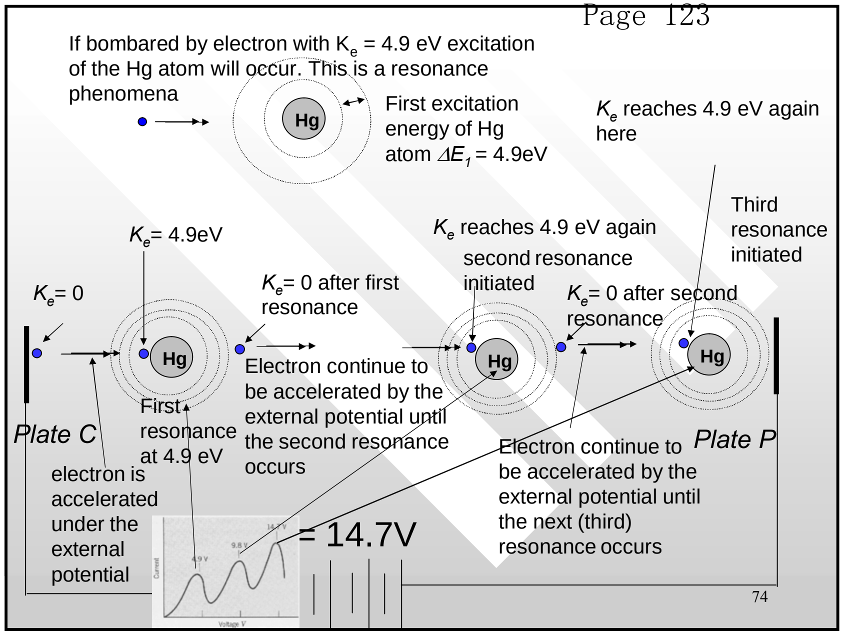
The electrons that arrive at the anode peaks at equal voltage intervals of 4.9 V

- As V increases, the current measured also increases
- The measured current drops at multiples of a critical potential
- V = 4.9 V, 9.8V, 14.7V



Interpretation

- As a result of inelastic collisions between the accelerated electrons of KE 4.9 eV with the the Hg atom, the Hg atoms are excited to an energy level above its ground state
- At this critical point, the energy of the accelerating electron equals to that of the energy gap between the ground state and the excited state
- This is a resonance phenomena, hence current increases abruptly
- After inelastically exciting the atom, the original (the bombarding) electron move off with too little energy to overcome the small retarding potential and reach the plate
- As the accelerating potential is raised further, the plate current again increases, since the electrons now have enough energy to reach the plate
- Eventually another sharp drop (at 9.8 V) in the current occurs because, again, the electron has collected just the same energy to excite the same energy level in the other atoms



- The higher critical potentials result from two or more inelastic collisions and are multiple of the lowest (4.9 V)
- The excited mercury atom will then de-excite by radiating out a photon of exactly the energy (4.9 eV) which is also detected in the Frank-Hertz experiment
- The critical potential verifies the existence of atomic levels

Bohr's correspondence principle

- The predictions of the quantum theory for the behaviour of any physical system must correspond to the prediction of classical physics in the limit in which the quantum number specifying the state of the system becomes very large:
- $\lim_{n \to \infty}$ quantum theory = classical theory
- At large n limit, the Bohr model must reduce to a "classical atom" which obeys classical theory

In other words…

- The laws of quantum physics are valid in the atomic domain; while the laws of classical physics is valid in the classical domain; where the two domains overlaps, both sets of laws must give the same result.

PYQ 20 Test II 2003/04

- Which of the following statements are correct?
- **I** Frank-Hertz experiment shows that atoms are excited to discrete energy levels
- **II** Frank-Hertz experimental result is consistent with the results suggested by the line spectra
- **III** The predictions of the quantum theory for the behaviour of any physical system must correspond to the prediction of classical physics in the limit in which the quantum number specifying the state of the system becomes very large
- **IV** The structure of atoms can be probed by using electromagnetic radiation
- **A. II,III** **B. I, II,IV** **C. II, III, IV**
- **D. I,II, III, IV** **E. Non of the above**
- **ANS:D, My own questions**

Example (Read it yourself)

- Classical EM predicts that an electron in a circular motion will radiate EM wave at the same frequency
- According to the correspondence principle, the Bohr model must also reproduce this result in the large *n* limit

More quantitatively

- In the limit, $n = 10^3 - 10^4$, the Bohr atom would have a size of 10^{-3} m
- This is a large quantum atom which is in classical domain
- The prediction for the photon emitted during transition around the $n = 10^3 - 10^4$ states should equals to that predicted by classical EM theory.

n **→ large**

v_n **(Bohr) = v (classical theory)**

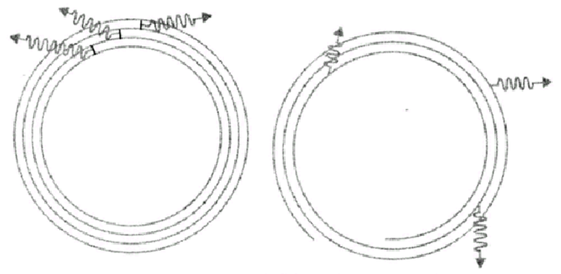


FIGURE 6.26 (Top) A large quantum atom. Photons are emitted in discrete transitions as the electron jumps to lower states. (Bottom) A classical atom. Photons are emitted continuously by the accelerated electron.

Classical physics calculation

- The period of a circulating electron is

$$T = 2\pi r/(2K/m)^{1/2}$$
$$= \pi r(2m)^{1/2}(8\pi e_0 r)^{1/2}/e$$

- This result can be easily derived from the mechanical stability of the atom as per

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{m_e v^2}{r}$$

- Substitute the quantised atomic radius $r_n = n^2 r_0$ into T, we obtain the frequency as per

$$\nu_n = 1/T = me^4/32\pi^3\varepsilon_0^2\hbar^3 n^3$$

Based on Bohr's theory

- Now, for an electron in the Bohr atom at energy level $n = 10^3$ - 10^4, the frequency of an radiated photon when electron make a transition from the n state to n–1 state is given by

$$\nu_n = (me^4/64\pi^3\varepsilon_0^2\hbar^3)[(n-1)^{-2} - n^{-2}]$$
$$= (me^4/64\pi^3\varepsilon_0^2\hbar^3)[(2n-1)/n^2(n-1)^2]$$

In the limit of large n,

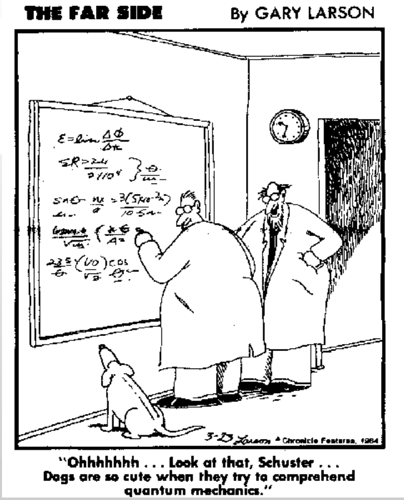
$$\nu \approx (me^4/64\pi^3\varepsilon_0^2\hbar^3)[2n/n^4]$$
$$= (me^4/32\pi^3\varepsilon_0^2\hbar^3)[1/n^3]$$

Classical result and Quantum calculation meets at $n \rightarrow \infty$

- Hence, in the region of large n, where classical and quantum physics overlap, the classical prediction and that of the quantum one is identical

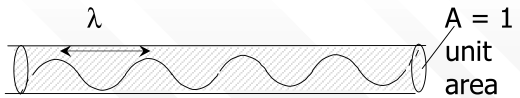
$$\nu_{classical} = \nu_{Bohr} = (me^4/32\pi^3\varepsilon_0^2\hbar^3)[1/n^3]$$

Introductory Quantum mechanics



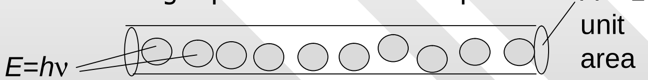
Probabilistic interpretation of matter wave

A beam of light if pictured as monochromatic wave (λ, ν)



Intensity of the light beam is $I = \varepsilon_0 c \overline{E^2}$

A beam of light pictured in terms of photons



Intensity of the light beam is $I = Nh\nu$

N = average number of photons per unit time crossing unit area perpendicular to the direction of propagation

Intensity = energy crossing one unit area per unit time. I is in unit of joule per m^2 per second

Probability of observing a photon

- Consider a beam of light
- In wave picture, $E = E_0 \sin(kx - \omega t)$, electric field in radiation
- Intensity of radiation in wave picture is

$$I = \varepsilon_0 c \overline{E^2}$$

- On the other hand, in the photon picture, $I = Nh\nu$
- Correspondence principle: what is explained in the wave picture has to be consistent with what is explained in the photon picture in the limit $N \rightarrow$ infinity:

$$I = \varepsilon_0 c \overline{E^2} = Nh\nu$$

Statistical interpretation of radiation

- The probability of observing a photon at a point in unit time is proportional to N
- However, since $Nh\nu = \varepsilon_0 c\overline{E^2} \propto \overline{E^2}$
- the probability of observing a photon must also
- This means that the probability of observing a photon at any point in space is proportional to the square of the averaged electric field strength at that point

$$Prob\ (x) \propto \overline{E^2}$$

Square of the mean of the square of the **wave field amplitude**

What is the physical interpretation of matter wave?

- we will call the mathematical representation of the de Broglie's wave / matter wave associated with a given particle (or an physical entity) as

The wave function, Ψ

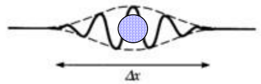


FIGURE 6.14 An idealized wave packet localized in space over a region Δx is the perposition of many waves of different amplitudes and frequencies.

- We wish to answer the following questions:
- Where is exactly the particle located within Δx? the locality of a particle becomes fuzzy when it's represented by its matter wave. We can no more tell for sure where it is exactly located.
- Recall that in the case of conventional wave physics, |field amplitude|^2 is proportional to the intensity of the wave). Now, what does $|\Psi|^2$ physically mean?

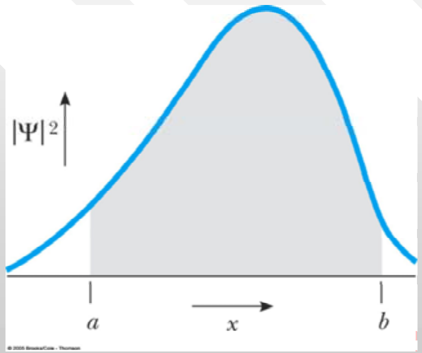
Probabilistic interpretation of (the square of) matter wave

- As seen in the case of radiation field, |electric field's amplitude|^2 is proportional to the probability of finding a photon
- In exact analogy to the statistical interpretation of the radiation field,
- $P(x) = |\Psi|^2$ is interpreted as the probability density of observing a material particle
- More quantitatively,
- Probability for a particle to be found between point a and b is

$$p(a \le x \le b) = \int_a^b P(x)dx = \int_a^b |\Psi(x,t)|^2\ dx$$

$p_{ab} = \int_a^b |\Psi(x,t)|^2\ dx$ is the probability to find the particle between a and b

- It value is given by the area under the curve of probability density between a and b



Expectation value

- Any physical observable in quantum mechanics, *O* (which is a function of position, *x*), when measured repeatedly, will yield an expectation value of given by

$$\langle O \rangle = \frac{\int_{-\infty}^{\infty} \Psi O \Psi^* dx}{\int_{-\infty}^{\infty} \Psi \Psi^* dx} = \frac{\int_{-\infty}^{\infty} O |\Psi|^2 dx}{\int_{-\infty}^{\infty} \Psi \Psi^* dx}$$

- Example, *O* can be the potential energy, position, etc.
- (Note: the above statement is not applicable to energy and linear momentum because they cannot be express explicitly as a function of *x* due to uncertainty principle)…

Example of expectation value: average position measured for a quantum particle

- If the position of a quantum particle is measured repeatedly with the same initial conditions, the averaged value of the position measured is given by

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x |\Psi|^2 dx}{1} = \int_{-\infty}^{\infty} x |\Psi|^2 dx$$

Example

- A particle limited to the *x* axis has the wave function Ψ = *ax* between *x*=0 and *x*=1; Ψ = 0 else where.
- (a) Find the probability that the particle can be found between *x*=0.45 and *x*=0.55.
- (b) Find the expectation value <*x*> of the particle's position

Solution

- (a) the probability is

$$\int_{-\infty}^{\infty} |\Psi|^2 dx = \int_{0.45}^{0.55} x^2 dx = a^2 \left[\frac{x^3}{3} \right]_{0.45}^{0.55} = 0.0251 a^2$$

- (b) The expectation value is

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx = \int_{0}^{1} x^3 dx = a^2 \left[\frac{x^3}{4} \right]_{0}^{1} = \frac{a^2}{4}$$

Max Born and probabilistic interpretation

- Hence, a particle's wave function gives rise to a *probabilistic interpretation* of the position of a particle
- Max Born in 1926

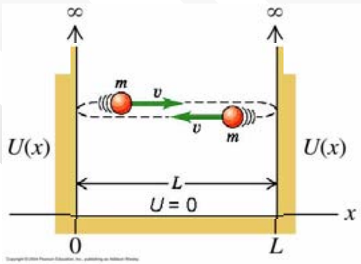
German-British physicist who worked on the mathematical basis for quantum mechanics. Born's most important contribution was his suggestion that the absolute square of the wavefunction in the Schrödinger equation was a measure of the probability of finding the particle at a given location. Born shared the 1954 Nobel Prize in physics with Bothe

PYQ 2.7, Final Exam 2003/04

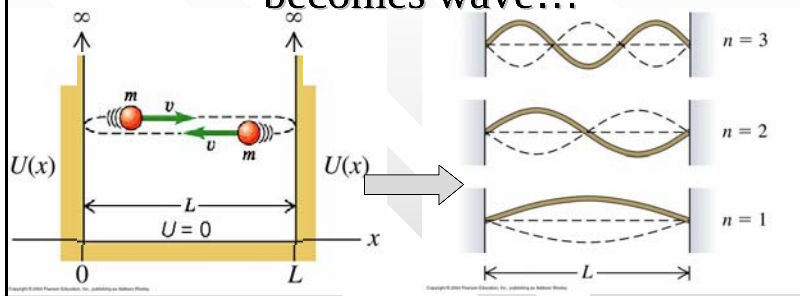
- A large value of the probability density of an atomic electron at a certain place and time signifies that the electron
- **A.** is likely to be found there
- **B.** is certain to be found there
- **C.** has a great deal of energy there
- **D.** has a great deal of charge
- **E.** is unlikely to be found there
- ANS:A, Modern physical technique, Beiser, MCP 25, pg. 802

Particle in in an infinite well (sometimes called particle in a box)

- Imagine that we put particle (e.g. an electron) into an "infinite well" with width L (e.g. a potential trap with sufficiently high barrier)
- In other words, the particle is confined within $0 < x < L$
- In Newtonian view the particle is traveling along a straight line bouncing between two rigid walls



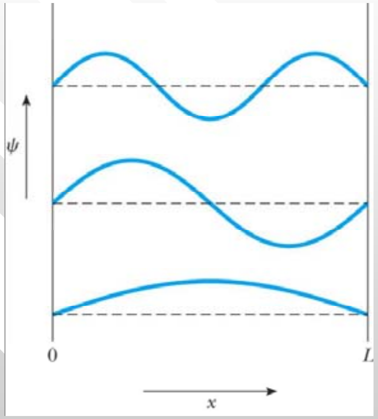
However, in quantum view, particle becomes wave…



- The 'particle' is no more pictured as a particle bouncing between the walls but a de Broglie wave that is trapped inside the infinite quantum well, in which they form standing waves

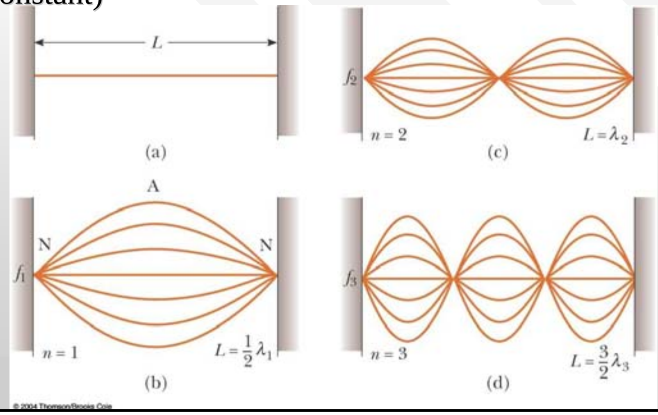
Particle forms standing wave within the infinite well

- How would the wave function of the particle behave inside the well?
- They form standing waves which are confined within $0 \leqslant x \leqslant L$



Standing wave in general

- Shown below are standing waves which ends are fixed at $x = 0$ and $x = L$
- For standing wave, the speed is constant), $v = \lambda f =$ constant)



Mathematical description of standing waves

- In general, the equation that describes a standing wave (with a constant width L) is simply:

$$L = n\lambda_n/2$$

  $n = 1, 2, 3, \ldots$ (positive, discrete integer)
- n characterises the "mode" of the standing wave
- $n = 1$ mode is called the 'fundamental' or the first harmonic
- $n = 2$ is called the second harmonics, etc.

  λ_n are the wavelengths associated with the n-th mode standing waves
- The lengths of λ_n is "quantised" as it can take only discrete values according to $\lambda_n = 2L/n$

Energy of the particle in the box

- Recall that

$$V(x) = \begin{cases} \infty, & x \le 0, x \ge L \\ 0, & 0 < x < L \end{cases}$$

- For such a free particle that forms standing waves in the box, it has no potential energy
- It has all of its mechanical energy in the form of kinetic energy only
- Hence, for the region $0 < x < L$, we write the total energy of the particle as

$$E = K + V = p^2/2m + 0 = p^2/2m$$

Energies of the particle are quantised

- Due to the quantisation of the standing wave (which comes in the form of $\lambda_n = 2L/n$),
- the momentum of the particle must also be quantised due to de Broglie's postulate:

$$p \to p_n = \frac{h}{\lambda_n} = \frac{nh}{2L}$$

It follows that the total energy of the particle is also quantised: $E \to E_n = \frac{p_n^2}{2m} = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$

$$E_n = \frac{p_n^2}{2m} = n^2 \frac{h^2}{8mL^2} = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$$

The n = 1 state is a characteristic state called the ground state = the state with lowest possible energy (also called zero-point energy)

$$E_n(n=1) \equiv E_0 = \frac{\pi^2 \hbar^2}{2mL^2}$$

Ground state is usually used as the reference state when we refer to ``excited states'' (n = 2, 3 or higher)

The total energy of the n-th state can be expressed in term of the ground state energy as

$$E_n = n^2 E_0 \quad \text{(n = 1,2,3,4…)}$$

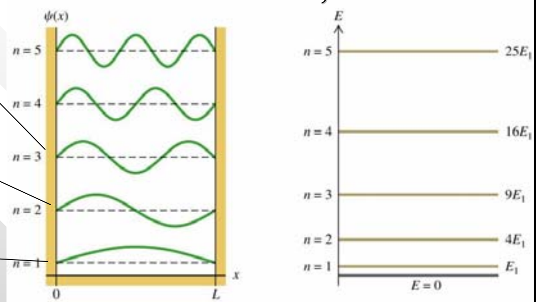
The higher n the larger is the energy level

- Some terminology
- n = 1 corresponds to the ground state
- n = 2 corresponds to the first excited state, etc

n = 3 is the second excited state, 4 nodes, 3 antinodes

n = 2 is the first excited state, 3 nodes, 2 antinodes

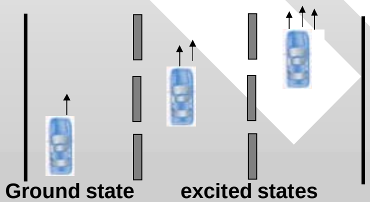
n = 1 is the ground state (fundamental mode): 2 nodes, 1 antinode



- Note that lowest possible energy for a particle in the box is not zero but E_0 (= E_1), the zero-point energy.
- This a result consistent with the Heisenberg uncertainty principle

Simple analogy

- Cars moving in the right lane on the highway are in 'excited states' as they must travel faster (at least according to the traffic rules). Cars travelling in the left lane are in the ``ground state'' as they can move with a relaxingly lower speed. Cars in the excited states must finally resume to the ground state (i.e. back to the left lane) when they slow down



Example on energy levels

- Consider an electron confined by electrical force to an infinitely deep potential well whose length L is 100 pm, which is roughly one atomic diameter. What are the energies of its three lowest allowed states and of the state with n = 15?
- **SOLUTION**
- For n = 1, the ground state, we have

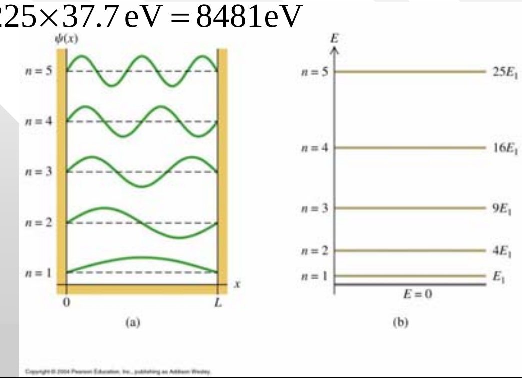
$$E_1 = (1)^2 \frac{h^2}{8m_e L^2} = \frac{(6.63\times10^{-34}\,\mathrm{Js})^2}{(9.1\times10^{-31}\,\mathrm{kg})(100\times10^{-12}\,\mathrm{m})^2} = 6.3\times10^{-18}\,\mathrm{J} = 37.7\mathrm{eV}$$

- The energy of the remaining states (n=2,3,15) are

$$E_2 = (2)^2 E_1 = 4\times37.7\,\mathrm{eV} = 150\,\mathrm{eV}$$

$$E_3 = (3)^2 E_1 = 9\times37.7\,\mathrm{eV} = 339\,\mathrm{eV}$$

$$E_{15} = (15)^2 E_1 = 225\times37.7\,\mathrm{eV} = 8481\mathrm{eV}$$



Question continued

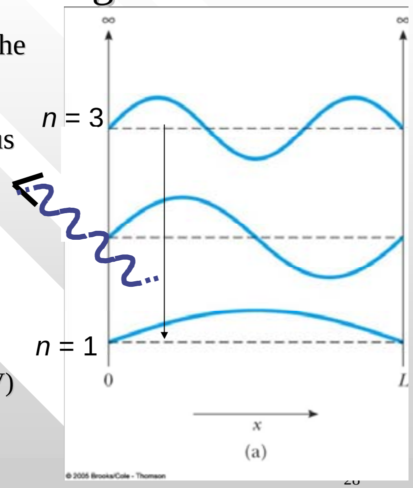
- When electron makes a transition from the n = 3 excited state back to the ground state, does the energy of the system increase or decrease?
- **Solution:**
- The energy of the system decreases as energy drops from 299 eV to 150 eV
- The lost amount $|\Delta E| = E_3 - E_1$ = 299 eV – 150 eV is radiated away in the form of electromagnetic wave with wavelength λ obeying $\Delta E = hc/\lambda$

Photon with

λ = 8.3 nm

Example
Radiation emitted during de-excitation

- Calculate the wavelength of the electromagnetic radiation emitted when the excited system at n = 3 in the previous example de-excites to its ground state



- **Solution**

  $\lambda = hc/|\Delta E|$
- = 1240 nm. eV / ($|E_3 - E_1|$)
- = 1240 nm. eV/(299 eV–150 eV) = 8.3 nm

Example
macroscopic particle's quantum state

- Consider a 1 microgram speck of dust moving back and forth between two rigid walls separated by 0.1 mm. It moves so slowly that it takes 100 s for the particle to cross this gap. What quantum number describes this motion?

Solution

- The energy of the particle is

$$E(=K)=\frac{1}{2}mv^2=\frac{1}{2}\left(1\times10^{-9}\,\text{kg}\right)\times\left(1\times10^{-6}\,\text{m/s}\right)^2=5\times10^{-22}\,\text{J}$$

- Solving for *n* in $E_n = n^2\frac{\pi^2\hbar^2}{2mL^2}$
- yields $n=\frac{L}{h}\sqrt{8mE}\approx3\times10^{14}$
- This is a very large number
- It is experimentally impossible to distinguish between the n = 3 x 10^{14} and n = 1 + (3 x 10^{14}) states, so that the quantized nature of this motion would never reveal itself

PYQ 4(a) Final Exam 2003/04

- An electron is contained in a one-dimensional box of width 0.100 nm. Using the particle-in-a-box model,
- (i) Calculate the *n* = 1 energy level and *n* = 4 energy level for the electron in eV.
- (ii) Find the wavelength of the photon (in nm) in making transitions that will eventually get it from the the *n* = 4 to *n* = 1 state
- Serway solution manual 2, Q33, pg. 380, modified

Solution

- 4a(i) In the particle-in-a-box model, standing wave is formed in the box of dimension *L*: $\lambda_n=\frac{2L}{n}$
- The energy of the particle in the box is given by

$$K_n=E_n=\frac{p_n^2}{2m_e}=\frac{\left(h/\lambda_n\right)^2}{2m_e}=\frac{n^2h^2}{8m_eL^2}=\frac{n^2\pi^2\hbar^2}{2m_eL^2}$$

$$E_1=\frac{\pi^2\hbar^2}{2m_eL^2}=37.7\text{ eV}\qquad E_4=4^2E_1=603\text{ eV}$$

- 4a(ii)
- The wavelength of the photon going from *n* = 4 to *n* = 1 is $\lambda = hc/(E_6 - E_1)$
- = 1240 eV nm/ (603 – 37.7) eV = **2.2 nm**

- The quantum states of a macroscopic particle cannot be experimentally discerned (as seen in previous example)
- Effectively its quantum states appear as a continuum

allowed energies in classical system – appear continuous (such as energy carried by a wave; total mechanical energy of an orbiting planet, etc.)

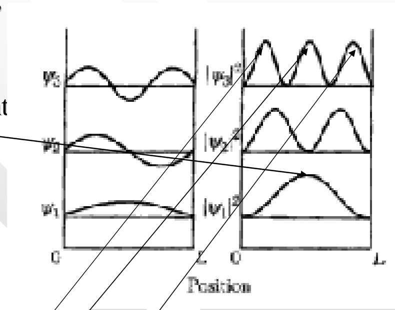
descret energies in quantised system – discrete (such as energy levels in an atom, energies carried by a photon)

Example on the probabilistic interpretation: Where in the well the particle spend most of its time?

- The particle spend most of its time in places where its probability to be found is largest
- Find, for the $n = 1$ and for n =3 quantum states respectively, the points where the electron is most likely to be found

Solution

- For electron in the n = 1 state, the probability to find the particle is highest at $x = L/2$
- Hence electron in the n =1 state spend most of its time there compared to other places



- For electron in the n = 3 state, the probability to find the particle is highest at $x = L/6, L/2, 5L/6$
- Hence electron in the n =3 state spend most of its time at this three places

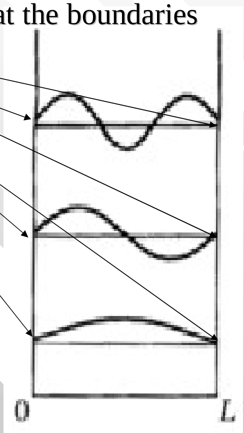
Boundary conditions and normalisation of the wave function in the infinite well

- Due to the probabilistic interpretation of the wave function, the probability density $P(x) = |\Psi|^2$ must be such that
- $P(x) = |\Psi|^2 > 0$ for $0 < x < L$
- The particle has no where to be found at the boundary as well as outside the well, i.e $P(x) = |\Psi|^2 = 0$ for $x \leqslant 0$ and $x \geqslant L$

- The probability density is zero at the boundaries

- Inside the well, the particle is bouncing back and forth between the walls
- It is obvious that it must exist within somewhere within the well
- This means:

$$\int_{-\infty}^{\infty} P(x)dx = \int_{0}^{L} |\Psi|^2 \, dx = 1$$



$$\int_{-\infty}^{\infty} P(x)dx = \int_{0}^{L} |\Psi|^2 \, dx = 1$$

- is called the normalisation condition of the wave function
- It represents the physical fact that the particle is contained inside the well and the integrated possibility to find it inside the well must be 1
- The normalisation condition will be used to determine the normalisaton constant when we solve for the wave function in the Schrodinder equation

Schrodinger Equation

Schrödinger, Erwin (1887-1961), Austrian physicist and Nobel laureate. Schrödinger formulated the theory of wave mechanics, which describes the behavior of the tiny particles that make up matter in terms of waves. Schrödinger formulated the Schrödinger wave equation to describe the behavior of electrons (tiny, negatively charged particles) in atoms. For this achievement, he was awarded the 1933 Nobel Prize in physics with British physicist Paul Dirac

What is the general equation that governs the evolution and behaviour of the wave function?

- Consider a particle subjected to some time-independent but space-dependent potential *V*(x) within some boundaries

- The behaviour of a particle subjected to a time-independent potential is governed by the famous (1-D, time independent, non relativistic) Schrodinger equation:

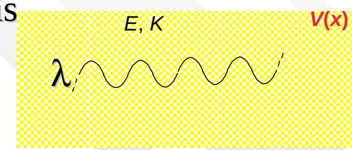
$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + (E - V)\psi(x) = 0$$

How to derive the T.I.S.E

- 1) Energy must be conserved: $E = K + U$
- 2) Must be consistent with de Brolie hypothesis that
  $p = h/\lambda$
- 3) Mathematically well-behaved and sensible (e.g. finite, single valued, linear so that superposition prevails, conserved in probability etc.)
- Read the msword notes or text books for more technical details (which we will skip here)

Energy of the particle

- The kinetic energy of a particle subjected to potential $V(x)$ is



- E is conserved if there is no net change in the total mechanical energy between the particle and the surrounding
  (Recall that this is just the definition of total mechanical energy)
- It is essential to relate the de Broglie wavelength to the energies of the particle:

$$\lambda = h / p = h / \sqrt{[2m(E-V)]}$$

- Note that, as $V \to 0$, the above equation reduces to the no-potential case (as we have discussed earlier)

$\lambda = h / p \to h / \sqrt{[2mE]}$, where $E = K$ only

Infinite potential revisited

- Armed with the T.I.S.E we now revisit the particle in the infinite well
- By using appropriate boundary condition to the T.I.S.E, the solution of T.I.S.E for the wave function Ψ should reproduces the quantisation of energy level as have been deduced earlier, i.e.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

In the next slide we will need to do some mathematics to solve for $\Psi(x)$ in the second order differential equation of TISE to recover this result. This is a more formal way compared to the previous standing waves argument which is more qualitative

Why do we need to solve the Shrodinger equation?

- The potential $V(x)$ represents the environmental influence on the particle
- Knowledge of the solution to the T.I.S.E, i.e. $\psi(x)$ allows us to obtain essential physical information of the particle (which is subjected to the influence of the external potential $V(x)$), e.g the probability of its existence in certain space interval, its momentum, energies etc.

Take a classical example: A particle that are subjected to a gravity field $U(x) = GMm/r^2$ is governed by the Newton equations of motion,

$$-\frac{GMm}{r^2} = m\frac{d^2r}{dt^2}$$

- Solution of this equation of motion allows us to predict, e.g. the position of the object m as a function of time, $r=r(t)$, its instantaneous momentum, energies, etc.

The infinite well in the light of TISE

$$V(x)=\begin{cases}\infty, & x\le 0, x\ge L\\ 0, & 0<x<L\end{cases}$$

Plug the potential function V(x) into the T.I.S.E

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+(E-V)\psi(x)=0$$

Within 0 < x < *L*, *V* (*x*) = 0, hence the TISE becomes

$$\frac{\partial^2\psi(x)}{\partial x^2}=-\frac{2m}{\hbar^2}E\psi(x)\equiv -B^2\psi(x)$$

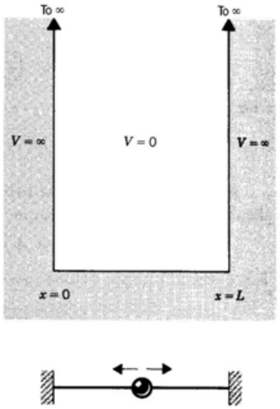


FIGURE 5.3 A particle moves freely in the one-dimensional region $0 \le x \le L$, but is excluded completely from $x < 0$ and $x > L$.

The behavior of the particle inside the box is governed by the equation

$$\frac{\partial^2\psi(x)}{\partial x^2}=-B^2\psi(x)$$

$$B^2=\frac{2mE}{\hbar^2}$$

This term contain the information of the energies of the particle, which in terns governs the behaviour (manifested in terms of its mathematical solution) of ψ (x) inside the well. Note that in a fixed quantum state *n*, *B* is a constant because *E* is conserved.

However, if the particle jumps to a state $n' \neq n$, *E* takes on other values. In this case, *E* is not conserved because there is an net change in the total energy of the system due to interactions with external environment (e.g. the particle is excited by external photon)

If you still recall the elementary mathematics of second order differential equations, you will recognise that the solution to the above TISE is simply

$$\psi(x)=A\sin Bx+C\cos Bx$$

Where *A*, *C* are constants to be determined by ultilising the boundary conditions pertaining to the infinite well system

You can prove that indeed

$$\psi(x)=A\sin Bx+C\cos Bx \quad \textbf{(EQ 1)}$$

is the solution to the TISE $\frac{\partial^2\psi(x)}{\partial x^2}=-B^2\psi(x)$ **(EQ 2)**

- I will show the steps in the following:
- Mathematically, to show that EQ 1 is a solution to EQ 2, we just need to show that when EQ1 is plugged into the LHS of EQ. 2, the resultant expression is the same as the expression to the RHS of EQ. 2.

Plug

$\psi(x)=A\sin Bx+C\cos Bx$ into the LHS of EQ 2:

$$\frac{\partial^2\psi(x)}{\partial x^2}=\frac{\partial^2}{\partial x^2}\left[A\sin Bx+C\cos Bx\right]$$

$$=\frac{\partial}{\partial x}\left[BA\cos Bx-BC\sin Bx\right]$$

$$=-B^2A\sin Bx-B^2C\cos Bx$$

$$=-B^2\left[A\sin Bx+C\cos Bx\right]$$

$$=-B^2\psi(x)=\text{RHS of EQ2}$$

Proven that EQ1 is indeed the solution to EQ2

Boundary conditions

- Next, we would like to solve for the constants *A*, *C* in the solution $\psi(x)$, as well as the constraint that is imposed on the constant *B*

- We know that the wave function forms nodes at the boundaries. Translate this boundary conditions into mathematical terms, this simply means

$$\psi(x = 0) = \psi(x = L) = 0$$

- First,
- Plug $\psi(x = 0) = 0$ into
  $\psi = A\sin Bx + C\cos Bx$, we obtain
  $\psi(x=0)) = 0 = A\sin 0 + C\cos 0 = C$
- i.e, $C = 0$
- Hence the solution is reduced to
  $\psi(x) = A\sin Bx$

- Next we apply the second boundary condition

  $\psi(x = L) = 0 = A\sin(BL)$

- Only either *A* or sin(*BL*) must be zero but not both

- *A* cannot be zero else this would mean $\psi(x)$ is zero everywhere inside the box, conflicting the fact that the particle must exist inside the box
- The upshot is: *A* cannot be zero

- This means it must be $\sin BL = 0$, or in other words
- $B = n\pi / L \equiv B_n$, $n = 1,2,3,\ldots$

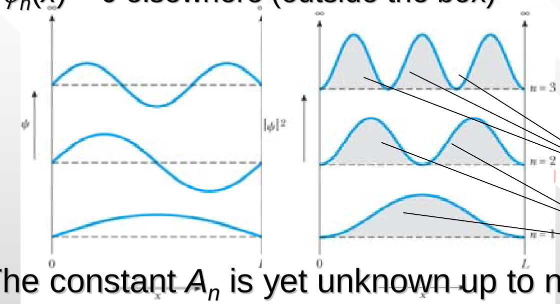
- *n* is used to characterise the quantum states of $\psi_n(x)$

- B is characterised by the positive integer *n*, hence we use B_n instead of B

- The relationship $B_n = n\pi/L$ translates into the familiar quantisation of energy condition:

- $(B_n = n\pi/L)^2$ → $B_n^2 = \frac{2mE_n}{\hbar^2} = \frac{n^2\pi^2}{L^2} \Rightarrow E_n = n^2\frac{\pi^2\hbar^2}{2mL^2}$

- Hence, up to this stage, the solution is
- $\psi_n(x) = A_n \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$ for $0 < x < L$
- $\psi_n(x) = 0$ elsewhere (outside the box)



- The constant A_n is yet unknown up to now
- We can solve for A_n by applying another "boundary condition" – the normalisation condition that:

$$\int_{-\infty}^{\infty} \psi_n^2(x)dx = \int_0^L \psi_n^2(x)dx = 1$$

Solve for A_n with normalisation

$$\int_{-\infty}^{\infty} \psi_n^2(x)dx = \int_0^L \psi_n^2(x)dx = A_n^2 \int_0^L \sin^2(\frac{n\pi x}{L})dx = \frac{A_n^2 L}{2} = 1$$

- thus

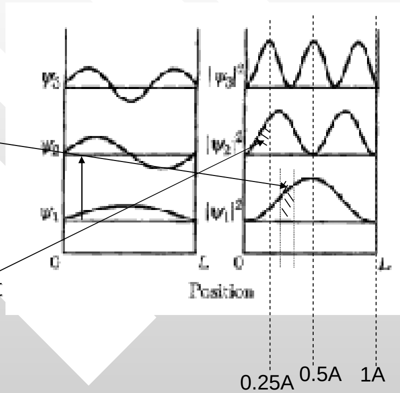
$$A_n = \sqrt{\frac{2}{L}}$$

- We hence arrive at the final solution that

- $\psi_n(x) = (2/L)^{1/2} \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$ for $0 < x < L$
- $\psi_n(x) = 0$ elsewhere (i.e. outside the box)

Example

- An electron is trapped in a one-dimensional region of length $L = 1.0 \times 10^{-10}$ m.
- (a) How much energy must be supplied to excite the electron from the ground state to the first state?
- (b) In the ground state, what is the probability of finding the electron in the region from $x = 0.090 \times 10^{-10}$ m to 0.110×10^{-10} m?
- (c) In the first excited state, what is the probability of finding the electron between $x = 0$ and $x = 0.250 \times 10^{-10}$ m?



Solutions

(a) $E_1 \equiv E_0 = \frac{\hbar^2\pi^2}{2mL^2} = 37\text{eV}$ $\quad E_2 = n^2 E_0 = (2)^2 E_0 = 148\text{eV}$

$$\Rightarrow \Delta E = |E_2 - E_0| = 111\text{eV}$$

(b) $P_{n=1}(x_1 \le x \le x_2) = \int_{x_1}^{x_2} \psi_0^2 dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2\frac{\pi x}{L} dx$

For ground state

$$= \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_{x_1 = 0.09\mathring{A}}^{x_2 = 0.11\mathring{A}} = 0.0038$$

On average the particle in the ground state spend only 0.04 % of its time in the region between x=0.11A and x=0.09 A

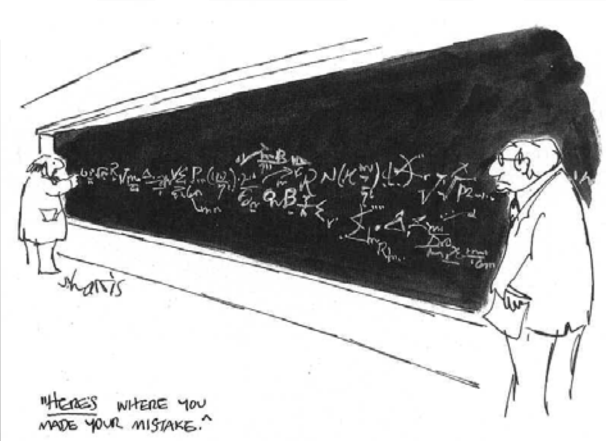
(c) **For n = 2,** $\psi_2 = \sqrt{\frac{2}{L}} \sin\frac{2\pi x}{L}$;

$$P_{n=2}(x_1 \le x \le x_2) = \int_{x_1}^{x_2} \psi_2^2 dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2\frac{2\pi x}{L} dx$$

$$= \left(\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\Bigg|_{x_1 = 0}^{x_2 = 0.25\mathring{A}} = 0.25$$

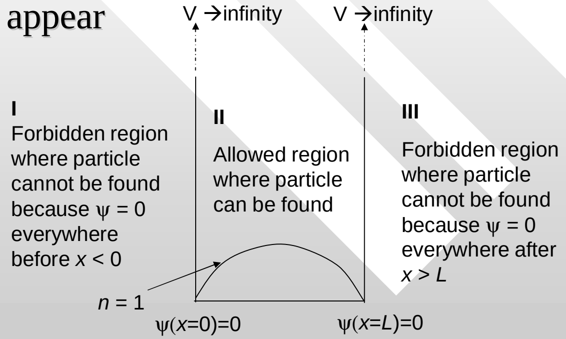
On average the particle in the n = 2 state spend 25% of its time in the region between x=0 and x=0.25 A

A nightmare for a difficult calculation



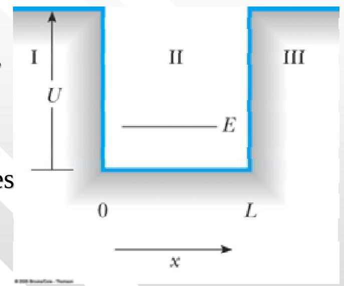
Quantum tunneling

- In the infinite quantum well, there are regions where the particle is "forbidden" to appear



Finite quantum well

- The fact that y is 0 everywhere x $\leqslant 0$, x $\geqslant L$ is because of the infiniteness of the potential, $V \to \infty$
- If V has only finite height, the solution to the TISE will be modified such that a non-zero value of y can exist beyond the boundaries at $x = 0$ and $x = L$
- In this case, the pertaining boundaries conditions are

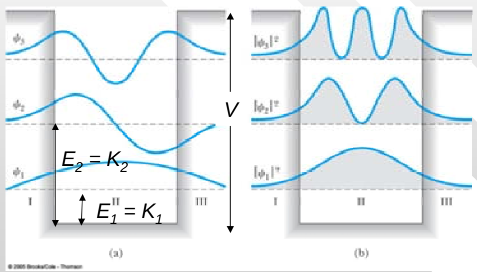


$$\psi_I(x=0) = \psi_{II}(x=0), \psi_{II}(x=L) = \psi_{III}(x=L)$$

$$\left.\frac{d\psi_I}{dx}\right|_{x=0} = \left.\frac{d\psi_{II}}{dx}\right|_{x=0}, \left.\frac{d\psi_{II}}{dx}\right|_{x=L} = \left.\frac{d\psi_{III}}{dx}\right|_{x=L}$$

- For such finite well, the wave function is not vanished at the boundaries, and may extent into the region I, III which is not allowed in the infinite potential limit
- Such ψ that penetrates beyond the classically forbidden regions diminishes very fast (exponentially) once x extents beyond x = 0 and $x = L$
- The mathematical solution for the wave function in the "classically forbidden" regions are
-

$$\psi(x) = \begin{cases} A_+ \exp(Cx) \neq 0, & x \leq 0 \\ A_- \exp(-Cx) \neq 0, & x \geq L \end{cases}$$



The total energy of the particle

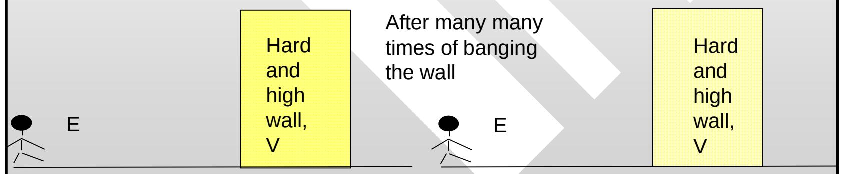
E = K inside the well.

The height of the potential well V is larger than E for a particle trapped inside the well

Hence, classically, the particle inside the well would not have enough kinetic energy to overcome the potential barrier and escape into the forbidden regions I, III

However, in QM, there is a slight chance to find the particle outside the well due to the quantum tunneling effect

- The quantum tunnelling effect allows a confined particle within a finite potential well to penetrate through the classically impenetrable potential wall



Quantum tunneling effect

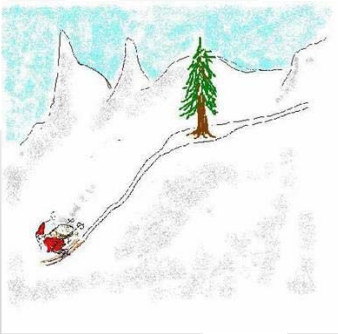
Why tunneling phenomena can happen

- It's due to the continuity requirement of the wave function at the boundaries when solving the T.I.S.E
- The wave function cannot just "die off" suddenly at the boundaries of a finite potential well
- The wave function can only diminishes in an exponential manner which then allow the wave function to extent slightly beyond the boundaries

$$\psi(x) = \begin{cases} A_+ \exp(Cx) \neq 0, & x \leq 0 \\ A_- \exp(-Cx) \neq 0, & x \geq L \end{cases}$$

- The quantum tunneling effect is a manifestation of the wave nature of particle, which is in turns governed by the T.I.S.E.
- In classical physics, particles are just particles, hence never display such tunneling effect

Quantum tunneling effect



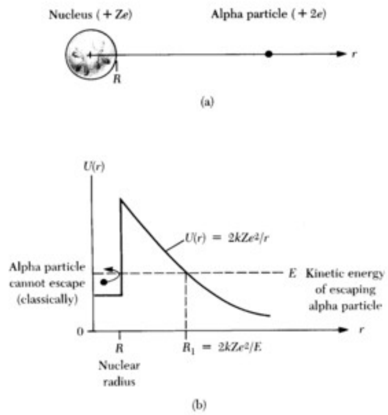


Figure 6.7 (a) Alpha decay of a radioactive nucleus. (b) The potential energy seen by an alpha particle emitted with energy E. R is the nuclear radius, about 10^{-14} m or 10 fm. Alpha particles tunneling through the potential barrier between R and R_1 escape the nucleus to be detected as radioactive decay products.

Real example of tunneling phenomena: alpha decay

Real example of tunneling phenomena: Atomic force microscope

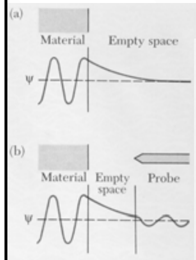


Figure 3 (a) The wavefunction of an electron in the surface of the material to be studied. The wavefunction extends beyond the surface into the empty region. (b) The sharp tip of a conducting probe is brought close to the surface. The wavefunction of a surface electron penetrates into the tip, so that the electron can "tunnel" from surface to tip.

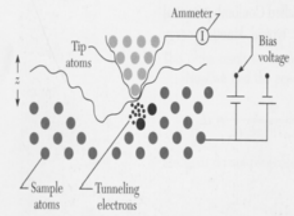


FIGURE A Highly schematic diagram of the scanning tunneling microscope process. Electrons, represented in the figure as small dots, tunnel across the gap between the atoms of the tip and sample. A feedback system that keeps the tunneling current constant causes the tip to move up and down tracing out the contours of the sample atoms.

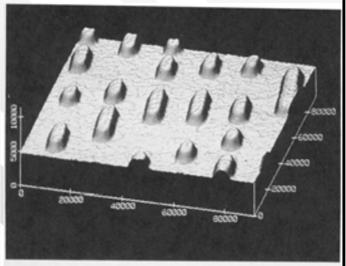


FIGURE D An atomic force microscope scan of a stamper used to mold compact disks. The numbers given are in nm. The bumps on this metallic mold stamp out 60 nm-deep holes in tracks that are 1.6 μm apart in the optical disks. *Photo courtesy of Digital Instruments.*

ZCT 104

MODERN PHYSICS

ACADEMIC SESSION 2005/06
(SECOND SEMESTER)

TOTORIAL PROBLEM SETS

TUTORIAL 1
SPECIAL RELATIVITY
(BASED ON UNDERSTANDING PHYSICS, CUMMINGS et al, John Wiley and Sons)

SEC. 38-2 ORIGINS OF SPECIAL RELATIVITY
1. Chasing Light.

What fraction of the speed of light does each of the following speeds *v* represent? That is, what is the value of the ratio *v/c*? (a) A typical rate of continental drift, 3 cm/y. (b) A high way speed limit of 100 km/h. (c) A supersonic plane flying at Mach 2.5 = 3100 km/h. (d) The Earth in orbit around the Sun at 30 km/s. (e) What conclusion(s) do you draw about the need for special relativity to describe and analyze most everyday phenomena? (Note: Some everyday phenomena can be derived from relativity. For example, magnetism can be described as arising from electrostatics plus special relativity applied to the slow-moving charges in wires.) ($v/c = 3.16\times10^{-18}.$; $v/c = 9.26\times10^{-8}.$; $v/c = 2.87\times10^{-6}.$; $v/c = 10^{-4}.$)

SEC. 38-3 . THE PRINCIPLE OF RELATIVITY
3. Examples of the Principles of Relativity

Identical experiments are carried out (1) in a high-speed train moving at constant speed along a horizontal track with the shades drawn and (2) in a closed freight container on the platform as the train passes. Copy the following list and mark with a “yes” quantities that will necessarily be the same as measured in the two frames. Mark with a “no” quantities that are not necessarily the same as measured in the two frames. (a) The time it takes for light to travel one meter in a vacuum; (b) the kinetic energy of an electron accelerated from rest through a voltage difference of one million volts; (c) the time for half the number of radioactive particles at rest to decay; (d) the mass of a proton; (e) the structure of DNA for an amoeba; (f) Newton’s Second Law of Motion: $F = ma$; (g) the value of the downward acceleration of gravity g.

SEC. 38-4. LOCATING EVENTS WITH AN INTELLIGENT OBSERVER
6. Eruption from the Sun

You see a sudden eruption on the surface of the Sun. From solar theory you predict that the eruption emitted a pulse of particles that is moving toward the Earth at one- eighth the speed of light. How long do you have to seek shelter from the radiation that will be emitted when the particle pulse hits the Earth? Take the light-travel time from the Sun to the Earth to be 8 minutes. (56 minutes)

SEC. 38-5 LABORATORY AND ROCKET LATTICEWORKS OF CLOCKS
7. Synchronizing a Clock

In a vast latticework of meter sticks and clocks, you stand next to a lattice clock whose coordinates are x = 8 km, y = 40 km, z = 44 km. When you receive the synchronizing flash, to what time do you quickly set your clock? (2×10^{-4} second)

10. Where and When?

Two firecrackers explode at the same place in the laboratory and are separated by a time of 12 years. (a) What is the spatial distance between these two events in a rocket in which the events are separated in time by 13 years? (b) What is the relative speed of the rocket and laboratory frames? Express your answer as a fraction of the speed of light. (4.7×10^{16} meters.; a little more than one-third the speed of light)

13. Fast-Moving Muons

The half-life of stationary muons is mea sured to be 1.6 microseconds. Half of any initial number of station ary muons decays in one half-life. Cosmic rays colliding with atoms in the upper atmosphere of the Earth create muons, some of which move downward toward the Earth’s surface. The mean lifetime of high-speed muons in one such burst is measured to be 16 microseconds. (a) Find the speed of these muons relative to the Earth. (b) Moving at this speed, how far will the muons move in one half-life? (c) How far would this pulse move in one half-life if there were no relativistic time stretching? (d) In the relativistic case, how far will the pulse move in 10 half-lives? (e) An initial pulse consisting of 10^8 muons is created at a distance above the Earth’s surface given in part (d). How many will remain at the Earth’s surface? Assume that the pulse moves vertically downward and none are lost to collisions. (Ninety-nine percent of the Earth’s atmosphere lies below 40 km altitude.)

($\frac{v}{c} = 0.995.$; 4.8×10^{3} meters.; 480 meters; 48 kilometers.; 9.8×10^{4})

15. Living a Thousand Years in One Year

Living a Thousand Years in One Year. You wish to make a round trip from Earth in a spaceship, traveling at constant speed in a straight line for 6 months on your watch and then returning at the same constant speed. You wish, further, to find Earth to be 1000 years older on your return. (a) What is the value of your constant speed with respect to Earth? (b) How much do you age during the trip? (c) Does it matter whether or not you travel in a straight line? For example, could you travel in a huge circle that loops back to Earth? ($v/c = 0.9999995.$; one year)

SEC 38-8 . CAUSE AND EFFECT

17. Relations between Events

The table shows the t and x coordinates of three events as observed in the laboratory frame.
Laboratory Coordinates of Three Events

Event	***t*** years	***x*** light-years
Event 1	**2**	**1**
Event 2	**7**	**4**
Event 3	**5**	**6**

On a piece of paper list vertically every pair of these events: (1,2), (1, 3), (2, 3). (a) Next to each pair write "time-like," "light-like," 0 "space-like" for the relationship between those two events. (b) Next to each pair, write "Yes" if it is possible for one of the events to cause the other event and "No" if a cause and effect relation between them is not possible. (For full benefit of this exercise construct and analyze your own tables.)

ANS: (1,2) timelike yes; (1,3) spacelike no; (2,3) lightlike yes

22. Proton Crosses Galaxy

Find the energy of a proton that crosses our galaxy (diameter 100 000 light-years) in one minute of its own time. ($5.27\times10^{10}\,mc^{2}$)

38-10 MOMENTUM AND ENERGY

23. Converting Mass to Energy

The values of the masses in the reaction $p + {}^{19}F \rightarrow \alpha + {}^{16}O$ have been determined by a mass spectrometer to have the values: $m(p)$ = 1.00782, $m(F)$ = 18.998405u, $m(\alpha)$ = 4.002603u, $m(O)$ = 15.994915u. Here u is the atomic mass unit (Section 1.7). How much energy is released in this reaction? Express your answer in both kilograms and MeV. (1.4467×10^{-29} kilogram ; 1.3020×10^{-12} joules)

27. Powerful Proton

A proton exits an accelerator with a kinetic energy equal to N times its rest energy. Find expressions for its (a) speed and (b) momentum. ($\frac{\left[N\left(N+2\right)\right]^{1/2}}{N+1}c.$; $p = \left[N\left(N+2\right)\right]^{1/2} mc.$)

30. A Box of Light

Estimate the power in kilowatts used to light a city of 8 million inhabitants. If all this light generated during one hour in the evening could be captured and put in a box, how much would the mass of the box increase? (16 million kilowatts; 0.64 g)

SEC. 38-11 . THE LORENTZ TRANSFORMATION

32. Really Simultaneous?

(a) Two events occur at the same time in the laboratory frame and at the laboratory coordinates (x_1 = 10 km, y_1 = 4 km, z_1 = 6 km) and (x_2 = 10 km, y_2 = 7 km, z_2 = -10 km). Will these two events be simultaneous in a rocket frame moving with speed v= 0.8c in the x direction in the laboratory frame? Explain your answer. (b) Three events occur at the same time in the laboratory frame and at the laboratory coordinates (x_0,y_1, z_1), (x_0,y_2, z_2) and (x_0,y_1, z_3) where x_0 has the same value for all three events. Will these three events be simultaneous in a rocket frame moving with speed v in the laboratory x direction? Explain your answer. (c) Use your results of parts (a) and (b) to make a general statement about simultaneity of events in laboratory and rocket frames.

ANS: (a) simultaneous—in the rocket frame ($\Delta t'$ = 0); (b) events are *simultaneous* in both the laboratory and rocket frames for the pair of events in part (a) and for all three pair of events in part (b);

38-12 LORENTZ CONTRACTION

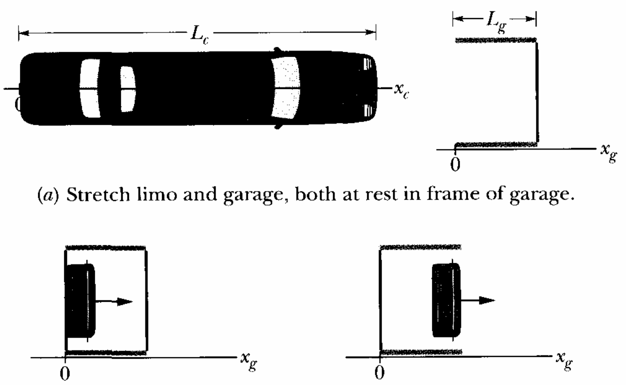
36. Electron Shrinks Distance

An evacuated tube at rest in the /laboratory has a length 3.00 m as measured in the laboratory. An electron moves at speed v = 0.999 987c in the laboratory along the axis of this evacuated tube. What is the length of the tube measured in the rest frame of the electron? (1.53 cm)

39. Traveling to the Galactic Center

(a) Can a person, in principle, /travel from Earth to the center of our galaxy, which is 23 000 ly distant, in one lifetime? Explain using either length contraction or time dilation arguments. (b) What constant speed with respect to the galaxy is required to make the trip in 30 y of the traveler's life time? (Yes; $v/c = 0.999\,999\,15$)

40. Limo in the Garage



(*a*) Stretch limo and garage, both at rest in frame of garage.
(*b*) Stretch limo at speed in frame of garage, just as front garage door closes.
(*c*) Stretch limo at speed in frame of garage, just as back garage door opens.

FIGURE 38-10 Problem 40.

Carman has just purchased the world's longest stretch limo, which has proper length L = 30.0 m. Part (a) of Figure 38-10 shows the limo parked at rest in front of a garage of proper length L_g = 6.00 m, which has front and back doors. Looking at the limo parked in front of the garage, Carman says there is no way that the limo can fit into the garage. "*Au con traire*!" shouts Garageman, "Under the right circumstances the limo can fit into the garage with both garage doors closed and room to spare!" Garageman envisions a fast-moving limo that takes up exactly one-third of the proper length of the garage. Part (b) of Figure 38-10 shows the speeding limo just as the front garage door closes behind it as recorded in the garage frame. Part (c) of Figure 38-10 shows the limo just as the back garage door opens in front of it as recorded in the garage frame. Find the sped of the limo with respect to the garage required for this scenario to take place. ($v = 0.99778c.$)

SEC 38-13 RELETIVITY OF VELOCITIES

42. Separating Galaxies.

Galaxy A is measured to be receding from us on Earth with a speed of 0.3c. Galaxy B, located in precisely the opposite direction, is also receding from us at the same speed. What recessional velocity will an observer on galaxy A measure (a) for our galaxy, and (b) for galaxy B? (0.3c., -0/55*c*)

44. Transit Time

An unpowered spaceship whose rest length 350 meters has a speed 0.82c with respect to Earth. A micrometeorite, also with speed of 0.82c with respect to Earth, passes spaceship on an antiparallel track that is moving in the opposite direction. How long does it take the micrometeorite to pass spaceship as measured on the ship? (1.2×10^{-6} second.)

TUTORIAL 2

Black Body, Photoelectricity, Compton Scattering, X-rays, Pair-production/annihilation

1. The total intensity $I(T)$ radiated from a blackbody (at all wavelengths λ) is equal to the integral over all wavelengths. $0 < \lambda < \infty$, of the Planck distribution $I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$. (a) By changing variables to $x = hc/\lambda kT$, show that $I(T)$ has the form $I(T) = \sigma T^4$, where σ is a constant independent of temperature. This result is called **Stefan's fourth-power law**, after the Austrian physicist Josef Stefan. (b) Given that $\int_0^\infty \frac{x^3 dx}{e^x - 1} = \pi^4/15$, show that the Stefan-Boltzmann Constant σ is $\sigma = \frac{2\pi^5 k^4}{15h^3c^2}$. (c) Evaluate σ numerically, and find the total power radiated from a red-hot (T = 1000 K) steel hail of radius 1 cm. (Such a ball is well approximated as a blackbody.) (Taylor, Problem 4.4, pg. 141,) **ANS: (c)** 71 W

2. If Planck constant were smaller than it is, would quantum phenomena be more or less conspicuous than they are now? (Beiser, Ex. 1, pg. 89)

3. The diameter of an atomic nucleus is about 10×10^{-15} m. Suppose you wanted to study the diffraction of photons by nuclei. What energy of photons would you choose? (Krane, Q.1, pg. 94)

4. Electric current is charge flowing per unit time. If we increase the kinetic energy of the electron by increasing the energy of the photons, shouldn't the current increase, because the charge flows more rapidly? Why doesn't it? (Krane, Q.6, pg. 94)

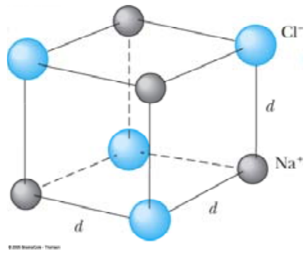
5. What would be the effects on a photoelectric effect if we were to double the frequency of the incident light? If we were to double the wavelength? If we were to double the intensity? (Krane, Q.7, pg. 94)

6. The Compton-scattering formula suggests that objects viewed from different angles should reflect light of different wavelengths. Why don't we observe a change in colour of objects as we change the viewing angle? (Krane, Q.16, pg. 95)

7. You have a monoenergetic source of X-rays of energy 84 keV, but for an experiment you need 70 keV X-rays. How would you convert the X-ray energy from 84 to 70 keV? (Krane, Q.16, pg. 95)

8. Show that a photon cannot transfer all of its energy to a free electron. (*Hint*: Note that energy and linear momentum must be conserved.) (Serway, Moses and Moyer, P27. pg. 103)

9. **The determination of Avogadro's number with x—rays**. X-rays. X-rays from a molybdenum (0.626 A) are incident on a NaCl crystal, which has the atomic arrangement shown in Figure below. If NaCl has a density of 2.17 g/cm^3 and the n= 1 diffraction maximum from planes separated by d is found at θ = 6.41°, compute Avogadro's number. (*Hint:* First determine d. Using Figure P3.39, determine the number of NaCl molecules per primitive cell and set the mass per unit volume of the primitive cell equal to the density. (Serway, Moses and Moyer, P39, pg./ 104)



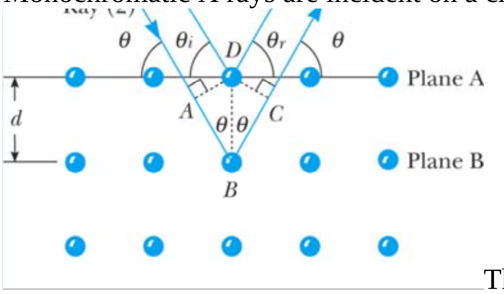
ANS: $N_A = 6.13\times10^{23}$/mole

10. Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp (λ= 546.1 nm) is used, a retarding potential of 1.70 V reduces the photocurrent to zero. (a) Based on this measurement, what is the work function for this metal? (b) What

stopping potential would be observed when using the yellow light from a helium discharge tube (λ = 587.5 nm)? (Serway, Moses and Moyer. P42, pg 104)

ANS: (a) 0.571 eV; (b) 1.54 V

11. Monochromatic X rays are incident on a crystal in the geometry of Figure below.



The first-order Bragg peak is observed when the angle of incidence is 34.0°. The crystal spacing is known to be 0.347 nm. (a) What is the wavelength of the X rays? (b) Now consider a set of crystal planes that makes an angle of 45° with the surface of the crystal (as in the Figure). For X rays of the same wavelength, find the angle of incidence measured from the surface of the crystal that produces the first-order Bragg peak. At what angle from the surface does the emerging beam appear in this case? (Krane, P3, pg 95)

12. The universe is filled with thermal radiation, which has a bla at an effective temperature of 2.7 K. What is the peak wavelength of this radiation? What is the energy (in eV) of a quanta at the peak wavelength? In what region of the electromagnetic spectrum is this peak wavelength? (Krane. P 20, pg 96)

13. Light from the sun arrives at the earth an average of 1.5×10^{11} m away, at the rate of 1.4×10^{13} W/m of area perpendicular to the direction of the light. Assume that sunlight is monochromatic with a frequency of 5×10^{14} Hz. *(a)* How many photons fall per second on each square meter of Earth's surface directly facing the sun? *(b)* What is the power output of the sun, and how many photons per second does it emit? (*c*) How many photons per cubic meter are there near the earth? (Beiser, Ex. 9, pg. 90)

ANS: (a) 4.2 x 10^{21}; (b) 4.2 x 10^{26}Watt; 1.2 x 10^{45} photon per second (c) 1.4 x 10^{13} photon/m^3

14. 1.5 mW of 400-nm light is directed at a photoelectric cell. If 0.10 percent of the incident photons produce photoelectrons, find the current in the cell. (Beiser, Ex. 15, pg. 90)

ANS: 0.48 μA

15. *(a)* Find the change in wavelength of 80-pm x-rays that are scattered 120° by a target, *(b)* Find the angle between the directions of the recoil electron and the incident photon. *(c)* Find the energy of the recoil electron. (Beiser, Ex. 34, pg. 90)
ANS: (a) 3.64 pm (b) 29.3° (c) 674 eV

16. A photon of frequency *v* is scattered by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $KE_{max} = (2h^2v^2/mc^2) / (1+ 2hv/mc^2)$ (Beiser, Ex. 35, pg. 90)

17. Show that, regardless of its initial energy, a photon cannot undergo Compton scattering through an angle of more than 60° and still be able to produce an electron-positron pair. *(Hint:* Start by expressing the Compton wavelength of the electron in terms of the maximum photon wavelength needed for pair production.) (Beiser, Ex. 41, pg. 91)

18. (a) Verily that the minimum energy a photon must have to create an electron-positron pair in the presence of a stationary nucleus of mass *M* is $2mc^2/(1 + m/M)$, where *m* is the electron rest mass. (**b**) Find the minimum energy needed for pair production in the presence of a proton. (Beiser, Ex. 42, pg. 91)
ANS: (b) 1.023 MeV

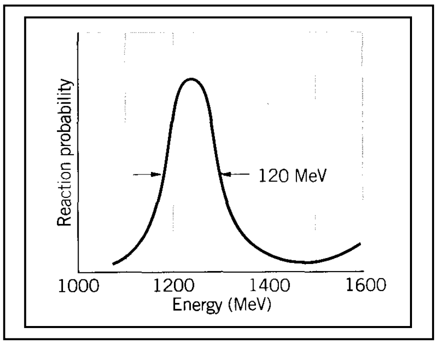
19. Why is it in a pair annihilation the resultant photons cannot be singly produced?

TUTORIAL 3
Wave properties of particles

1. Suppose we cover one slit in the two-slit experiment with a very thin sheet of fluorescent material that emits a photon of light whenever an electron passes through. We then fire electrons one at a time at the double slit; whether or not we see a flash of light tells us which slit the electron went through. What effect does this have on the interference pattern? Why? (*Suggestion: Read Chap. 1, Feynman Lectures on Physics Vol. 3)* (Krane, Q13, pg. 131,)

2. The speed of an electron is measured to within an uncertainty of 2.0×10^4 m/s. What is the size of the smallest region of space in which the electron can be confined? (Krane, P14, pg. 133)

3. A pi meson (pion) and a proton can briefly join together to form a Δ particle. A measurement of the energy of the πp system (see Figure) shows a peak at 1236 MeV, corresponding to the rest energy of the Δ particle, with an experimental spread of 120 MeV. What is the lifetime of the Δ? (Krane, P17, pg. 133)



4. A proton or a neutron can sometimes "violate" conservation of energy emitting and then reabsorbing a pi meson, which has a mass of 135 MeV/c^2. This is possible as long as the pi meson is reabsorbed within a short enough time Δt consistent with the uncertainty principle. (*a*) Consider p→ p + π. By what amount ΔE is energy conservation violated? (ignore any kinetic energies.) (*b*) For how long a time Δt can the pi meson exist? (*c*) Assuming pi meson to travel at very nearly the speed of light, how far from the proton can it go? (This procedure gives us an estimate of the range of the nuclear force, because we believe that protons and neutron are held together in the nucleus by exchanging pi mesons.) (Krane, P22, pg. 133)

5. Show that the formula for low-energy electron diffraction (LEED), when electrons are incident perpendicular to a crystal surface, may be written as $\sin\phi = \dfrac{nhc}{d\left(2m_e c^2 K\right)^{1/2}}$, where n is the order of the maximum, d is the atomic spacing, m_e is the electron mass, K is the electron's kinetic energy, and ϕ is the angle between the incident and diffracted beams, (b) Calculate the atomic spacing in a crystal that has consecutive diffraction maxima at $\phi = 24.1°$ and $\phi = 54.9°$ for 100-eV electrons. (Serway, M & M, P 14, pg. 188) **ANS:** 33×10^{-10} m for n=1, 33×10^{-10} m for n=2)

6. A woman on a ladder drops small pellets toward a spot on the floor, (a) Show that, according to the uncertainty principle, the miss distance must be at least $\Delta x = \left(\dfrac{\hbar}{2m}\right)^{1/2}\left(\dfrac{H}{2g}\right)^{1/4}$, where H is the initial height of each pellet above the floor and m is the mass of each pellet, (b) If H = 2.0 m and m = 0.50 g, what is Δx? (Serway & M & M, P 21, pg. 188) **ANS: (b)** $\Delta x_{\text{total}} = 5.2\times10^{-16}$ m

7. An excited nucleus with a lifetime of 0.100 ns emits a γ ray of energy 2.00 MeV. Can the energy width (uncertainty in energy, ΔE) of this 2.00-MeV γ emission line be directly measured if the best gamma detectors can measure energies to ±5 eV? (Serway & M & M, P 25, pg. 188)

ANS: NO

8. Find the de Broglie wavelength of a 1.00-MeV proton. Is a relativistic calculation needed? (Beiser, Ex. 6, pg. 117) **ANS:** 2.86×10^{-14} m; No need.

9. Show that the de Broglie wavelength of a particle of mass *m* and kinetic energy KE is given by
$\lambda = \dfrac{hc}{\sqrt{KE(KE + 2mc^2)}}$ (Beiser, Ex. 10, pg. 117)

10. What effect on the scattering angle in the Davisson-Germer experiment does increasing the electron energy have? (Beiser, Ex. 23, pg. 117)

11. A beam of 50-keV electrons is directed at a crystal and diffracted electrons are found at an angle of 50° relative to the original beam. What is the spacing of the atomic planes of the crystal? A relativistic calculation is needed for λ. (Beiser, Ex. 26, pg. 117) **ANS:** 3.0 pm

12. The lowest energy possible for a certain particle trapped in a certain box is 1.00 eV. (*a*) What are the next two higher energies the particle can have? (*b*) If the particle is an electron, how wide is the box? (Beiser, Ex. 29, pg. 118)

    ANS: (a) 4 eV, 9 eV; (b) 45 fm

13. Discuss the prohibition of $E = 0$ for a particle trapped in a box L wide in terms of the uncertainty principle. How does the minimum momentum of such a particle compare with the momentum uncertainty required by the uncertainty principle if we take $\Delta x = L$? (Beiser, Ex. 30, pg. 118)

14. (*a*) How much time is needed to measure the kinetic energy of an electron whose speed is 10.0 m/s with an uncertainty of no more than 0.100 percent? How far will the electron have travelled in this period of time? (b) Make the same calculations for a 1.00-g insect whose speed is the same. What do these sets of figures indicate? (Beiser, Ex. 34, pg. 118)

    ANS: (a) 1.2 ms, 1.2 cm; (b) 9.5×10^{-29} s, 9.5×10^{-28} m.

15. How accurately can the position of a proton with $v << c$. be determined without giving it more than 1.00 keV of kinetic energy? (Beiser, Ex. 35, pg. 118). **ANS:** 0.144 pm

16. (*a*) Find the magnitude of the momentum of a particle in a box in its *n*th state. (*b*) The minimum change in the particles momentum that a measurement can cause corresponds to a change of ±1 in the quantum number *n*. If $\Delta x = L$. show that $\Delta p \, \Delta x \geq \hbar / 2$. (Beiser, Ex. 36, pg. 118)

    ANS: (a) $nh/2L$; (b) $h/2L$;

TUTORIAL 4
Atomic models

1. How is the quantization of the energy in the hydrogen atom similar to the quantization of the systems discussed in the 1-D infinite quantum well? How is it different? Do the quantizations originate from similar causes? (Krane, Q8, pg. 201)

2. In both the Rutherford theory and the Bohr theory, we used the classical expression for the kinetic energy. Estimate the velocity of an electron in the Bohr atom and of an alpha particle in a typical scattering experiment, and decide if the use of the classical formula is justified. (Krane, Q14, pg. 201)

3. The lifetimes of the levels in a hydrogen atom are of the order of 10^{-8} s. Find the energy uncertainty of the first excited state and compare it with the energy of the state. (Krane, P29, pg. 204)

4. A long time ago, in a galaxy far, far away, electric charge had not yet been invented, and atoms were held together by gravitational forces. Compute the Bohr radius and the n=2 to n = 1 transition energy in a gravitationally hound hydrogen atom. (Krane, P33, pg. 204)

5. The **fine structure constant** is defined as $\alpha = e^2/2\varepsilon_o hc$. This quantity got its name because it first appeared in a theory by the German physicist Arnold Sommerfeld that tried to explain the line structure in spectral lines (multiple lines close together instead of single lines) by assuming that elliptical as well as circular orbits are possible in the Bohr model. Sommerfeld's approach was on the wrong track, but α has nevertheless turned out to be a useful quantity in atomic physics, (*a*) Show that $\alpha = v_1/c$, where v_1 is the velocity of the electron in the ground state of the Bohr atom, (*b*) Show that the value of α is very close to 1/137 and is a pure number with no dimensions. Because the magnetic behavior of a moving charge depends on its velocity, the small value of α is representative of the relative magnitudes of the magnetic and electric aspects of electron behavior in an atom (*c*) Show that $\alpha\, a_0 = \lambda_c / 2\pi$, where a_0 is the radius of the ground-state Bohr orbit and λ_c is the Compton wavelength of the electron. (Beiser Ex. 9, pg. 158)

6. Show that the energy of the photon emitted when a hydrogen atom makes a transition from state n to state n - 1 is, when n is very large, $\Delta E \cong \alpha^2(mc^2/n^3)$ where α is the fine structure constant. (Krane, P38, pg. 205)

7. Can the electron in the ground state of the hydrogen atom absorb a photon of energy (a) less than 13.6 eV and (b) greater than 13.6 eV? (c) What is the minimum photon energy that can be absorbed by the ground state of the hydrogen atom? (Serway, M & M, Q3, pg. 145)

   ANS: (a) Yes (b) No (c) 10.2 eV

8. Four possible transitions for a hydrogen atom are listed here.

   (A) $n_i = 2; n_f = 5$ (B) $n_i = 5; n_f = 3$ (C) $n_i = 7; n_f = 4$ (D) $n_i = 4; n_f = 7$

   (a) Which transition emits the photons having the shortest wavelength?

   (b) For which transition does the atom gain the most energy? (c) For which transition(s; does the atom lose energy? (Serway, M & M. Q11, pg. 145)

9. An electron initially in the n = 3 state of a one-electron atom of mass M at rest undergoes a transition to the n = 1 ground state. (a) Show that tile recoil speed of the atom from emission of a photon is given approximately by $v=8hR/9M$. (b) Calculate the percent of the 3 → 1 transition energy that is carried off by the recoiling atom if the atom is deuterium. (Serway, M & M. Q29, pg. 148)

   ANS: (b) 3.23×10^{-8} %

10. *The Auger process*. An electron in chromium makes a transition from the n = 2 state to the n = 1 state with out emitting a photon. Instead, the excess energy is transferred to an outer electron (in the n = 4 state), which is ejected by the atom. (This is called an Auger process, and the ejected electron is referred to as an Auger electron.) Use the Bohr theory to find the kinetic energy of the Auger electron. (Serway, M & M. Q28, pg. 148)

    ANS: 5.385 keV

11. In a hot star, a multiply ionized atom with a single remaining electron produces a series of spectral lines as described by the Bohr model. The series corresponds to electronic transitions that terminate in the same final state. The longest and shortest wavelengths of the series are 63.3 nm and 22.8 nm, respectively. (*a*) What is the ion? (*b*) Find the wave lengths of the next three spectral lines nearest to the line of longest wavelength. (Serway, M & M. Q44, pg. 150) **ANS:** (a) O^{7+} ; (b) 41.0 nm , 33.8 nm, 30.4 nm

12. Find the frequency of revolution of the electron in the classical model of the hydrogen atom. In what region of the spectrum are electromagnetic waves of this frequency? (Beiser, Ex. 4, pg. 157) **ANS:** 6.6×10^{15} Hz, ultraviolet

13. What is the shortest wavelength present in the Bracken series of spectral lines? (Beiser, Ex.5, pg. 158)

    ANS: 1459 nm

14. A beam of 13.0-eV electrons is used to bombard gaseous hydrogen. What series of wavelengths will be emitted? (Beiser Ex. 16, pg. 158). **ANS:** Excited to the n = 4 level but no higher

15. The longest wavelength in the Lyman series is 121.5 nm and the shortest wavelength in the Balmer series is 364.6 nm. Use the figures to find the longest wavelength of light that could ionize hydrogen. (Beiser, Ex. 23, pg. 158) **ANS:** 91.13 cm

16. When an excited atom emits a photon, the linear momentum of the photon must be balanced by the recoil momentum of the atom. As a result, some of the excitation energy of the atom goes into the kinetic energy of its recoil. (*a*) Modify $E_i - E_f = h\nu$ to include this effect, (*b*) Find the ratio between the recoil energy and the photon energy for the $n = 3 \rightarrow n \rightarrow$ 2transition in hydrogen, for which $E_f - E_i$ = 1.9 eV. Is the effect a major one? A nonrelativistic calculation is sufficient here. (Beiser, Ex. 27, pg. 158)

    ANS: (a)
    $$E_f - E_i = h\nu \left(1 + \frac{1}{2}\left(1 + \frac{Mc^2}{h\nu}\right)\right.$$

    (b) 1.0 x 10^{-9}; nonrelativistic is sufficient

TUTORIAL 5
Introductory Quantum Mechanics

1. Which of the following wave functions cannot be solutions of Schrodinger's equation for all values of x? Why not? *(a)* $\psi = A \sec x$; *(b)* $\psi = A \tan x$; (c) $\psi = A e^{x^2}$; (d) $\psi = A e^{-x^2}$ (Beiser, Ex. 3, pg. 197)

   ANS: only (d) could be a solution

2. The wave function of a certain particle is $\psi = A \cos^2 x$ for $\pi/2 < x < \pi/2$. *(a)* Find the value of A. (*b*) Find the probability that the panicle be found between $x = 0$ and $x = \pi/4$ (Beiser, Ex. 5, pg. 197)

   ANS: (a) $\sqrt{\frac{8}{3}}$ **;** (b) 0.462

3. The expectation value $\langle x \rangle$ of a particle trapped in a box a wide is $a/2$ $(0 \le x \le a)$, which means that its average position is the middle of the box. Find the expectation value $\langle x^2 \rangle_n$ in the stationary state n. What is the behaviour of $\langle x^2 \rangle_n$ as n becomes infinity. Is this consistent with classical physics? (Beiser, Ex. 17, pg. 198). **ANS:** $a^2\left(\frac{1}{3}-\frac{1}{2\pi^2 n^2}\right)$.

4. Find the probability that a particle in a box L wide can be found between $x = 0$ and $x = L/n$ when it is in the nth state. (Beiser, Ex. 19, pg. 198)

   ANS: $1/n$.

5. What is the physical meaning of $\int_{-\infty}^{\infty} |\psi|^2\, dx = 1$ ψ dx = 1? (Krane, Q3, pg/ 168)

6. What are the dimensions of $\psi(x)$? (Krane, Q4, pg. 168)

   ANS: $\sqrt{\mathrm{L}}$.

7. What happens to the probability density in the infinite well when n → ∞? Is this consistent with classical physics? (Krane, Q6, pg. 168)

8. How would the solution to the one-dimensional infinite potential energy well be different if the potential energy were not zero for $0 \le x \le L$ but instead had a constant value U_0. What would be the energies of the excited states? What would be the wavelengths of the standing de Broglie waves? Sketch the behavior of the lowest two wave functions. (Krane, Q6, pg. 168)

9. A particle in an infinite well is in the ground state with an energy of 1.26 eV. How much energy must be added to the particle to reach the second excited state (n = 3)? The third excited state (n = 4)? (Krane, P4, pg. 170)

10. An electron is trapped in a one-dimensional well of width 0.132 nm. The electron is in the n = 10 state. (*a*) What is the energy of the electron? (*b*) What is the uncertainty in its momentum? (*c*) What is the uncertainty in its position? How do these results change as n → ∞? Is this consistent with classical behavior? (Krane, P9, pg. 170)

11. Consider a particle moving in a one-dimensional box with walls at $x = -L/2$ and $x = +L/2$. (a) Write the wave functions and probability densities for the states n = 1, n = 2, and n = 3. (b) Sketch the wave function and probability densities. (Hint: Make an analogy to the case of a particle in a box with walls at x = 0 and $x = L$) (Serway, M & M, P11, pg. 228)

12. A particle of mass m is placed in a one-dimensional box of length L. The box is so small that the particle's motion is relativistic, so that $E = p^2/2m$ is not valid. (*a*) Derive an expression for the energy levels of the particle using the relativistic energy- momentum relation and the quantization of momentum that derives from confinement. (*b*) If the particle is an electron in a box of length $L = 1.00\times10^{-12}$m, find its lowest possible kinetic energy. By what percent is the nonrelativistic formula for the energy in error? (Serway, M & M, P14, pg. 228)

ANS: (a) $K_n = \left[\left(\frac{nhc}{2L}\right)^2 + \left(mc^2\right)^2\right]^{1/2} - mc^2$; (b) 0.29 MeV, 29% too big

ZCT 104

MODERN PHYSICS

PAST YEAR QUESTIONS AND TUTORIAL PROBLEM SETS

(2003/04 – 2004/2005)

Tutorial 1

Special Relativity

Conceptual Questions

1) What is the significance of the negative result of Michelson-Morley experiment?

ANS
The negative result of the MM experiment contradicts with the prediction of the absolute frame (the Ether frame) of reference, in which light is thought to propagate with a speed c. In the Ether postulate, the speed of light that is observed in other initial reference frame (such as the Earth that is moving at some constant speed relative to the Absolute frame), according to the Galilean transformation, would be different than that of the Ether frame. In other words, the MM negative result provides the first empirical evidence to the constancy of light postulate by Einstein.

2) Is it possible to have particles that travel at the speed of light?

ANS
Particle travelling at the speed of light would have an infinite mass, as per $m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$. Hence it is physically not possible to supply infinite amount of energy to boost a particle from rest to the speed of light.

postulate by Einstein.

3) A particle is moving at a speed less that $c/2$. If the speed of the particle is doubled, what happens to its momentum?

ANS

According to $\mathbf{p} = \gamma m \mathbf{u}$, doubling the speed u will make the momentum of an object increase by the factor $2\left[\frac{c^2-u^2}{c^2-4u^2}\right]^{1/2}$. Here's the working:

SESSI 04/05/ TUTORIAL 1

$$p = \gamma m_0 u \rightarrow p' = \gamma' m_0 u'$$

$$\left(\frac{p'}{p}\right)^2 = \left(\frac{u'}{u}\right)^2\left(\frac{\gamma'}{\gamma}\right)^2 = \left(\frac{u'}{u}\right)^2\left[\frac{1-\left(\frac{u}{c}\right)^2}{1-\left(\frac{u'}{c}\right)^2}\right] = \left(\frac{u'}{u}\right)^2\frac{c^2-u^2}{c^2-u'^2} \Rightarrow \frac{p'}{p} = \left(\frac{u'}{u}\right)\sqrt{\frac{c^2-u^2}{c^2-u'^2}}$$

$$\text{Let } u' = 2u \Rightarrow \frac{p'}{p} = \left(\frac{2u}{u}\right)\sqrt{\frac{c^2-u^2}{c^2-(2u)^2}} = 2\sqrt{\frac{c^2-u^2}{c^2-4u^2}}$$

4. The rest energy and total energy respectively, of three particles, expressed in terms of a basic amount A are (1) A, $2A$; (2) A, $3A$; (3) $3A$, $4A$. Without written calculation, rank the particles according to their (a) rest mass, (b) Lorentz factor, and (c) speed, greatest first.

ANS
Case 1: $\{m_0c^2, E\} = \{A, 2A\}$; Case 2: $\{m_0c^2, E\} = \{A, 3A\}$; Case 3: $\{m_0c^2, E\} = \{3A, 4A\}$

(a) Rest mass = m_0. Hence for case 1: m_0 m_0c^2=A; Case 2:m_0c^2=A; Case 3: m_0c^2 =3A. Therefore, the answer is: mass in (3) > mass in (2) = mass in (1);

(b) Lorentz factor $\gamma = E/m_0c^2$. Hence for case 1: γ = 2A/A=2; case 2: γ = 3A/A=3; case 3: γ = 4A/3A=4/3 = 1.33. Therefore, the answer is: γ in (2) > γ in (1) > γ in (3)

(c) $\gamma^{-2} = 1 - v^2/c^2 \Rightarrow v^2/c^2 = 1-\gamma^{-2}$. Hence for case 1: v^2/c^2 = 1 – 1/4 = 0.75; case 2: v^2/c^2 = 1 – 1/9 = 0.89; case 3: v^2/c^2 = 1 – 9/16 = 0.4375. Therefore, the answer is: v^2/c^2 in (2) > v^2/c^2 in (1) > v^2/c^2 in (3)

PROBLEMS

1. Space Travel (from Cutnell and Johnson, pg 861,863)
Alpha Centauri, a nearby star in our galaxy, is 4.3 light-years away. If a rocket leaves for Alpha Centauri and travels at a speed of v = 0.95c relative to the Earth, (i) by how much will the passengers have aged, according to their own clock, when they reach their destination? ii) What is the distance between Earth and Alpha Centauri as measured by the passengers in the rocket? Assume that the Earth and Alpha Centauri are stationary with respect to one another.

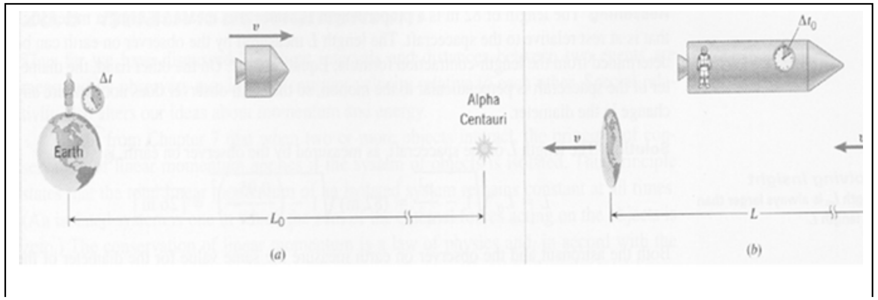


Figure: (a) As measured by an observer on the earth, the distance to Alpha Centauri is L_0, and the time required to make the trip is Δt. (b) According to the passenger on the spacecraft, the earth and Alpha Centauri move with speed v relative to the craft. The passenger measures the distance and time of the trip to be L and Δt_0 respectively, both quantities being less than those in part (a).

Reasoning

The two events in this problem are the departure from Earth and the arrival at Alpha Centauri. At departure, Earth is just outside the spaceship. Upon arrival at the destination, Alpha Centauri is just outside. Therefore, relative to the passengers, the two events occur at the same place - namely, 'just outside the spaceship. Thus, the passengers measure the proper time interval Δt_0 on their clock, and it is this interval that we must find. For a person left behind on Earth, the events occur at different places, so such a person measures the dilated time interval Δt rather than the proper time interval. To find Δt we note that the time to travel a given distance is inversely proportional to the speed. Since it takes 4.3 years to traverse the distance between earth and Alpha Centauri at the speed of light, it would take even longer at the slower speed of v = 0.95c. Thus, a person on earth measures the dilated time interval to be Δt = (4.3 years)/0.95 = 4.5 years. This value can be used with the time-dilation equation to find the proper time interval Δt_0.

Solution

Using the time-dilation equation, we find that the proper time interval by which the Passengers judge their own aging is
$\Delta t_0 = \Delta t\sqrt{(1-v^2/c^2)} = 4.5 \text{ years } \sqrt{(1-0.95^2)} = 1.4$ years.

Thus, the people aboard the rocket will have aged by only 1.4 years when they reach Alpha Centauri, and not the 4.5 years an earthbound observer has calculated.

Both the earth-based observer and the rocket passenger agree that the relative speed between the rocket and earth is v = 0.95c. Thus, the Earth observer determines the distance to Alpha Centauri to be $L_0 = v\Delta t$ = (0.95c)(4.5 years) = 4.3 light-years. On the other hand, a passenger aboard the rocket finds

SESSI 04/05/ TUTORIAL 1

the distance is only $L = v\Delta t_0$ = (0.95c)(1.4 years) = 1.3 light-years. The passenger, measuring the shorter time, also measures the shorter distance - length contraction.

Problem solving insight

In dealing with time dilation, decide which interval is the proper time interval as follows: (1) Identify the two events that define the interval. (2) Determine the reference frame in which the events occur at the same place; an observer at rest in this frame measures the proper time interval Δt_0.

2) The Contraction of a Spacecraft (Cutnell, pg 863)

An astronaut, using a meter stick that is at rest relative to a cylindrical spacecraft, measures the length and diameter of the spacecraft to be 82 and 21 m respectively. The spacecraft moves with a constant speed of v = 0.95c relative to the Earth. What are the dimensions of the spacecraft, as measured by an observer on Earth?

Reasoning

The length of 82 m is a proper length L_o since it is measured using a meter stick that is at rest relative to the spacecraft. The length L measured by the observer on Earth can be determined from the length-contraction formula. On the other hand, the diameter of the spacecraft is perpendicular to the motion, so the Earth observer does not measure any change in the diameter.

Solution

The length L of the spacecraft, as measured by the observer on Earth, is

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = 82m\sqrt{1-\frac{(0.95c)^2}{c^2}} = 26 \text{ m}$$

Both the astronaut and the observer on Earth measure the same value for the diameter of the spacecraft: Diameter = 21 m

Problem solving insight The proper length L_0 is always larger than the contracted length L.

3) *Additional problem 36, Cutnell pg. 879.*
Two spaceship A and B are exploring a new planet. Relative to this planet, spaceship A has a speed of 0.60c, and spaceship B has a speed of 0.80c. What is the ratio D_A/D_B of the values for

the planet's diameter that each spaceship measures in a direction that is parallel to its motion?

Solution

Length contraction occurs along the line of motion, hence both spaceship observe length contraction on the diameter of the planet. The contracted length measures by a moving observer is inversely proportional to the Lorentz factor γ. Hence,

$$\frac{L_A}{L_B}=\frac{\gamma_B}{\gamma_A}=\sqrt{\frac{1-\left(\frac{v_A}{c}\right)^2}{1-\left(\frac{v_B}{c}\right)^2}}=\sqrt{\frac{1-(0.6)^2}{1-(0.8)^2}}=4/3.$$

4) The Energy Equivalent of a Golf Ball (Cutnell, pg 866)
A 0.046-kg golf ball is lying on the green. (a) Find the rest energy of the golf ball. (b) If this rest energy were used to operate a 75-W light bulb, for how many years could the bulb stay on?

Reasoning
The rest energy E_0 that is equivalent to the mass m of the golf ball is found from the relation $E_0 = mc^2$. The power used by the bulb is 75 W, which means that it consumes 75 J of energy per second. If the entire rest energy of the ball were available for use, the bulb could stay on for a time equal to the rest energy divided by the power.

Solution
(a) The rest energy of the ball is

$E_0 = mc^2 = (0.046\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2 = 4.1 \times 10^{15}$ J

(b) This rest energy can keep the bulb burning for a time t given by t = Rest energy/ Power = 4.1×10^{15} J/75 W = 5.5×10^{13} s = 1.7 million years!

5) A High-Speed electron (Cutnell pg. 867)
An electron (mass = 9.1×10^{-31} kg) is accelerated to a speed of $0.9995c$ in a particle accelerator. Determine the electron's (a) rest energy, (b) total energy, and (c) kinetic energy in MeV

(a) $E_0 = mc^2 = 9.109\times10^{-31}\,kg\times(3\times10^8)^2\,m/s = 8.19\times10^{-14}\,J = 0.51MeV$

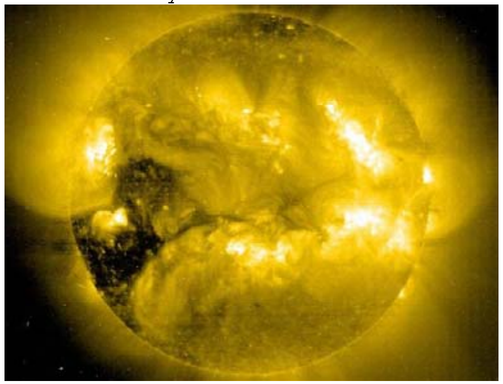
(b) Total energy of the traveling electron,

SESSI 04/05/ TUTORIAL 1

$$E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{0.51MeV}{\sqrt{1-0.995^2}}=16.2MeV$$

(c) The kinetic energy = $E - E_0$ = 15.7 MeV

6) The Sun Is Losing Mass (Cutnell, pg 868)
The sun radiates electromagnetic energy at the rate of 3.92×10^{26} W. (a) What is the change in the sun's mass during each second that it is radiating energy? (b) The mass of the sun is 1.99×10^{30} kg. What fraction of the sun's mass is lost during a human lifetime of 75 years?



Reasoning
Since a W = I J/s the amount of electromagnetic energy radiated during each second is 3.92×10^{26} J. Thus, during each second, the sun's rest energy decreases by this amount. The change ΔE_0 in the sun's rest energy is related to the change Δm in its mass by $\Delta E_0=\Delta m\, c^2$.
Solution
(a) For each second that the sun radiates energy, the change in its mass is $\Delta m = \Delta E_0/c^2 = 3.92\times10^{26}\ \text{J}/(3\times10^8\ \text{m/s})^2 = (4.36\times10^9)$ kg. Over 4 billion kilograms of mass are lost by the sun during each second.

(b) The amount of mass lost by the sun in 75 years is
$\Delta m = (4.36\times10^9)\,\text{kg} \times (3.16\times10^7\ \text{s/year}) \times (75\ \text{years}) = 1\times10^{19}$ kg
Although this is an enormous amount of mass, it represents only a tiny fraction of the sun's total mass:
$\Delta m/m = 1.0\times10^{19}\ \text{kg}/1.99\times10^{30}\ \text{kg} = 5.0\times10^{-12}$

Figure 28.12 An intergalactic cruiser, closing in on a hostile spacecraft, fires a beam of laser light.

7) The Speed of a Laser Beam (Cutnell, pg 871)
Figure below shows an intergalactic cruiser approaching a hostile spacecraft. The velocity of the cruiser relative to the spacecraft is v_{CS} = +0.7c. Both vehicles are moving at a constant velocity. The cruiser fires a beam of laser light at the enemy. The velocity of the laser beam relative to the cruiser is v_{LC} = +c. (a) What is the velocity of the laser beam v_{LS} relative to the renegades aboard the spacecraft? (b) At what velocity do the renegades aboard the spacecraft see the laser beam move away from the cruiser?

Reasoning and Solution

(a) Since both vehicles move at a constant velocity, each constitutes an inertial reference frame. According to the speed of light postulate, all observers in inertial reference frames measure the speed of light in a vacuum to be c. Thus, the renegades aboard the hostile spacecraft see the laser beam travel toward them at the speed of light, even though the beam is emitted from the cruiser, which itself is moving at seven-tenths the speed of light.

More formally, we can use Lorentz transformation of velocities to calculate v_{LS}. We will take the direction as +ve when a velocity is pointing from left to right. We can take view that the hostile spacecraft is at rest (as the stationary frame, O) while the cruiser is approaching it with velocity v_{CS} = + 0.7c (according to our choice of the sign). In this case, the cruiser is the moving frame, O'. The light beam as seen in the moving frame O' is v_{LC} = +c. We wish to find out what is the speed of this laser beam from O point of view, e.g. what v_{LS} is.

We may like to identify v_{LS}, v_{LC} and v_{CS} with the definitions used in the Lorentz formula: $u_x = \dfrac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}}$. In fact, a little contemplation would allow us to make the identification that, with our choice of frames (that the hostile spacecraft as the stationary frame): $v_{LC} \equiv u_{x'}$ = +c; $v_{CS} \equiv u$ = + 0.7c and $v_{LS} = u_x$ = the speed of laser beam as seen by the stationary frame O (the quantity we are seeking). Hence, we have

$u_x = \dfrac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}} \equiv v_{LS} = \dfrac{v_{LC} + v_{CS}}{1 + \dfrac{v_{LC} v_{CS}}{c^2}} = \dfrac{(+c) + (+0.7c)}{1 + \dfrac{(+c)(+0.7c)}{c^2}} = \dfrac{1.7c}{1.7c} = +c$, i.e. the laser beam is seen, from the view point of the hostile spacecraft, to be approaching it with a velocity +c (+ve means the velocity is from left to right).

(b) The renegades aboard the spacecraft see the cruiser approach them at a relative velocity of v_{CS} = +0.7c, and they also see the laser beam approach them at a relative velocity Of v_{LS} +c. Both these velocities are measured relative to the same inertial reference frame-namely, that of the spacecraft. Therefore, the renegades aboard the spacecraft see the laser beam move away from the cruiser at a velocity that is the difference between these two velocities, or +c - (+0.7c) = +0.3c. The relativistic velocity-addition formula, is not applicable here because both velocities are measured relative to the same inertial reference frame (the spacecraft's reference frame). The relativistic velocity-addition formula can be used only when the velocities are measured relative to different inertial reference frames.

8) The Relativistic Momentum of a High-Speed Electron (Cutnell, pg 865)
The particle accelerator at Stanford University is three kilometers long and accelerates electrons to a speed of 0.999 999 999 7c, which is very nearly equal to the speed of light. Find the magnitude of the relativistic momentum of an electron that emerges from the accelerator, and compare it with the non-relativistic value.

Reasoning and Solution

The magnitude of the electron's relativistic momentum can be obtained from $p = \gamma m_0 v = 1\times10^{-17}$ Ns, where $m_0 = 9.1\times10^{-31}$ kg, $v\gamma = \dfrac{0.999999997c}{\sqrt{1-\dfrac{(0.999999997c)^2}{c^2}}} = 1.09989\times10^{13}$ m/s. The relativistic momentum is greater than the non-relativistic momentum by a factor of $\gamma = \dfrac{1}{\sqrt{1-\dfrac{(0.999999997c)^2}{c^2}}} = 4\times10^4$.

9) Resnick and Halliday, Sample problem 37-8, pg. 1047.
The most energetic proton ever detected in the cosmic rays coming to Earth from space had an astounding kinetic energy of

3.0 x 10^{20} eV. (a) What were the proton's Lorentz factor γ and speed v (both relative to the ground-based detector)?

Solution

$$\gamma = \frac{E}{m_0c^2} = \frac{m_0c^2 + K}{m_0c^2} = 1 + \frac{K}{m_0c^2} \Rightarrow \gamma = 1 + \frac{3.0\times10^{20}\,\text{eV}}{938\times10^{6}\,\text{eV}} \approx 3.2\times10^{11}$$

$$\gamma^{-1} = \sqrt{1-\left(\frac{v}{c}\right)^2} = \sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}.\ \text{But}\ 1+\frac{v}{c} \approx 2 \Rightarrow \gamma^{-2} \approx 2\left(1-\frac{v}{c}\right) = 9.766\times10^{-24}$$

$$\Rightarrow v \approx \left(1-5\times10^{-24}\right)c = 0.999999999999999999999995c$$

Tutorial 2

Matter and Wave; Blackbody radiation

Conceptual Questions

1. **What is ultraviolet catastrophe? What is the significance of it in the development of modern physics? (My own question)**

   ANS
   The classical theory explanation of the blackbody radiation by Rayleigh-Jeans fails in the limit $\lambda \to 0$ (or equivalently, when frequency $\to \infty$), i.e. $R(\lambda) \to \infty$ at $\lambda \to 0$. The failure prompted Planck to postulate that the energy of electromagnetic waves is quantised (via $\varepsilon = h\nu$) as opposed to the classical thermodynamics description ($\varepsilon = \kappa T$). With Planck's postulate, radiation now has particle attributes instead of wave.

2. **What assumptions did Planck make in dealing with the problem of blackbody radiation? Discuss the consequences of the assumptions.**

ANS
Planck made two new assumptions: (1) Radiation oscillator energy is quantized and (2) they emit or absorb energy in discrete irreducible packets. The "oscillator" here actually refers to the molecules or atoms that made up the walls of the blackbody cavity. These assumptions contradict the classical idea of energy as continuously divisible.

3. **The classical model of blackbody radiation given by the Rayleigh-Jeans law has two major flaws. Identify them and explain how Planck's law deals with them.**

   ANS
   The first flaw is that the Rayleigh–Jeans law predicts that the intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases. This is known as the *ultraviolet catastrophe*. The second flaw is the prediction much more power output from a black-body than is shown experimentally. The intensity of radiation from the blackbody is given by the area under the red $I(\lambda,\ T)$ *vs.* λ curve in Figure 40.5 in the text, not by the area under the blue curve. Planck's Law dealt with both of these issues and brought the theory into agreement with the experimental data by adding an exponential term to the denominator that

depends on $\frac{1}{\lambda}$. This both keeps the predicted intensity from approaching infinity as the wavelength decreases and keeps the area under the curve finite.

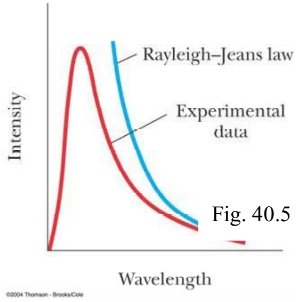


Fig. 40.5

4. What are the most few distinctive physical characteristics, according to your point of view, that exclusively differentiate a classical particle from a wave? Construct a table to compare these two.

ANS (my suggestions)

Particle	**Wave**
Complete localized	Cannot be confined to any particular region of space. A wave can be "simultaneously everywhere" at a given instance in time
Mass and electric charge can be identified with infinite precision	No mass is associated with a wave.
Energy carried by a particle is concentrated in it and is not spreading over the boundary that define its physical location	Energy carried by wave spreads over an infinite regions of space along the direction the wave propagates
Momentum and position can be identified with infinite precision.	Wavelength and position of a wave cannot be simultaneously measured to infinite precision, they must obey the classical wave uncertainty relation $\Delta\lambda\,\Delta x \geq \lambda^2$
There is not definition of wavelength for a particle	There is not definition of momentum for waves
Does not undergo diffraction and interference	Waves undergo diffraction and interference

(others)	(others)

Problems

1. For a blackbody, the total intensity of energy radiated over all wavelengths, *I*, is expected to rise with temperature. In fact one find that the total intensity increases as the fourth power of the temperature. We call this the *Stefan's law*: $I = \sigma T^4$, where σ is the Stefan's constant $\sigma = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}$. How does the total intensity of thermal radiation vary when the temperature of an object is doubled?

ANS

Intensity of thermal radiation $I \propto T^4$. Hence, when T is double, ie. $T\rightarrow 2T$, $I\rightarrow I'(2)^4 = 16I$, i.e. the total intensity of thermal radiation increase by 16 times.

2. (Krane, pg. 62)

In the spectral distribution of blackbody radiation, the wavelength λ_{max} at which the intensity reaches its maximum value decreases as the temperature is increased, in inverse proportional to the temperature: $\lambda_{max} \propto 1/T$. This is called the *Wein's displacement law*. The proportional constant is experimentally determined to be $\lambda_{max}T = 2.898\times10^{-3}\,\mathrm{m\cdot K}$

(a) At what wavelength does a room-temperature (T = 20°C) object emit the maximum thermal radiation?

(b) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum?

(c) How many times as much thermal radiation does it emit at the higher temperature?

ANS

(a) Converting to absolute temperature, T = 293 K, and from Wien's displacement law, $\lambda_{max}T = 2.898\times10^{-3}\,\mathrm{m\cdot K}$

λ_{max} = 2.898×10^{-3} m·K/293K = 9.89 μm

(b) Taking the wavelength of red light to be =650 nm, we again use Wien's displacement law to find T:

T = 2.898×10^{-3} m·K/650×10^{-9} m = 4460 K

(c) Since the total intensity of radiation is proportional to T^4, the ratio of the total thermal emissions will be

$$\frac{T_2^4}{T_1^4} = \frac{4460^4}{293^4} = 5.37\times10^4$$

Be sure to notice the use of absolute (Kelvin) temperatures.

3. **Show that the spectral distribution derived by Planck,** $I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)}$ **reduces to the Rayleigh-Jeans law,** $I(\lambda,T) = \frac{2\pi ck_B T}{\lambda^4}$ **in the long wavelength limit.**

ANS

In long wavelength limit, $hc \ll \lambda k_B T$, the exponential term is approximated to

$e^{hc/\lambda k_B T} = 1 + \frac{hc}{\lambda k_B T} + \frac{1}{2!}\left(\frac{hc}{\lambda k_B T}\right)^2 + \ldots \approx 1 + \frac{hc}{\lambda k_B T}$. Hence, substituting $e^{hc/\lambda k_B T} \approx 1 + \frac{hc}{\lambda k_B T}$ into the Planck's distribution, we have

$$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)} \approx \frac{2\pi hc^2}{\lambda^5\left[\left(1+\frac{hc}{\lambda k_B T}\right)-1\right]} = \frac{2\pi hc^2}{\lambda^5\left[\frac{hc}{\lambda k_B T}\right]} = \frac{2\pi hc^2 \lambda k_B T}{\lambda^5 hc} = \frac{2\pi ck_B T}{\lambda^4},$$

which is nothing but just the RJ's law.

Tutorial 3

Photoelectricity, Compton scatterings, pair-production/annihilation, X-rays

Conceptual Questions

1. **What is the significance of the Compton wavelength of a given particle (say an electron) to a light that is interacting with the particle? (Own question)**

ANS

The Compton wavelength (a characteristic constant depend solely on the mass of a given particle) characterises the length scale at which the quantum property (or wave) of a given particle starts to show up. In an interaction that is characterised by a length scale larger than the Compton wavelength, particle behaves classically. For interactions that occur at a length scale comparable than the Compton wavelength, the quantum (or, wave) nature of the particle begins to take over from classical physics.

In a light-particle interaction, if the wavelength of the light is comparable to the Compton wavelength of the interacting particle, light displays quantum (granular/particle) behaviour rather than like a wave.

2. **Why doesn't the photoelectric effect work for free electron? (Krane, Question 7, pg 79)**

ANS (verify whether the answer make sense)

Essentially, Compton scattering is a two-body process. The free electron within the target sample (e.g. graphite) is a unbounded elementary particle having no internal structure that allows the photons to be `absorbed'. Only elastic scattering is allowed here.

Whereas PE effect is a inelastic scattering, in which the absorption of a whole photon by the atom is allowed due to the composite structure (the structure here refers the system of the orbiting electrons and nuclei hold together via electrostatic potential) of the atom. A whole photon is allowed to get absorbed by the atom in which the potential energy acts like a medium to transfer the energy absorbed from the photon, which is then `delivered' to the bounded electrons (bounded to the atoms) that are then `ejected' out as photoelectrons.

3. **How is the wave nature of light unable to account for the observed properties of the photoelectric effect? (Krane, Question 5, pg 79)**

ANS

See lecture notes

4. In the photoelectric effect, why do some electrons have kinetic energies smaller than K_{max}? (Krane, Question 6, pg 79)

ANS

By referring to $K_{max} = h\nu - \phi$, K_{max} corresponds to those electrons knocked loose from the surface by the incident photon whenever $h\nu > \phi$. Those below the surface required an energy greater than ϕ and so come off with less kinetic energy.

5. Must Compton scattering take place only between x-rays and free electrons? Can radiation in the visible (say, a green light) Compton scatter a free electron? (My own question)

ANS

In order to Compton scatter the electron, the wavelength of the radiation has to be comparable to the Compton wavelength of the electron. If such criterion is satisfied the cross section (the probability for which a scattering process can happen) of Compton scattering between the radiation and the electron would be highly enhanced. It so happen that the Compton wavelength of the electron, $\lambda_e = \frac{h}{m_e c} \sim 10^{-12}\text{m}$ is ~ the order the X-rays', $\lambda_{X-ray} \sim 10^{-12}\text{m}$, hence X-rays' Compton scattering with electrons is most prominent compared to radiation at other wavelengths. This means that at other wavelength (such as in the green light region, where $\lambda_{green} << \lambda_e$) the cross section of Compton scattering would be suppressed.

Problems

1. The diameter of an atomic nucleus is about 10×10^{-15} m. Suppose you wanted to study the diffraction of photons by nuclei. What energy of photons would you choose? Why? (Krane, Question 1, pg 79)

Solution

Diffraction of light by the nucleus occurs only when the wavelength of the photon is smaller or of the order of the size of the nucleus, $\lambda \sim D$ (D = diameter of the nucleus). Hence, the minimum energy of the photon would be $E = hc/\lambda \sim hc/D \sim 120$ MeV.

2. Photons from a Light Bulb (Cutnell, pg884)
In converting electrical energy into light energy, a sixty-watt incandescent light bulb operates at about 2.1% efficiency. Assuming that all the light is green light (vacuum wavelength 555 nm), determine the number of photons per second given off by the bulb.

Reasoning

The number of photons emitted per second can be found by dividing the amount of light energy emitted per second by the energy E of one photon. The energy of a single photon is $E = hf$. The frequency of the photon is related to its wavelength λ by $\nu = c/\lambda$.

SESSI 04/05/ TUTORIAL 3

Solution

At an efficiency of 2.1%, the light energy emitted per second by a sixty-watt bulb is (0.021)(60.0 J/s)=1.3 J/s. The energy of a single photon is

$E = hc/\lambda = (6.63\times10^{-34}\text{Js})(3\times10^{8}\text{ m/s})/555\times10^{-9}\text{ nm} = 3.58\times10^{-19}\text{ J}$

Therefore, number of photons emitted per second =
1.3 J/s/ (3.58×10^{-19} J/photon) = 3.6×10^{18} photon per second

3. Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (a) Find the maximum KE of the photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2? (Beiser, pg. 63)

Solution

(a) The energy of the photons is, $E_p = hc/\lambda$ = 3.5eV. The work function of potassium is 2.2 eV. So,
KE = $h\nu - \phi$ = 3.5 eV - 2.2 eV = 5.68×10^{-19} J

(b) The photon energy in joules is 5.68×10^{-19} J. Hence the number of photons that reach the surface per second is
$n_p = (E/t)/E_p = (E/A)(A)/E_p$
$= (1.00\text{ W/m}^2)(1.00\times10^{-4}\text{ m}^2)/5.68\times10^{-19}\text{ J}$
$= 1.76\times10^{14}$ photons/s
The rate at which photoelectrons are emitted is therefore $n_e = (0.0050)n_p = 8.8\times10^{11}$ photoelectrons/s

4. The work function for tungsten metal is 4.53 eV. (a) What is the cut-off wavelength for tungsten? (b) What is the maximum kinetic energy of the electrons when radiation of wavelength 200.0 nm is used? (c) What is the stopping potential in this case? (Krane, pg. 69)

Solution

(a) The cut-off frequency is given by $\lambda_c = \frac{hc}{\phi} = \frac{1240\text{eV}\cdot\text{nm}}{200\text{nm}} = 274\text{nm}$, in the uv region

(b) At the shorter wavelength,

$$K_{\max} = h\frac{c}{\lambda} - \phi = \frac{1240eV\cdot nm}{200nm} - 4.52eV = 1.68eV$$

(c) The stopping potential is just the voltage corresponding to $K_{\max}$:

$$V_s = K_{\max}/e = \frac{1.68eV}{e} = 1.68\text{V}$$

5. X-rays of wavelength 10.0 pm (1 pm = 10^{-12} m) are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons. (Beiser, pg. 75)

Solution

(a) The Compton shift is given by $\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\varphi)$, and so $\lambda' = \lambda + \lambda_c(1-\cos 45^o) = 10.0\,pm + 0.293\lambda_c = 10.7$ pm

(b) $\Delta\lambda$ is a maximum when $1-\cos\varphi$ = 2, in which case, $\Delta\lambda = \lambda + 2\lambda_c$ = 10.0 pm + 4.9 pm = 14.9 pm

(c) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$KE_{max} = h(\nu - \nu') = hc(\frac{1}{\lambda} - \frac{1}{\lambda'})$ =40.8 eV

6. Gautreau and Savin, page 70, Q 9.28

A photon of wavelength 0.0030 Å in the vicinity of a heavy nucleus produces an electron-positron pair. Determine the kinetic energy of each of the particles if the kinetic energy of the positron is twice that of the electron.

Solution:
From (total relativistic energy before) = (total relativistic energy after),

$$\frac{hc}{\lambda} = 2m_ec^2 + K_+ + K_- = 2m_ec^2 + 3K_-$$

$$\frac{12.4\times10^{-3}\,\text{MeV}\cdot\text{Å}}{0.0030\,\text{Å}} = 2(0.511\text{MeV})\cdot 3K_-$$

$$K_- = 1.04\text{MeV};\ K_+ = 2K_- = 2.08\text{MeV}$$

7. Gautreau and Savin, page 71, Q 9.32

Annihilation occurs between an electron and positron at rest, producing three photons. Find the energy of the third photon of the energies of the two of the photons are 0.20 MeV and 0.30 MeV.

Solution:
From conservation of energy, 2(0.511 MeV) = 0.20 MeB + 0.30 MeV = E_3 or E_3 = 0.522 MeV

8. Gautreau and Savin, page 71, Q 9.33
How Many positrons can a 200 MeV photon produce?

Solution:
The energy needed to produce an electron-positron pair at rest is twice the rest energy of an electron, or 1.022 MeV. Therefore, Maximum number of positrons =

$$(200\text{ MeV})\left(\frac{1\text{ pair}}{1.022\text{ MeV}}\right)\left(1\frac{\text{positron}}{\text{pair}}\right) = 195\text{ positrons}$$

Tutorial 4
Wave particle duality, de Brolie postulate, Heisenberg Uncertainty principle

Conceptual Questions

1. **What difficulties does the uncertainty principle cause in trying to pick up an electron with a pair of forceps? (Krane, Question 4, pg. 110)**

ANS
When the electron is picked up by the forceps, the position of the electron is ``localised' (or fixed), i.e. $\Delta x$ = 0. Uncertainty principle will then render the momentum to be highly uncertainty. In effect, a large $\Delta p$ means the electron is ``shaking'' furiously against the forceps' tips that tries to hold the electron ``tightly''.

2. **An electron and a proton both moving at nonrelativistic speeds have the same de Broglie wavelength. Which of the following are also the same for the two particles?**
(a) speed (b) kinetic energy (c) momentum
(d) frequency

ANS
(c). According to de Broglie's postulate, $\lambda = \frac{h}{p} = \frac{h}{mv}$, two particles with the same de Broglie wavelength will have the same momentum $p = mv$. If the electron and proton have the same momentum, they cannot have the same speed (a) because of the difference in their masses. For the same reason, because $K = p^2/2m$, they cannot have the same kinetic energy (b). Because the particles have different kinetic energies, Equation $\lambda = \frac{h}{p} = \frac{h}{mv}$ tells us that the particles do not have the same frequency (d).

3. **The location of a particle is measured and specified as being exactly at x = 0, with *zero* uncertainty in the x direction. How does this affect the uncertainty of its velocity component in the y direction?**
(a) It does not affect it.
(b) It makes it infinite.
(c) It makes it zero.

ANS
(a). The uncertainty principle relates uncertainty in position and velocity along the same axis. The zero uncertainty in

position along the x axis results in infinite uncertainty in its velocity component in the x direction, but it is unrelated to the y direction.

4. **You use a large potential difference to accelerate particles from rest to a certain kinetic energy. For a certain potential difference, the particle that will give you the highest resolution when used for the application as a microscope will be a) an electron, b) a proton, c) a neutron, or d) each particle will give you the same resolution under these circumstances. (Serway QQ)**

ANS

(b). The equation $\lambda = h/(2mq\Delta V)^{1/2}$ determines the wavelength of a particle. For a given potential difference and a given charge, the particle with the highest mass will have the smallest wavelength, and can be used for a microscope with the highest resolution. Although neutrons have the highest mass, their neutral charge would not allow them to be accelerated due to a potential difference. Therefore, protons would be the best choice. Protons, because of their large mass, do not scatter significantly off the electrons in an atom but can be used to probe the structure of the nucleus.

5. **Why was the demonstration of electron diffraction by Davisson and Germer and important experiment? (Serway, Q19, pg. 1313)**

ANS

The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

6. **If matter has wave nature why is this wave-like character not observed in our daily experiences? (Serway, Q21, pg. 1313)**

ANS

Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength and does not exhibit quantum behavior.

Problems

1. **Beiser, pg. 100, example 3.3**

An electron has a de Broglie wavelength of 2.00 pm. Find its kinetic energy and the phase and the group velocity of its de Broglie waves.

Solution

(a) First calculate the pc of the electron

$pc = hc/\lambda$ = 1.24 keV.nm / 2.00 pm = 620 keV

The rest energy of the electron is E_0=511 keV, so the KE of the electron is

$KE = E - E_0 = [E_0^2-(pc)^2]^{1/2} - E_0 = \ldots$ 292 keV

(b) The electron's velocity is to be found from

$$\gamma = E/E_0 = (KE+E_0)/E_0 = 803/511 = 1.57$$

$$\frac{1}{\gamma^2} = 1-\frac{v^2}{c^2} = 0.405 \Rightarrow \frac{v^2}{c^2} = 1-0.405 = 0.595$$

$$\Rightarrow v = 0.771c$$

Hence, the phase velocity is $v_p = \dfrac{c^2}{v} = \dfrac{c^2}{0.771c} = 1.29c$

The group velocity is $v_g = v = 0.771c$

2. **Find the de Broglie wave lengths of (a) a 46-g ball with a velocity of 30 m/s, and (b) an electron with a velocity of 10^7 m/s (Beiser, pg. 92)**

Solution

(a) Since $v << c$, we can let $m = m_o$. Hence

$\lambda = h/mv = 6.63\times10^{-34}$ Js/(0.046 kg)(30 m/s)
$= 4.8\times10^{-34}$ m

The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behaviour.

(b) Again $v << c$, so with $m = m_o = 9.1\times10^{-31}$ kg, we have

$\lambda = h/mv = 6.63\times10^{-34}$ Js/(9.1×10^{-31} kg)(10^7 m/s)
$= 7.3\times10^{-11}$ m

The dimensions of atoms are comparable with this figure - the radius of the hydrogen atom, for instance, is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behaviour.

3. **The de Broglie Wavelength (Cutnell, pg. 897)**
An electron and a proton have the same kinetic energy and are moving at non-relativistic speeds. Determine the ratio of the de Broglie wavelength of the electron to that of the proton.

ANS
Using the de Broglie wavelength relation $p = h/\lambda$ and the fact that the magnitude of the momentum is related to the kinetic energy by $p = (2mK)^{1/2}$, we have

$$\lambda = h/p = h/(2mK)^{1/2}$$

Applying this result to the electron and the proton gives

$$\lambda_e/\lambda_p = (2m_pK)^{1/2}/(2m_eK)^{1/2}$$
$$= (m_p/m_e)^{1/2} = (1.67\times10^{-27}\,\text{kg}/9.11\times10^{-31}\,\text{kg})^{1/2} = 42.8$$

As expected, the wavelength for the electron is greater than that for the proton.

4. **Find the kinetic energy of a proton whose de Broglie wavelength is 1.000 fm = 1.000×10^{-15} m, which is roughly the proton diameter (Beiser, pg. 92)**

ANS
A relativistic calculation is needed unless pc for the proton is much smaller than the proton rest mass of E_o = 0.938 GeV.

So we have to first compare the energy of the de Broglie wave to E_o:

$E = pc = \frac{hc}{\lambda} = \frac{1242 eV\cdot nm}{10^{-6} nm} = 1.24$ GeV, c.f. E_o = 0.938 GeV. Since the energy of the de Broglie wave is larger than the rest mass of the proton, we have to use the relativistic kinetic energy instead of the classical $K = p^2/2m$ expression.

The total energy of the proton is
$E = \sqrt{E_0^2 + (pc)^2} = \sqrt{(0.938\,GeV)^2 + (1.24\,GeV)^2} = 1.555$ GeV.
The corresponding kinetic energy is
KE = $E - E_o$ = (1.555 - 0.938) GeV = 0.617 GeV = 617 MeV

5. **A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom. (Beiser, pg 114)**

ANS
Here we find that with $\Delta x = 5.3\times10^{-11}$ m.
$\Delta p \geq \frac{\hbar}{2\pi} = 9.9\times10^{-25}$ Ns.
An electron whose momentum is of this order of magnitude behaves like a classical particle, an its kinetic energy is $K = p^2/2m \geq (9.9\times10^{-25}\text{ Ns})^2/2\times9.110^{-31}$ kg = 5.4×10^{-19} J, which is 3.4 eV. The kinetic energy of an electron in the lowest energy level of a hydrogen atom is actually 13.6 eV.

6. **A measurement established the position of a proton with an accuracy of $\pm1.00\times10^{-11}$m. Find the uncertainty in the proton's position 1.00 s later. Assume $v << c$. (Beiser, pg. 111)**

ANS
Let us call the uncertainty in the proton's position Δx_0 at the time t = 0. The uncertainty in its momentum at this time is therefore $\Delta p \geq \frac{\hbar}{2\Delta x_0}$. Since $v << c$, the momentum uncertainty is $\Delta p \geq \Delta(mv) = m_0\Delta v$ and the uncertainty in the proton's velocity is $\Delta v \geq \frac{\Delta p}{m_0} \geq \frac{\hbar}{2m_0\Delta x_0}$. The distance x of the proton covers in the time t cannot be known more accurately than $\Delta x \geq t\Delta v \geq \frac{\hbar t}{2m_0\Delta x_0}$. Hence Δx is inversely proportional to Δx_0: the more we know about the proton's position at t = 0 the les we know about its later position at t. The value of Δx at t = 1.00 s is $\Delta x \geq \frac{(1.054\times10^{-34} Js)(1.00s)}{2(1.672\times10^{-27} kg)(1.00\times10^{-11} m)} = 3.15\times10^3$ m. This is 3.15 km! What has happened is that the original wave group has spread out to a much wider one because the phase velocities of the component wave vary with wave number and a large range of wave numbers must have been present to produce the narrow original wave

7. **Broadening of spectral lines due to uncertainty principle: An excited atom gives up it excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time is radiates is**

1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon. (Beiser, pg. 115)

ANS

The photon energy is uncertain by the amount $\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054\times10^{-34} Js}{2(1.0\times10^{-8} s)} = 5.3\times10^{-27}$ J. The corresponding uncertainty in the frequency of light is $\Delta\nu = \frac{\Delta E}{h} \geq 8\times10^{6}$Hz. This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom. As a result, the radiation from a group of excited atoms does not appear with the precise frequency ν. For a photon whose frequency is, say, 5.0×10^{14} Hz, $\frac{\Delta\nu}{\nu} = 1.6\times10^{-8}$.

Tutorial 5
Atomic models

Conceptual Questions

1. **What is the ONE essential difference between the Rutherford model and the Bohr's model? (My own question)**

   ANS
   Rutherford's model is a classical model, in which EM wave will be radiated rendering the atom to collapse. Whereas the Bohr's model is a semi-classical model in which quantisation of the atomic orbit happens.

2. **Conventional spectrometers with glass components do not transmit ultraviolet light ($\lambda \lesssim$ 380 nm). Explain why non of the lines in the Lyman series could be observed with a conventional spectrometer. (Taylor and Zafiratos, pg. 128)**

   ANS

   For Lyman series, n_f = 1. According to $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, the wavelength corresponding to n_i = 2 in the Lyman series is predicted to be $\lambda = \frac{4}{3R_\infty} = \frac{4}{3(109{,}737\text{cm}^{-1})}$=121.5 nm. Similarly, for n_i = 3, one finds that λ=102 nm, and inspection of $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ shows that the larger we take n, the smaller the corresponding wavelength. Therefore, all lines in the Lyman series lie well into the ultraviolet and are unobservable with a conventional spectrometer.

3. **Does the Thompson model fail at large scattering angles or at the small scattering angle? Why? (Krane, Questions 1, pg. 173)**

   ANS
   Thompson model fails at large angle (but is consistent with scattering experiments at small angle). Thompson model predicts that the average scattered angle is given by a small value of θ_{ave} ~ 1°. However, in the experiment, alpha particles are observed to be scattered at angle in excess of 90°. This falsifies Thompson model at large angle.

4. **In which Bohr orbit does the electron have the largest velocity? Are we justified in treating the electron non-relativistically? (Krane, Questions 6. pg. 174)**

ANS

The velocity in an orbit n is given by $v = h/2\pi mnr_0$, which means that the velocity is inversely proportional to the n number. Hence the largest velocity corresponds to the $n = 1$ state,

$v(n=1)/c = h/2c\pi mr_0$

$= 6.63\times10^{-34}/2\pi(9.1\times10^{-31})(0.53\times10^{-10})/c$

$= 0.007.$

Hence, nonrelativistic treatment is justified.

5. **How does a Bohr atom violate the $\Delta x\Delta p \geq \frac{\hbar}{2}$ uncertainty relation? (Krane, Question 11, pg. 174)**

ANS

The uncertainty relation in the radial direction of an electron in a Bohr orbit is $\Delta r\Delta p_r \geq \frac{\hbar}{2}$. However, in the Bohr model, the Bohr orbits are assumed to be precisely known $(=r_n = n^2 r_0)$ for a given n. This tantamount to $\Delta r = 0$, which must render the momentum in the radial direction to become infinite. But in the Bohr atom the electron does not have such radial motion caused by this uncertainty effect. So in this sense, the discrete Bohr orbit violates the uncertainty relation $\Delta x\Delta p \geq \frac{\hbar}{2}$.

Problem

1. **If we assume that in the ground of the hydrogen the position of the electron along the Bohr orbit is not known and not knowable, then the uncertainty in the position is about $\Delta x \approx 2r_0 = 10^{-10}$ m, (a) what is the magnitude of the momentum of the electron at the ground state? (b) What is the corresponding quantum uncertainty in the momentum? (Ohanian, pg. 152)**

ANS

(a) Angular momentum, $|L| \equiv |p|r = n\hbar$. Hence, in the ground state, $|p| = \hbar/r_0 = 2.1\times10^{-24}$ Ns

(b) $\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2(2r_0)} = 5.3\times10^{-25}$ Ns.

2. **Serway and Mosses, Problem 13(a), page 148**
What value of n is associated with the Lyman series line in hydrogen whose wavelength is 102.6 nm?

Solution:

$$\lambda = 102.6 \text{ nm}; \ \frac{1}{\lambda} = R\left(1-\frac{1}{n^2}\right) \Rightarrow n = \frac{R}{\left(R-\frac{1}{\lambda}\right)^{1/2}} = \frac{R}{\left(R-\frac{1}{102.6\times10^{-9}\text{ m}}\right)^{1/2}} = 2.99 \approx 3$$

3. **Serway and Moses, Problem 22**
Find the potential energy and kinetic energy of an electron in the ground state of the hydrogen atom.

Solution:

$E = K + U = \frac{mv^2}{2} - \frac{ke^2}{r}$. But $\frac{mv^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r}$. Thus $E = \left(\frac{1}{2}\right)\left(\frac{-ke^2}{r}\right) = \frac{U}{2}$, so $U = 2E = 2(-13.6 \text{ eV}) = -27.2 \text{ eV}$ and $K = E - U = -13.6 \text{ eV} - (-27.2 \text{ eV}) = 13.6 \text{ eV}$.

4. **Serway and Moses, Problem 21**
Calculate the longest and shortest wavelengths for the Paschen series. (b) Determine the photon energies corresponding to these wavelengths.

Solution

(a) For the Paschen series; $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$; the maximum wavelength corresponds to $n_i = 4$, $\frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$; $\lambda_{max} = 1\,874.606$ nm. For minimum wavelength, $n_i \to \infty$, $\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$; $\lambda_{min} = \frac{9}{R} = 820.140$ nm.

(b) $\frac{hc}{\lambda_{min}} = \frac{\left(\frac{hc}{1874.606 \text{ nm}}\right)}{1.6\times10^{-19} \text{ J/eV}} = 0.662\,7$ nm, $\frac{hc}{\lambda_{min}} = \frac{\left(\frac{hc}{820.140 \text{ nm}}\right)}{1.6\times10^{-19} \text{ J/eV}} = 1.515$ nm

5. **Hydrogen atoms in states of high quantum number have been created in the laboratory and observed in space. (a) Find the quantum number of the Bohr orbit in a hydrogen atom whose radius is 0.0199 mm. (b) What is the energy of a hydrogen atom in this case? (Beiser, pg. 133)**

Solution

(a) From $r_n = n^2 r_0$, we have $n = \sqrt{\frac{r_n}{r_0}} = \sqrt{\frac{0.0100\times10^{-3}}{5.3\times10^{-11}}} = 434$

(b) From $E_n = -\frac{13.6}{n^2}$eV, we have $E_n = -\frac{13.6}{434^2}$eV = -0.000072 eV. Such an atom would obviously be extremely fragile and be easily ionised (compared to the kinetic energy of the atom at temperature T, $kT \sim (1.38\times10^{-23}$ J/K$)\times(300$ K$)$ $=0.03$ eV)

6. **(a) Find the frequencies of revolution of electrons in n = 1 and n = 3 Bohr orbits. (b) What is the frequency of the photon emitted when an electron in the n = 2 orbit drops to an n = 1 orbit? (c) An electron typically spends about 10^{-8} s in an excited state before it drops to a lower state by emitting a photon. How many revolutions does an electron in an n = 2 Bohr orbit make in 10^{-8} s? (Beiser, pg. 137)**

Solution

(a) Derive the frequency of revolution from scratch: Forom Bohr's postulate of quantisation of angular momentum, $L = (mv)r = nh/2\pi$, the velocity is related to the radius as $v = nh/2mr\pi$. Furthermore, the quantised radius is given in terms of Bohr's radius as $r_n = n^2 r_0$. Hence, $v = h/2\pi mnr_0$.

The frequency of revolution $f = 1/T$ (where T is the period of revolution) can be obtained from $v = 2\pi r/T = 2\pi n^2 r_0 f$. Hence, $f = v/2\pi r = (h/2\pi mnr_0)/2\pi r = h/4\pi^2 mn^3(r_0)^2$.

For n = 1, $f_1 = h/4\pi^2 m(r_0)^2 = 6.56\times10^{15}$ Hz.
For n = 2, $f_2 = h/4\pi^2 m(2)^3(r_0)^2 = 6.56\times10^{15}/8$ Hz $= 8.2\times10^{14}$.

(b) $\nu = \frac{\Delta E}{h} = \frac{13.6eV}{h}\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3c}{4}\frac{13.6eV}{1242eV\cdot nm} = 0.00821\times(3\times10^8 m/s)/10^{-9}m$

$= 2.463\times10^{15}$ Hz. The frequency is intermediate between f_1 and f_2.

(c) The number of revolutions the electron makes is $N = f_2\Delta t = (8.2\times10^{14})\times10^8 = 8.2\times10^{22}$ rev.

Special Relativity

Conceptual Questions

1) **The speed of light in water is c/n, where n = 1.33 is the index of refraction of water. Thus the speed of light in water is less than c. Why doesn't this violate the speed of light postulate?**

ANS
The constancy of light postulate only applies to light propagating in vacuum. So, a light propagating in a medium which is otherwise could still has a travelling speed other than c.

2) **What is the significance of the negative result of Michelson-Morley experiment?**

ANS
The negative result of the MM experiment contradicts with the prediction of the absolute frame (the Ether frame) of reference, in which light is thought to propagate with a speed c. In the Ether postulate, the speed of light that is observed in other initial reference frame (such as the Earth that is moving at some constant speed relative to the Absolute frame), according to the Galilean transformation, would be different than that of the Ether frame. In other words, the MM negative result provides the first empirical evidence to the constancy of light postulate by Einstein.

3) **Is it possible to have particles that travel at the speed of light?**

ANS
Particle travelling at the speed of light would have an infinite mass, as per $m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$. Hence it is physically not possible to supply infinite amount of energy to boost a particle from rest to the speed of light.

4) **What is the twin-paradox? What is the solution to the paradox?**

ANS
Refer to page 43-44, Krane.

PROBLEMS

1) **Space Travel (from Cutnell and Johnson, pg 861,863)**
Alpha Centauri, a nearby star in our galaxy, is 4.3 light-years away. If a rocket leaves for Alpha Centauri and travels at a speed of v = 0.95c relative to the Earth, (i) by how much will the passengers have aged, according to their own clock, when they reach their destination? ii) What is the distance between Earth and Alpha Centauri as measured by the passengers in the rocket? Assume that the Earth and Alpha Centauri are stationary with respect to one another.

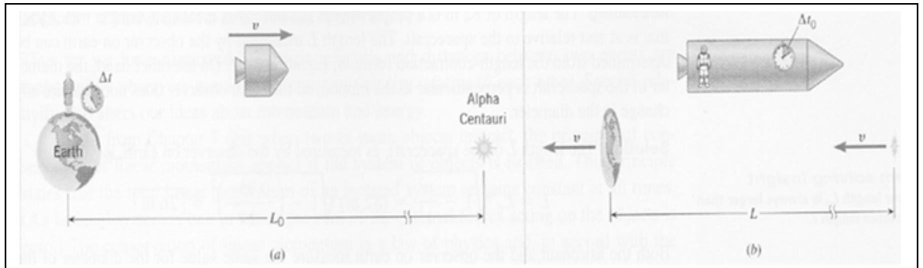


Figure: (a) As measured by an observer on the earth, the distance to Alpha Centauri is L_0, and the time required to make the trip is Δt. (b) According to the passenger on the spacecraft, the earth and Alpha Centauri move with speed v relative to the craft. The passenger measures the distance and time of the trip to be L and Δt_0 respectively, both quantities being less than those in part (a).

Reasoning

The two events in this problem are the departure from Earth and the arrival at Alpha Centauri. At departure, Earth is just outside the spaceship. Upon arrival at the destination, Alpha Centauri is just outside. Therefore, relative to the passengers, the two events occur at the same place - namely, 'just outside the spaceship. Thus, the passengers measure the proper time interval Δt_0 on their clock, and it is this interval that we must find. For a person left behind on Earth, the events occur at different places, so such a person measures the dilated time interval Δt rather than the proper time interval. To find Δt we note that the time to travel a given distance is inversely proportional to the speed. Since it takes 4.3 years to traverse the distance between earth and Alpha Centauri at the speed of light, it would take even longer at the slower speed of $v = 0.95c$. Thus, a person on earth measures the dilated time interval to be Δt = (4.3 years)/0.95 = 4.5 years. This value

SESSI 03/04/ TUTORIAL 1

can be used with the time-dilation equation to find the proper time interval Δt_0.

Solution

Using the time-dilation equation, we find that the proper time interval by which the Passengers judge their own aging is

$\Delta t_0 = \Delta t\sqrt{(1-v^2/c^2)}$ = 4.5 years $\sqrt{(1-0.95^2)}$ = 1.4 years.

Thus, the people aboard the rocket will have aged by only 1.4 years when they reach Alpha Centauri, and not the 4.5 years an earthbound observer has calculated.

Both the earth-based observer and the rocket passenger agree that the relative speed between the rocket and earth is $v = 0.95c$. Thus, the Earth observer determines the distance to Alpha Centauri to be $L_0 = v\Delta t = (0.95c)(4.5 \text{ years}) = 4.3$ light-years. On the other hand, a passenger aboard the rocket finds the distance is only $L = v\Delta t_0 = (0.95c)(1.4 \text{ years}) = 1.3$ light-years. The passenger, measuring the shorter time, also measures the shorter distance - length contraction.

Problem solving insight

In dealing with time dilation, decide which interval is the proper time interval as follows: (1) Identify the two events that define the interval. (2) Determine the reference frame in which the events occur at the same place; an observer at rest in this frame measures the proper time interval Δt_0.

2) **The Contraction of a Spacecraft (Cutnell, pg 863)**

An astronaut, using a meter stick that is at rest relative to a cylindrical spacecraft, measures the length and diameter of the spacecraft to be 82 and 21 m respectively. The spacecraft moves with a constant speed of $v = 0.95c$ relative to the Earth. What are the dimensions of the spacecraft, as measured by an observer on Earth?

Reasoning

The length of 82 m is a proper length *Lo* since it is measured using a meter stick that is at rest relative to the spacecraft. The length *L* measured by the observer on Earth can be determined from the length-contraction formula. On the other hand, the diameter of the spacecraft is perpendicular to the motion, so the Earth observer does not measure any change in the diameter.

Solution

The length L of the spacecraft, as measured by the observer on Earth, is

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = 82m\sqrt{1-\frac{(0.95c)^2}{c^2}} = 26\ \text{m}$$

Both the astronaut and the observer on Earth measure the same value for the diameter of the spacecraft: Diameter = 21 m

Problem solving insight The proper length L_0 is always larger than the contracted length L.

3) ***Additional problem 36, Cutnell pg. 879.***
Two spaceship A and B are exploring a new planet. Relative to this planet, spaceship A has a speed of 0.60*c*, and spaceship B has a speed of 0.80*c*. What is the ratio D_A/D_B of the values for the planet's diameter that each spaceship measures in a direction that is parallel to its motion?

Solution

Length contraction occurs along the line of motion, hence both spaceship observe length contraction on the diameter of the planet. The contracted length measures by a moving observer is inversely proportional to the Lorentz factor γ. Hence,

$$\frac{L_A}{L_B} = \frac{\gamma_B}{\gamma_A} = \sqrt{\frac{1-\left(\frac{v_A}{c}\right)^2}{1-\left(\frac{v_B}{c}\right)^2}} = \sqrt{\frac{1-(0.6)^2}{1-(0.8)^2}} = 4/3.$$

4) **The Relativistic Momentum of a High-Speed Electron (Cutnell, pg 865)**
The particle accelerator at Stanford University is three kilometers long and accelerates electrons to a speed of 0.999 999 999 7*c*, which is very nearly equal to the speed of light. Find the magnitude of the relativistic momentum of an electron that emerges from the accelerator, and compare it with the non-relativistic value.

SESSI 03/04/ TUTORIAL 1

Reasoning and Solution

The magnitude of the electron's relativistic momentum can be obtained from $p = \gamma m_0 v = 1\times10^{-17}$ Ns, where $m_0 = 9.1\times10^{-31}$ kg, $v\gamma = \frac{0.999999997c}{\sqrt{1-\frac{(0.999999997c)^2}{c^2}}} = 1.09989\times10^{13}$ m/s. The relativistic momentum is greater than the non-relativistic momentum by a factor of $\gamma = \frac{1}{\sqrt{1-\frac{(0.999999997c)^2}{c^2}}} = 4\times10^4$.

5) **The Energy Equivalent of a Golf Ball (Cutnell, pg 866)**
A 0.046-kg golf ball is lying on the green. (a) Find the rest energy of the golf ball. (b) If this rest energy were used to operate a 75-W light bulb, for how many years could the bulb stay on?

Reasoning

The rest energy E_0 that is equivalent to the mass m of the golf ball is found from the relation $E_0 = mc^2$. The power used by the bulb is 75 W, which means that it consumes 75 J of energy per second. If the entire rest energy of the ball were available for use, the bulb could stay on for a time equal to the rest energy divided by the power.

Solution

(a) The rest energy of the ball is

$E_0 = mc^2 = (0.046\ \text{kg})(3.0\times10^8\ \text{m/s})^2 = 4.1\times10^{15}$ J

(b) This rest energy can keep the bulb burning for a time t given by

t = Rest energy/ Power = 4.1×10^{15} J/75 W = 5.5×10^{13} s = 1.7 million years!

6) **A High-Speed electron (Cutnell pg. 867)**
An electron (mass = 9.1×10^{-31} kg) is accelerated to a speed of 0.9995*c* in a particle accelerator. Determine the electron's (a) rest energy, (b) total energy, and (c) kinetic energy in MeV

(a) $E_0 = mc^2 = 9.109\times10^{-31}kg\times(3\times10^8)^2 m/s = 8.19\times10^{-14}J = 0.51MeV$

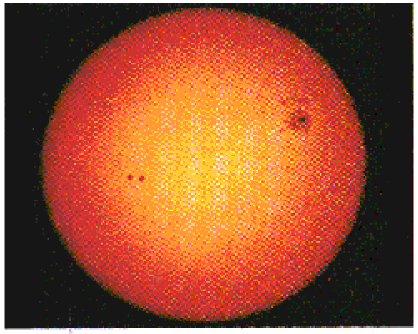
(b) Total energy of the traveling electron,

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0.51MeV}{\sqrt{1-0.995^2}} = 16.2MeV$$

(c) The kinetic energy = $E - E_0$ = 15.7 MeV

7) **The Sun Is Losing Mass (Cutnell, pg 868)**

The sun radiates electromagnetic energy at the rate of 3.92×10^{26} W. (a) What is the change in the sun's mass during each second that it is radiating energy? (b) The mass of the sun is 1.99×10^{30} kg. What fraction of the sun's mass is lost during a human lifetime of 75 years?



Visible light image.

Reasoning

Since a W = I J/s the amount of electromagnetic energy radiated during each second is 3.92×10^{26} J. Thus, during each second, the sun's rest energy decreases by this amount. The change ΔE_0 in the sun's rest energy is related to the change Δm in its mass by $\Delta E_0 = \Delta m\ c^2$.

Solution

(a) For each second that the sun radiates energy, the change in its mass is

$\Delta m = \Delta E_0/c^2 = 3.92\times10^{26}\ \text{J}/(3\times10^8\ \text{m/s})^2 = (4.36\times10^9)$ kg.

Over 4 billion kilograms of mass are lost by the sun during each second.

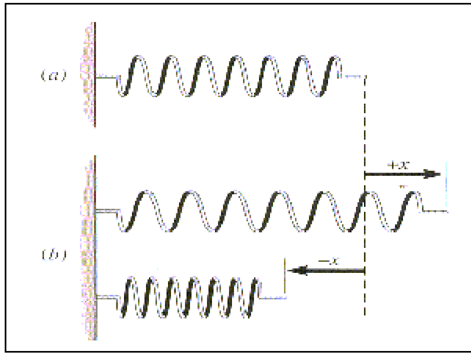
(b) The amount of mass lost by the sun in 75 years is

$\Delta m = (4.36\times10^9)\,\text{kg} \times (3.16\times10^7\ \text{s/year}) \times (75\ \text{years}) = 1\times10^{19}$ kg

Although this is an enormous amount of mass, it represents only a tiny fraction of the sun's total mass:

$$\Delta m/m = 1.0\times10^{19}\ \text{kg}/1.99\times10^{30}\ \text{kg} = 5.0\times10^{-12}$$

8) **Figure below shows the top view of a spring lying on a horizontal table. The spring is initially unstrained. Suppose that the spring is either stretched or compressed by an amount *x* from its unstrained length, as part (b) of the drawing shows. Has the mass of the spring changed? If so, is the change greater, smaller, or the same when the spring is stretched rather than compressed? (Cutnell, pg 868)**



(a) This spring is unstrained. (b) When the spring is either stretched or compressed by an amount *x*, it gains elastic potential energy and hence, mass.

Reasoning and Solution

Whenever a spring is stretched or compressed, its elastic potential energy changes. The elastic potential energy of an ideal spring is equal to $1/2kx^2$ where k is the spring constant and x is the amount of stretch or compression. Consistent with the theory of special relativity, any change in the total energy of a system, including a change in the elastic potential energy, is equivalent to a change in the mass of the system. Thus, the mass of a strained spring is greater than that of an unstrained spring. Furthermore, since the elastic potential energy depends on x^2, the increase in mass of the spring is the same whether it is compressed or stretched, provided the magnitude of x is the same in both cases. The increase is exceedingly small because the factor c^2 is so large.

9) **The Speed of a Laser Beam (Cutnell, pg 871)**
Figure below shows an intergalactic cruiser approaching a hostile spacecraft. The velocity of the cruiser relative to the spacecraft is v_{CS} = +0.7*c*. Both vehicles are moving at a constant velocity. The cruiser fires a beam of laser light at the enemy. The velocity of the laser beam relative to the cruiser is v_{LC} = +*c*. (a) What is the velocity of the laser beam v_{LS} relative to the renegades aboard the spacecraft? (b) At what velocity do the renegades aboard the spacecraft see the laser beam move away from the cruiser?

Figure 28.12 An intergalactic cruiser, closing in on a hostile spacecraft, fires a beam of laser light.

Reasoning and Solution

(a) Since both vehicles move at a constant velocity, each constitutes an inertial reference frame. According to the speed of light postulate, all observers in inertial reference frames measure the speed of light in a vacuum to be *c*. Thus, the renegades aboard the hostile spacecraft see the laser beam travel toward them at the speed of light, even though the beam is emitted from the cruiser, which itself is moving at seven-tenths the speed of light.

More formally, we can use Lorentz transformation of velocities to calculate v_{LS}. We will take the direction as +ve when a velocity is pointing from left to right. We can take view that the hostile spacecraft is at rest (as the stationary frame, O) while the cruiser is approaching it with velocity v_{CS} = + 0.7c (according to our choice of the sign). In this case, the cruiser is the moving frame, O'. The light beam as seen in the moving frame O' is v_{LC} = +c. We wish to find out what is the speed of this laser beam from O point of view, e.g. what v_{LS} is.

We may like to identify v_{LS}, v_{LC} and v_{CS} with the definitions used in the Lorentz formula: $u_x = \dfrac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}}$. In fact, a little contemplation would allow us to make the identification that, with our choice of frames (that the hostile spacecraft as the stationary frame): $v_{LC} \equiv u_{x'}$ = +c; $v_{CS} \equiv u$ = + 0.7c and $v_{LS} = u_x$ = the speed of laser beam as seen by the stationary frame O (the quantity we are seeking). Hence, we have

SESSI 03/04/ TUTORIAL 1

$$u_x = \frac{u'_x + u}{1 + \frac{u'_x u}{c^2}} \equiv v_{LS} = \frac{v_{LC} + v_{CS}}{1 + \frac{v_{LC} v_{CS}}{c^2}} = \frac{(+c) + (+0.7c)}{1 + \frac{(+c)(+0.7c)}{c^2}} = \frac{1.7c}{1.7c} = +c$$

, i.e. the laser beam is seen, from the view point of the hostile spacecraft, to be approaching it with a velocity +c (+ve means the velocity is from left to right).

(b) The renegades aboard the spacecraft see the cruiser approach them at a relative velocity of v_{CS} = +0.7c, and they also see the laser beam approach them at a relative velocity Of v_{LS} +c. Both these velocities are measured relative to the same inertial reference frame-namely, that of the spacecraft. Therefore, the renegades aboard the spacecraft see the laser beam move away from the cruiser at a velocity that is the difference between these two velocities, or +*c* - (+0.7*c*) = +0.3*c*. The relativistic velocity-addition formula, is not applicable here because both velocities are measured relative to the same inertial reference frame (the spacecraft's reference frame). The relativistic velocity-addition formula can be used only when the velocities are measured relative to different inertial reference frames.

10) **Mass and Energy (Cutnell, pg 873)**
The rest energy E_0 and the total energy *E* of three particles, expressed in terms of a basic amount of energy *E'* = 5.98 × 10^{-10} J, are listed in the table below. The speeds of these particles are large, in some cases approaching the speed of light. For each particle, determine its mass and kinetic energy.

Particle	Rest Energy	Total Energy
a	E'	2E'
b	E'	4E'
c	5E'	6E'

Concept Questions and Answers

Given the rest energies specified in the table, what is the ranking (largest first) of the masses of the particles?

Answer

The rest energy is the energy that an object has when its speed is zero. According to special relativity, the rest energy E_0 and the mass *m* are equivalent. Thus, the rest energy is directly proportional to the mass. From the table it can be seen that particles a and b have identical rest energies, so they have identical masses. Particle c has the greatest rest energy, so it has the greatest mass. The ranking of the masses, largest first, is c, then a and b.

What is the ranking (largest first) of the kinetic energies of the particles?

According to special relativity, the kinetic energy is the difference between the total energy E and the rest energy E_0, so KE = $E - E_0$. Therefore, we can examine the table and determine the kinetic energy of each particle in terms of E'. The kinetic energies of particles a, b, and c are, respectively, $2E' - E' = E'$, $4E' - E' = 3E'$, and $6E' - 5E' = E'$. The ranking of the kinetic energies, largest first, is b, then a and c.

Solution

(a) The mass of particle a can be found from its rest energy $E_0 = mc^2$. Since $E_0 = E'$ (see the table), its mass is $m_a = E'/c^2 = 5.98\text{x}10^{-10}$ J/(3x10^8 m/s)^2 = $6.64\text{x}10^{-27}$ kg

In a similar manner, we find that the masses of particles b and c are

$$m_b = 6.64\text{x}10^{-27}\text{ kg},\ m_c = 33.2\text{x}10^{-27}\text{ kg},$$

As expected, the ranking is $m_c > m_a = m_b$

(b) The kinetic energy KE of a particle is KE = $E - E_0$. For particle a, its total energy is $E = 2E'$ and its rest energy is $E_0 = E'$, so its kinetic energy is

$$KE_a = 2E' - E' = E' = 5.98\text{x}10^{-10}\text{ J}.$$

The kinetic energies of particles b and c can be determined in a similar fashion:

$$KE_b = 17.9\text{x}10^{-10}\text{ J},\ KE_c = 5.98\text{x}10^{-10}\text{ J}$$

As anticipated, the ranking is $KE_b > KE_a = KE_c$.

Tutorial 2
Preliminaries, Blackbody radiation, particle nature of waves

Conceptual Questions

1. **Explain in your own words the essential differences between the concept of wave from that of particle (Own question)**

ANS
Particle is finite in size and is localised both in space and time, whereas wave is not.

2. **What is ultraviolet catastrophe? What is the significance of it in the development of modern physics? (Own question)**

ANS
The classical theory explanation of the blackbody radiation by Rayleigh-Jeans fails in the limit $\lambda \to 0$ (or equivalently, when frequency $\to \infty$), i.e. $R(\lambda) \to \infty$ at $\lambda \to 0$. The failure prompted Planck to postulate that the energy of electromagnetic waves is quantised (via $\varepsilon = h\nu$) as opposed to the classical thermodynamics description ($\varepsilon = kT$). With Planck's postulate, radiation now has particle attributes instead of wave.

3. **What is the significance of the Compton wavelength of a given particle? What does the Compton wavelength of a particle mean to light that interacts with it? (Own question)**

ANS
The Compton wavelength (a characteristic constant depend solely on the mass of a given particle) characterises the length scale at which the quantum property (or wave) of a given particle starts to show up. In an interaction that is characterised by a length scale larger than the Compton wavelength, particle behaves classically. For interaction that occurs at a length scale comparable or smaller than the Compton wavelength, the quantum (or, wave) nature starts of the particle begins to take over from classical physics.

In a light-particle interaction, if the wavelength of the light is comparable to the Compton wavelength of the interacting particle, light displays quantum (granular/particle) behaviour rather than as a wave.

4. **How does the Rayleigh scattering could be explained by the Compton scattering relation, $\Delta\lambda = \lambda_c(1-\cos\theta)$? In the γ-ray region, which effect, Compton scattering or Rayleigh scattering is dominant? Explain. (Own question)**

ANS
Rayleigh scattering refers to unresolved peaks of the scattered x-ray, ie. $\Delta\lambda = 0$, which is due to the extremely small Compton wavelength of the whole ATOM, as seen by the x-ray $\lambda_c = h/Mc \to 0$, where M = mass of the atom (instead of $m_e << M$).

5. **Why doesn't the photoelectric effect work for free electron? (Krane, Question 7, pg 79)**

ANS (to be verified)
Essentially, Compton scattering is a two-body process. The free electron within the target sample (e.g. graphite) is a unbounded elementary particle having no internal structure that allows the photons to be `absorbed'. Only elastic scattering is allowed here.

Whereas PE effect is a inelastic scattering, in which the absorption of a whole photon by the atom is allowed due to the composite structure (the structure here refers the system of the orbiting electrons and nuclei hold together via electrostatic potential) of the atom. A whole photon is allowed to get absorbed by the atom in which the potential energy acts like a medium to transfer the energy absorbed from the photon, which is then `delivered' to the bounded electrons (bounded to the atoms) that are then `ejected' out as photoelectrons.

6. **How is the wave nature of light unable to account for the observed properties of the photoelectric effect? (Krane, Question 5, pg 79)**

ANS
See lecture notes

7. **In the photoelectric effect, why do some electrons have kinetic energies smaller than K_{max}? (Krane, Question 6, pg 79)**

ANS
By referring to $K_{max} = h\nu - \phi$, K_{max} corresponds to those electrons knocked loose from the surface by the incident photon whenever $h\nu > \phi$. Those below the surface required an energy greater than ϕ and so come off with less kinetic energy.

Problems

1. **The diameter of an atomic nucleus is about 10×10^{-15} m. Suppose you wanted to study the diffraction of photons by nuclei. What energy of photons would you choose? Why? (Krane, Question 1, pg 79)**

ANS
Diffraction of light by the nucleus occurs only when the wavelength of the photon is smaller or of the order of the size of the nucleus, $\lambda \sim D$ (D = diameter of the nucleus). Hence, the minimum energy of the photon would be $E = hc/\lambda \sim hc/D \sim 120$ MeV.

2. **How does the total intensity of thermal radiation vary when the temperature of an object is doubled? (Krane, Question 4, pg 79)**

ANS
Intensity of thermal radiation $I \propto T^4$. Hence, when T is double, ie. $T \to 2T$, $I \to I'(2)^4 = 16I$, i.e. the total intensity of thermal radiation increase by 16 times.

3. **Photons from a Light Bulb (Cutnell, pg884)**
In converting electrical energy into light energy, a sixty-watt incandescent light bulb operates at about 2.1% efficiency. Assuming that all the light is green light (vacuum wavelength 555 nm), determine the number of photons per second given off by the bulb.

Reasoning
The number of photons emitted per second can be found by dividing the amount of light energy emitted per second by the energy E of one photon. The energy of a single photon is $E = hf$. The frequency of the photon is related to its wavelength λ by $\nu = c/\lambda$.

Solution
At an efficiency of 2. 1%, the light energy emitted per second by a sixty-watt bulb is (0.021)(60.0 J/s)=1.3 J/s. The energy of a single photon is

$E = hc/\lambda$
$= (6.63\times10^{-34}\text{Js})(3\times10^{8}\text{ m/s})/555\times10^{-9}\text{ nm} = 3.58\times10^{-19}\text{ J}$

Therefore,
Number of photons emitted per second =
1.3 J/s/ 3.58×10^{-19} J/photon = 3.6×10^{18} photon per second

4. **Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (a) Find the maximum KE of the photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2? (Beiser, pg. 63)**

(a) The energy of the photons is, $E_P = hc/\lambda$ = 3.5eV. The work function of potassium is 2.2 eV. So, KE = $h\nu - \phi$ = 3.5 eV - 2.2 eV = 5.68×10^{-19} J
(b) The photon energy in joules is 5.68×10^{-19} J. Hence

the number of photons that reach the surface per second is

$n_p = (E/t)/E_p = (E/A)(A)/E_p$
$= (1.00\ W/m^2)(1.00\times10^{-4}\ m^2)/5.68\times10^{-19}\ J$
$= 1.76\times10^{14}$ photons/s

The rate at which photoelectrons are emitted is therefore

$n_e = (0.0050)n_p = 8.8\times10^{11}$ photoelectrons/s

5. **(Krane, pg. 62)**
(a) At what wavelength does a room-temperature (T = 20°C) object emit the maximum thermal radiation?
(b) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum?
(c) How many times as much thermal radiation does it emit at the higher temperature?

ANS

(a) Converting to absolute temperature, T = 293 K, and from Wien's displacement law,

$\lambda_{max}T = 2.898\times10^{-3}$ m·K

$\lambda_{max} = 2.898\times10^{-3}$ m·K/ 293K = 9.89 μm

(b) Taking the wavelength of red light to be =650 nm, we again use Wien's displacement law to find T:

$T = 2.898\times10^{-3}$ m·K/650×10^{-9} m = 4460 K

(c) Since the total intensity of radiation is proportional to T^4, the ratio of the total thermal emissions will be

$$\frac{T_2^4}{T_1^4} = \frac{4460^4}{293^4} = 5.37\times10^4$$

Be sure to notice the use of absolute (Kelvin) temperatures.

6. **The work function for tungsten metal is 4.53 eV. (a) What is the cut-off wavelength for tungsten? (b) What is the maximum kinetic energy of the electrons when radiation of wavelength 200.0 nm is used? (c) What is the stopping potential in this case? (Krane, pg. 69)**

ANS

(a) The cut-off frequency is given by

$\lambda_c = \frac{hc}{\phi} = \frac{1240\text{eV}\cdot\text{nm}}{200\text{nm}} = 274\text{nm}$, in the uv region

(b) At the shorter wavelength,

$$K_{max} = h\frac{c}{\lambda} - \phi = \frac{1240eV\cdot nm}{200nm} - 4.52eV = 1.68eV$$

(c) The stopping potential is just the voltage corresponding to K_{max}:

$$V_s = K_{max}/e = \frac{1.68eV}{e} = 1.68\,\text{V}$$

7. **X-rays of wavelength 10.0 pm (1 pm = 10^{-12} m) are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons. (Beiser, pg. 75)**

Solution

(a) The Compton shift is given by

$\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\varphi)$, and so

$\lambda' = \lambda + \lambda_c(1-\cos45^o) = 10.0\,pm + 0.293\lambda_c = 10.7$ pm

(b) $\Delta\lambda$ is a maximum when $1-\cos\varphi$ = 2, in which case,

$\Delta\lambda = \lambda + 2\lambda_c$ = 10.0 pm + 4.9 pm = 14.9 pm

(C) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$KE_{max} = h(\nu - \nu') = hc(\frac{1}{\lambda} - \frac{1}{\lambda'}) = 40.8$ eV

Tutorial 3
Matter waves, The Uncertainty Principle and Schrodinger Equation

Conceptual Questions

1. **What difficulties does the uncertainty principle cause in trying to pick up an electron with a pair of forceps? (Krane, Question 4, pg. 110)**

   ANS
   When the electron is picked up by the forceps, the position of the electron is ``localised' (or fixed), i.e. $\Delta x = 0$. Uncertainty principle will then render the momentum to be highly uncertainty. In effect, a large $\Delta p$ means the electron is ``shaking'' furiously against the forceps' tips that tries to hold the electron ``tightly''.

2. **Is it possible for v_{phase} to be greater than c? Can v_{group} be greater than c? (Krane, Question 12, pg. 111)**

   ANS
   Is it possible for v_{phase} to be greater than c but not so for v_{group}. This is because the group velocity is postulated to be associated with the physical particle. Since a physical particle (with mass) can never move greater than the speed of light (according to SR), so is v_{group}.

3. **Why is it important for a wave function to be normalised? Is an unrenomalised wave function a solution to the Schrodinger equation? (Krane, Question 2, pg. 143)**

   ANS
   Due to the probabilistic interpretation of the wave function, the particle must be found within the region in which it exists. Statistically speaking, this means that the probability to find the particle in the region where it exists must be 1. Hence, the square of the wave function, which is interpreted as the probably density to find the particle at an intervals in space, integrated over all space must be one in accordance with this interpretation. Should the wave function is not normalised, that would lead to the consequence that the probability to find the particle associated with the wave function in the integrated region where the particle is suppose to be in is not one, which violates the probabilistic interpretation of the wave function.

   A wave function that is not normalised is also a solution to the Schrodinger equation. However, in order for the wave function to be interpreted in accordance to the probabilistic interpretation (so that the wave function could has a physical meaning) it must be normalised.

SESSI 03/04/ TUTORIAL 3

4. How would the solution to the infinite potential well be different if the width of the well is extended from L to $L + x_0$, where x_0 is a nonzero value of x? How would the energies be different?
   (Krane, Question 7, pg. 143)

   ANS
   The form of the solutions to the wave functions inside the well remains the same. They still exist as stationary states described by the same sinusoidal functions, except that in the expressions of the observables, such as the quantised energies and the expectation values, the parameter L be replaced by $L + x_0$. For the quantised energies, they will be modified as per

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \rightarrow \frac{n^2\pi^2\hbar^2}{2m(L+x_0)^2}.$$

5. The infinite quantum well, with width L, as defined in the lecture notes is located between $x = 0$ and $x = L$. If we define the infinite quantum well to be located between $x = -L/2$ to $x = +L/2$ instead (the width remains the same, L), find the solution to the time-independent Schrodinger equation. Would you expect the normalised constant to the wave function and the energies be different than that discussed in the notes? Explain. **(Brehm and Mullin, pg. 234 - 237)**

   ANS
   By applying the boundary conditions that the solution must vanish at both ends, i.e. $\psi(x=-L/2)=\psi(x=L/2)=0$, the solution takes the form

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\cos\frac{n\pi x}{L} \ (odd\ \ n) \\ \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \ (even\ n) \end{cases} \text{ for } -\frac{L}{2} \le x \frac{L}{2}$$

   This question is tantamount to re-analyse the same physical system in a shifted coordinates, $x \rightarrow x - L/2$. The normalisation and energies shall remain unchanged under the shift of coordinate system $x \rightarrow x - L/2$. Both of these quantities depends only on the width of the well but not on the coordinate system used.

Problems

1. **Find the de Broglie wave lengths of (a) a 46-g ball with a velocity of 30 m/s, and (b) an electron with a velocity of 10^7 m/s (Beiser, pg. 92)**

ANS

(a) Since $v << c$, we can let $m = m_o$. Hence

$$\lambda = h/mv = 6.63\times10^{-34}\ \text{Js}/(0.046\ \text{kg})(30\ \text{m/s}) = 4.8\times10^{-34}\ \text{m}$$

The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behaviour.

(b) Again $v << c$, so with $m = m_o = 9.1\times10^{-31}$ kg, we have

$$\lambda = h/mv = 6.63\times10^{-34}\ \text{Js}/(9.1\times10^{-31}\ \text{kg})(10^7\ \text{m/s}) = 7.3\times10^{-11}\ \text{m}$$

The dimensions of atoms are comparable with this figure - the radius of the hydrogen atom, for instance, is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behaviour.

2. **The de Broglie Wavelength (Cutnell, pg. 897)**
An electron and a proton have the same kinetic energy and are moving at non-relativistic speeds. Determine the ratio of the de Broglie wavelength of the electron to that of the proton.

ANS

Using the de Broglie wavelength relation $p = h/\lambda$ and the fact that the magnitude of the momentum is related to the kinetic energy by $p = (2mK)^{1/2}$, we have

$$\lambda = h/p = h/(2mK)^{1/2}$$

Applying this result to the electron and the proton gives

$$\lambda_e/\lambda_p = (2m_pK)^{1/2}/(2m_eK)^{1/2} = (m_p/m_e)^{1/2} = (1.67\times10^{-27}\ \text{kg}/9.11\times10^{-31}\ \text{kg})^{1/2} = 42.8$$

As expected, the wavelength for the electron is greater than that for the proton.

3. **Find the kinetic energy of a proton whose de Broglie wavelength is 1.000 fm = 1.000×10^{-15} m, which is roughly the proton diameter (Beiser, pg. 92)**

ANS

A relativistic calculation is needed unless pc for the proton is much smaller than the proton rest mass of E_o = 0.938 GeV.

So we have to first compare the energy of the de Broglie wave to E_o:

$E = pc = \dfrac{hc}{\lambda} = \dfrac{1242 eV\cdot nm}{10^{-6} nm} = 1.24$ GeV, c.f. E_o = 0.938 GeV. Since the energy of the de Broglie wave is larger than the rest mass of the proton, we have to use the relativistic kinetic energy instead of the classical $K = p^2/2m$ expression.

The total energy of the proton is

$E = \sqrt{E_0^2 + (pc)^2} = \sqrt{(0.938\,GeV)^2 + (1.24\,GeV)^2} = 1.555$ GeV.

The corresponding kinetic energy is

KE = $E - E_o$ = (1.555 - 0.938) GeV = 0.617 GeV = 617 MeV

4. **An electron is in a box 0.10 nm across, which is the order of atomic dimensions. Find its permitted energies. (Beiser, pg. 106)**

ANS

Here $m = 9.1\times10^{-31}$ kg and $L = 1\times10^{-10}$ m, so that the permitted electron energies are

$E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2} = 6.0\times10^{-18} n^2$ J = $38n^2$ eV.

The minimal energy the electron can have is 38 eV, corresponding to $n = 1$. The sequence of energy levels continues with E_2 = 152 eV, E_3 = 342 eV, E_4 = 608 eV and so on. If such a box existed, the quantisation of a trapped electron's energy would be a prominent feature of the system. (And indeed energy quantisation is prominent in the case of an atomic electron.)

5. **A 10-g marble is in a box 10 cm across. Find its permitted energies.**

ANS

With $m = 1.0\times10^{-2}$ kg and $L = 1.0\times10^{-1}$ m,

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = 5.5\times10^{-64} n^2 \text{ J}$$

The minimum energy the marble can have is 5.5×10^{-64} J, corresponding to n = 1. A marble with this kinetic energy has a speed of only 3.3×10^{-31} m/s and therefore cannot be experimentally distinguished from a stationary marble. A reasonable speed a marble might have is, say, 1/3 m/s - which corresponds to the energy level of quantum number $n = 10^{30}$! The permissible energy levels are so very close together, then, that there is no way to determine whether the marble can take on only those energies predicted by $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ or any energy whatever. Hence in the domain of everyday experience, quantum effects are imperceptible, which accounts for the success of Newtonian mechanics in this domain.

6. **A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom. (Beiser, pg 114)**

ANS

Here we find that with $\Delta x = 5.3\times10^{-11}$ m.

$\Delta p \geq \frac{\hbar}{2\pi} = 9.9\times10^{-25}$ Ns.

An electron whose momentum is of this order of magnitude behaves like a classical particle, an its kinetic energy is $K = p^2/2m \geq (9.9\times10^{-25} \text{ Ns})^2/2\times9.110^{-31}$ kg $= 5.4\times10^{-19}$ J which is 3.4 eV. The kinetic energy of an electron in the lowest energy level of a hydrogen atom actually 13.6 eV.

7. **A measurement established the position of a proton with an accruracy of $\pm1.00\times10^{-11}$m. Find the uncertainty in the proton's position 1.00 s later. Assume $v << c$. (Beiser, pg. 111)**

ANS

Let us call the uncertainty in the proton's position Δx_0 at the time $t = 0$. The uncertainty in its momentum at this time is therefore

$\Delta p \geq \frac{\hbar}{2\Delta x_0}$. Since $v << c$, the momentum uncertainty is $\Delta p \geq \Delta(mv) = m_0\Delta v$ and the uncertainty in the proton's velocity is $\Delta v \geq \frac{\Delta p}{m_0} \geq \frac{\hbar}{2m_0\Delta x_0}$. The distance x of the proton covers in the time t cannot be known more accurately than

$\Delta x \geq t\Delta v \geq \frac{\hbar t}{2m_0\Delta x_0}$. Hence Δx is inversely proportional to Δx_0: the more we know about the proton's position at $t = 0$ the les we know about its later position at t. The value of Δx at $t = 1.00$ s is

$$\Delta x \geq \frac{(1.054\times10^{-34} Js)(1.00s)}{2(1.672\times10^{-27} kg)(1.00\times10^{-11} m)} = 3.15\times10^3 \text{ m}$$

This is 3.15 km! What has happened is that the original wave group has spread out to a much wider one because the phase velocities of the component wave vary with wave number and a large range of wave numbers must have been present to produce the narrow original wave

8. **Broadening of spectral lines due to uncertainty principle: An excited atom gives up it excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time is radiates is 1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon. (Beiser, pg. 115)**

ANS

The photon energy is uncertain by the amount

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054\times10^{-34} Js}{2(1.0\times10^{-8} s)} = 5.3\times10^{-27} \text{ J}$$

The corresponding uncertainty in the frequency of light is $\Delta\nu = \frac{\Delta E}{h} \geq 8\times10^6$ Hz.

This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom. As a result, the radiation from a group of excited atoms does not appear with the precise frequency ν. For a photon whose frequency is, say, 5.0×10^{14} Hz, $\frac{\Delta\nu}{\nu} = 1.6\times10^{-8}$. In practice, other phenomena such as the doppler effect contribute more ian this to the broadening of spectral lines.

9. **If we assume that in the ground of the hydrogen the position of the electron along the Bohr orbit is not known and not knowable, then the uncertainty in the position is about $\Delta x \approx 2r_0 = 10^{-10}$ m, (a) What is the magnitude of the momentum of the electron at the ground state? (b) What is the corresponding quantum uncertainty in the momentum? (Ohanian, pg. 152)**

ANS

(a) Angular momentum, $|L| \equiv |p|r = n\hbar$. Hence, in the ground state, $|p| = \hbar / r_0 = 2.1\times10^{-24}$ Ns

(b) $\Delta p_x \geq \dfrac{\hbar}{2\Delta x} = \dfrac{\hbar}{2(2r_0)} = 5.3\times10^{-25}$ Ns.

10. **Show that $\psi = A\exp(kx - \omega t)$ is solution to the time-independent Schrodinger equation.**

ANS

Taking the partial derivative of ψ wrp to x,

$$\frac{\partial^2}{\partial x^2}\psi = (ik)^2 A\exp(kx-\omega t) = -k^2\psi . \qquad (1)$$

The total energy of the particle is

$E = K + U = p^2/2m + U = \dfrac{\hbar^2 k^2}{2m} + U$

$\Rightarrow k^2 = \dfrac{2m(E-U)}{\hbar^2}$.

Hence, Eq. (1) becomes $\dfrac{\partial^2}{\partial x^2}\psi = -\dfrac{2m(E-U)}{\hbar^2}\psi$. This shows that $\psi = A\exp(kx-\omega t)$ is the solution to the Schrodinger equation.

11. **Consider a quantum particle trapped in an infinite well with width *a*. Assuming that the particle is in the ground state, calculate the expectation values of its position <x> and <x^2>. Obtain the uncertainty in its position, Δx, given**

SESSI 03/04/ TUTORIAL 3

by standard statistical definition, Δx = <x^2> - <x>^{2}. (Brehm and Mullin, pg.265)

ANS

The solution of the ground state wave function for a particle in an infinite box is $\psi_n(x) = \sqrt{\dfrac{2}{a}}\sin\dfrac{\pi x}{a}$.

$$\langle x\rangle = \int_{-\infty}^{\infty}\psi x\psi dx = \frac{2}{a}\int_0^a x\sin^2\frac{\pi x}{a}dx = \frac{2a}{\pi^2}\int_0^{\pi} y\sin^2 y dy$$

$$\int_0^{\pi} y\sin^2 y dy = \frac{y^2}{4} - \frac{y\sin 2y}{4} - \frac{\cos 2y}{8}\Bigg|_0^{\pi} = \frac{\pi^2}{4}$$

$\therefore \langle x\rangle = \dfrac{a}{2}$. Likewise,

$$\langle x^2\rangle = \int_{-\infty}^{\infty}\psi x^2\psi dx = \frac{2}{a}\int_0^a x^2\sin^2\frac{\pi x}{a}dx = \frac{2a^2}{\pi^3}\int_0^{\pi} y^2\sin^2 y dy$$

$$\int_0^{\pi} y^2\sin^2 y dy = \frac{x^3}{6} - \frac{x\cos 2x}{4} + \frac{(1-2x^2)\sin 2x}{8}\Bigg|_0^{\pi} = \frac{\pi^3}{6} - \frac{\pi}{4}$$

$$\therefore \langle x^2\rangle = \int_{-\infty}^{\infty}\psi x^2\psi dx = a^2\left(\frac{1}{3} - \frac{1}{2\pi^2}\right)$$

Δx = <x^2> - <x>^{2} = $a^2\left(\dfrac{1}{3} - \dfrac{1}{2\pi^2}\right) - \dfrac{a^2}{4} = a^2\left(\dfrac{1}{12} - \dfrac{1}{2\pi^2}\right)$

Tutorial 4
Atomic model

Conceptual Questions

1. **What is the ONE essential difference between the Rutherford model and the Bohr's model? (Own question)**

ANS
Rutherford's model is a classical model, in which EM wave will be radiated rendering the atom to collapse. Whereas the Bohr's model is a semi-classical model in which quantisation of the atomic orbit happens.

2. **Conventional spectrometers with glass components do not transmit ultraviolet light ($\lambda \lesssim$ 380 nm). Explain why non of the lines in the Lyman series could be observed with a conventional spectrometer. (Taylor and Zafiratos, pg. 128)**

ANS
For Lyman series, n_f = 1. According to $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$, the wavelength corresponding to n_i = 2 in the Lyman series is predicted to be $\lambda = \frac{4}{3R_\infty} = \frac{4}{3(109{,}737\text{cm}^{-1})}$=121.5 nm. Similarly, for n_i = 3, one finds that λ=102 nm, and inspection of $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$ shows that the larger we take n, the smaller the corresponding wavelength. Therefore, all lines in the Lyman series lie well into the ultraviolet and are unobservable with a conventional spectrometer.

3. **Does the Thompson model fail at large scattering angles or at the small scattering angle? Why? (Krane, Questions 1, pg. 173)**

ANS
Thompson model fails at large angle (but is consistent with scattering experiments at small angle). Thompson model predicts that the average scattered angle is

given by $\theta_{ave} = \sqrt{N} \cdot \frac{\pi}{4} \cdot z \left(\frac{Ze^2}{4\pi\varepsilon_0 R^3} \right) R^2 \cdot \left(\frac{1}{mv^2} \right)$. One can estimate the order of θ_{ave} in an atomic scattering experiment: R ~ 0.1 nm (a typical atomic radius), N ~ 10^4 (no. of collisions in the target metal foil), kinetic energy of the alpha particle, mv^2 ~ 10 MeV, z = 2 (charge of alpha particle); Z ~ 79 for gold. Putting in all figures, one expects that alpha particle is scattered only for a small scattering angle of θ_{ave} ~ 1°. However, in the experiment, alpha particles are observed to be scattered at angle in excess of 90°. This falsifies Thompson model at large angle.

4. **In which Bohr orbit does the electron have the largest velocity? Are we justified in treating the electron non-relativistically? (Krane, Questions 6. pg. 174)**

ANS
The velocity in an orbit n is given by $v = h/2\pi mnr_0$, which means that the velocity is inversely proportional to the n number. Hence the largest velocity corresponds to the n = 1 state,

$v(n=1)/c = h/2c\pi mr_0$
$= 6.63\times10^{-34}/2\pi(9.1\times10^{-31})(0.53\times10^{-10})/c$
$= 0.007.$

Hence, nonrelativistic treatment is justified.

5. **How does a Bohr atom violate the $\Delta x \Delta p \geq \frac{\hbar}{2}$ uncertainty relation? (Krane, Question 11, pg. 174)**

ANS
The uncertainty relation in the radial direction of an electron in a Bohr orbit is $\Delta r \Delta p_r \geq \frac{\hbar}{2}$. However, in the Bohr model, the Bohr orbits are assumed to be precisely known $(= r_n = n^2 r_0)$ for a given n. This tantamount to $\Delta r = 0$, which must render the momentum in the radial direction to become infinite. But in the Bohr atom the electron does not have such radial motion caused by this uncertainty effect. So in this

sense, the discrete Bohr orbit violates the uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2}$.

Problem

1. **Hydrogen atoms in states of high quantum number have been created in the laboratory and observed in space. (a) Find the quantum number of the Bohr orbit in a hydrogen atom whose radius is 0.0199 mm. (b) What is the energy of a hydrogen atom in this case? (Beiser, pg. 133)0**

ANS

(a) From $r_n = n^2 r_0$, we have $n = \sqrt{\frac{r_n}{r_0}} = \sqrt{\frac{0.0100\times10^{-3}}{5.3\times10^{-11}}} = 434$

(b) From $E_n = -\frac{13.6}{n^2}$eV, we have $E_n = -\frac{13.6}{434^2}$eV = -0.000072 eV. Such an atom would obviously be extremely fragile and be easily ionised (compared to the kinetic energy of the atom at temperature T, kT ~ $(1.38\times10^{-23}$ J/K) × (300 K) =0.03 eV)

2. **(a) Find the frequencies of revolution of electrons in n = 1 and n = 3 Bohr orbits. (b) What is the frequency of the photon emitted when an electron in the n = 2 orbit drops to an n = 1 orbit? (c) An electron typically spends about 10^{-8} s in an excited state before it drops to a lower state by emitting a photon. How many revolutions does an electron in an n = 2 Bohr orbit make in 10^{-8} s? (Beiser, pg. 137)**

ANS

(a) Derive the frequency of revolution from scratch: Forom Bohr's postulate of quantisation of angular momentum,

$L = (mv)r = nh/2\pi$, the velocity is related to the radius as $v = nh/2mr\pi$. Furthermore, the quantised radius is given in terms of Bohr's radius as $r_n = n^2 r_0$. Hence, $v = h/2\pi mnr_0$.

The frequency of revolutionm $f = 1/T$ (where T is the period of revolution) can be obtained from $v = 2\pi r/T = 2\pi n^2 r_0 f$. Hence, $f = v/2\pi r = (h/2\pi mnr_0)/2\pi r = h/4\pi^2 mn^3 (r_0)^2$.

For n = 1, $f_1 = h/4\pi^2 m(r_0)^2 = 6.56\times10^{15}$ Hz.
For n = 2, $f_2 = h/4\pi^2 m(2)^3 (r_0)^2 = 6.56\times10^{15}/8$ Hz = 8.2×10^{14}.

(b)
$$v = \frac{\Delta E}{h} = \frac{13.6eV}{h}\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3c}{4}\frac{13.6eV}{1242eV\cdot nm} = 0.00821\times(3\times10^8 m/s)/10^{-9} m =$$
2.463×10^{15}s. The frequency is intermediate between f_1 and f_2.

(c) The number of revolutions the electron makes is $N = f_2 \Delta t = (8.2\times10^{14})\times10^8 = 8.2\times10^{22}$ rev.

3. **Consider a positronium atom consisting of a positron and electron revolving about their common centre of mass, which lies halfway between them. (a) If such a system were a normal atom, how would its emission spectrum compared to that of hydrogen atom? (b) What would be the electron-positron separation, r, in the ground state orbit of positronium? (Eisberg, pg. 106)**

ANS

(a) The emission spectrum is described by the general form of $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$, where $R \equiv \frac{\mu e^4}{4c\pi\hbar^3(4\pi\varepsilon_0)^2}$, the reduced mass of the positronium is $\mu = \frac{mM}{M+m} = \frac{m_e \cdot m_e}{m_e + m_e} = \frac{m_e}{2}$. Compared to the emission spectrum of hydrogen, which is given by $\frac{1}{\lambda_H} = Z^2 R_\infty\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$. Hence we have $\frac{\lambda_{positronium}}{\lambda_H} = \frac{R_\infty}{R_{positronium}} = \frac{m_e}{\mu_{positronium}} = 2$. That is, the spacing between the spectral lines in the positronium is doubled as compared to the corresponding spacing in that of the hydrogen.

(b) The ground state radius is

$$r_0(positronium) = \frac{4\pi\hbar^2\varepsilon_0}{Ze^2\mu} = 2\left(\frac{4\pi\hbar^2\varepsilon_0}{e^2 m_e}\right) = 2r_0$$

4. **Ordinary hydrogen atom contains about one part in 6000 of deuterium, or heavy hydrogen. This is a hydrogen atom whose nucleus contains a proton and a neutron. How does the doubled nuclear mass affect the atomic spectrum? (Eisberg, pg 102)**

ANS

The reduced mass is $\mu = \frac{mM}{M+m} = \frac{m_e \cdot 2M}{2M+m}$. The numerical ratio $\frac{\lambda_d}{\lambda_H} = \frac{R_\infty}{R_d} = \frac{m_e}{\mu_d} = m_e \frac{2M+m_e}{m_e \cdot 2M} = \frac{2M+m_e}{2M} \approx m_e$ is the same for both limits $2M >> m$ (for deuterium) or $M >> m$ (for hydrogen). Hence the double nuclear mass does not affect the atomic spectrum in a significant sense. To be more quantitative, the ratio $\frac{m_e}{\mu_d} = \frac{2M+m_e}{2M} = \frac{2(934\text{MeV}) + (0.51\text{MeV})}{2(934\text{MeV})} = 1.0003$. The nuclear mass to the atomic spectrum only cases a 0.03% shift to the wavelengths of the spectral lines.

5. A muonic atom contains a nucleus of charge *e* and a negative muon, μ^-, moving about it. The μ^- is an elementary particle with charge −*e* and a mass 207 times as large as an electron. (a) Calculate the biding energy of the muonic atom. (b) What is the wavelength of the first line in the Lyman series for such an atom? (Eisberg, pg. 106)

ANS

(a) $\mu = \frac{mM}{M+m} = \frac{m_\mu \cdot m_\mu}{m_\mu + m_\mu} = \frac{m_\mu}{2} = \frac{207}{2} m_e = 103.5 m_e$. The energy levels are given by

$E_n^{muon} = \frac{\mu e^4}{(4\pi\varepsilon_0)^2 2\hbar^2} \frac{1}{n^2} = 103.5 E_n = 103.5 \times \frac{-13.6eV}{n^2}$. Hence the biding energy is $\Delta E = E_\infty - E_{n=1} = 0 - (-1407.6)eV = 1407.6$ eV.

(b) $\frac{1}{\lambda} = \frac{E_i^{muon} - E_f^{muon}}{hc} = 103.5 \frac{m_e e^4}{(4\pi\varepsilon_0)^2 2\hbar^2 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = 103.5 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$,

where $R_\infty \equiv \frac{m_e e^4}{4c\pi\hbar^3 (4\pi\varepsilon_0)^2} = 109{,}737\text{cm}^{-1}$. The first line in Lyman series correspond to $n_i = 2$, $n_f = 1$. Hence this wavelength is given by

$\frac{1}{\lambda} = 103.5 R_\infty \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3 \times 103.5}{4} R_\infty = 8518334.625\,\text{cm}^{-1}$, or

$\lambda = 117.4\,\text{nm}$

ZCT 104/3E Modern Physics
Semester Test I, Sessi 2003/04
Duration: 1 hour

Answer all questions

1. A radar antenna is rotating at an angular speed of 0.25 rad/s, as measured on Earth. To an observer moving past the antenna at a speed of 0.8*c*, what is its angular speed in rad/s?

   A. 0.42 **B.** 0.09 **C.**1.92 **D.** 0.15
   E. Non of the above
   ANS: D, Cutnell, Q1, pg. 877

2. Suppose that you are travelling on board a spacecraft that is moving with respect to the Earth at a speed of 0.975*c*. You are breathing at a rate of 8.0 breaths per minute. As monitored on Earth, what is your breathing rate?
   A. 13.3 **B.** 2.88 **C.**22.2 **D.** 1.77
   E. Non of the above
   ANS: D, Cutnell, Q4, pg. 877

3. At what speed is the magnitude of the relativistic momentum of a particle three times the magnitude of the non-relativistic momentum?
   A. 0.999*c* **B.** 0.900*c* **C.**0.911*c* **D.** 0.94 3*c*
   E. Non of the above
   ANS: D, Cutnell, Q17, pg. 878

4. An electron and a positron collide and undergo pair-annihilation. If each particle is moving at a speed of 0.8*c* relative to the laboratory before the collision, determine the energy of each of the resultant photon.

   A. 0.85MeV **B.** 1.67 MeV **C.** 0.51 MeV **D.** 0.72MeV
   E. Non of the above
   ANS: A, Cutnell, Q17, pg. 878, modified

5. Ultraviolet light with a frequency of 3.0×10^{15} Hz strikes a metal surface and ejects electrons that have a maximum kinetic energy of 6.1 eV. What is the work function of the metal?

   A. 13.6 eV **B.** 1.67 eV **C.** 0.51 eV **D.** 6.3 eV
   E. Non of the above
   ANS: D, Cutnell, Q5, pg. 900, modified

6. X-ray of wavelength 1.2 $\mathring{A}$ strikes a crystal of *d*-spacing 4.4 $\mathring{A}$. Where does the diffraction angle of the second order occur?

   A.16^o **B.** 33^o **C.** 55^o **D.** 90^o
   E. Non of the above
   ANS: B, Schaum's 3000 solved problems, Q38.46, pg. 715

7. A honeybee (mass 1.3×10^{-4} kg) is crawling at a speed of 0.020 m/s. What is the de Broglie wavelength of the bee?
   A. 1.6×10^{-28} m **B.** 4.6×10^{-28} m **C.** 2.6×10^{-28} m
   D. 3.06×10^{-28} m **E.** Non of the above
   ANS: C, Cutnell, Q21, pg. 901, modified

8. An electron is trapped within a sphere whose diameter is 6×10^{-15} m. Estimate the minimum uncertainty in the electron's momentum in MeV/*c*.

   A. 16 **B.** 1 **C.** 50 **D.** 2 **E.** 10
   ANS:A , Cutnell, Q32, pg. 901, modified

9. Incident x-rays have a wavelength of 0.3120 nm and are scattered by the "free electron" in a graphite target. The angle of the scattered x-ray photon is 135 degree. What is the magnitude of the momentum of the incident photon?

   A. 0.01300 MeV/*c* **B.** 0.00391 MeV/*c* **C.** 0.03450 MeV/*c*
   D. 0.01315 MeV/*c* **E.** 0.00397 MeV/*c*
   ANS:E , Cutnell, Q15, pg. 900

10. What is the magnitude of the momentum of the scattered photon in Question 9?
    A. 0.01300 MeV/*c* **B.** 0.00391 MeV/*c* **C.** 0.03450 MeV/*c*
    D. 0.01315 MeV/*c* **E.** 0.00397 MeV/*c*
    ANS:B , Cutnell, Q15, pg. 900

11. Which of the following statement(s) is (are) true?
    I(T) When two observer who are moving relative to each other measure the same physical quantity, they may obtain different values
    II(T) The laws of physics are the same for observers in all inertial frames
    III (T) The speed of light in free space has the same value in all direction and in all inertial frames
    IV(F) Maxwell theory of electromagnetic radiation is inconsistent with special theory of relativity

    A. II,III **B. I, II,III** **C. II, III, IV**
    D. I only **E. I,II,III,IV**

    ANS:B, Christman's pocket companion, pg. 291.292

12. Which of the following statement(s) is (are) true?
    I(T) Relativity theory requires a revision of the definition of momentum if it were to be consistent with conservation of momentum
    II(F) The kinetic energy of a relativistic particle with rest mass m_0 moving with speed *v* is given by $m_0c^2(1-\gamma)$, where γ is the Lorentz factor
    III (F) The total energy of a relativistic particle is given by m_0c^2 (m_0 is the rest mass)
    IV(F) The classical expression of kinetic energy $K=\dfrac{p^2}{2m_0^2}$, where *p* is the linear momentum of the particle, is a special case of the relativistic energy $E=\sqrt{(pc)^2+(m_0c^2)^2}$

    A. II,III **B. I, II,III** **C. II, III, IV**
    D. I, IV **E. I,II, III,IV**

    ANS:**I only** (free mark will be given for this question since the correct answer is not in the option)
    Christman's pocket companion, pg. 299.300

13. Which of the following statement(s) is (are) true?

I(T) Photon carries momentum
II(F) The Compton shift $\Delta\lambda$ is greater for higher-energy photons
III(F) The Compton shift $\Delta\lambda$ is smaller for lower-energy photons

A. I only **B. I, II** **C. II, III**
D. I,III **E. I,II, III**

ANS:A, Machlup, pg. 497

14. Which of the following statements correctly describe the following experiments?
I(T) Photoelectricity exhibits particle nature of light
II(F) Electron diffraction exhibits wave nature of light
III (T) Compton effect exhibits particle nature of electron
IV(T) Compton effect exhibits particle nature of light
A. II,III **B. I, II,III** **C. II, III, IV**
D. I,III, IV **E. I,II, III,IV**

ANS:D, My own questions

15. Which of the following statements correctly describe light?
I(T) According to Einstein, the energy in an electromagnetic beam is concentrated in discrete bundles called photon
II(T) According to the classical Maxwell theory of radiation, light is described as electromagnetic wave
III (F) The energy of the photon is proportional to the root-mean-square of the amplitude of the electromagnetic fields
IV (*) The intensity of a beam of light is proportional to the root-mean-square of the amplitude of the electromagnetic fields
A. II,III **B. I, II,IV** **C. II, III, IV**
D. I,III, IV **E. I,II, III,IV**

ANS:**C** (Free mark will be given for this question because statement IV may appear confusing and ill-stated).

(*) Rigorously speaking, statement IV is correct because the "root-mean-square of the amplitude" is equal to the square of the amplitude. The amplitude is a constant independent of time and space, hence whether you average its square over a complete period or simply squaring it without taking its "average" the answer is still the same. Mathematically this is stated as $<E_0^2> = \frac{1}{T}\int_0^T E_0^2 dt = E_0^2$.

My own questions

16. Which of the following statements correctly describe photoelectricity?
I(T) If the frequency is unchanged the number of electrons ejected depends on the incident intensity
II(F) If the frequency is unchanged the kinetic energies of electrons ejected depends on the incident intensity
III (T) In photoeletricity the fundamental event is the interaction of a single quantum of light with a single particle of matter
IV(T) Electrons are ejected immediately when photoelectricity occurs
A. II,III **B. I, II,III** **C. II, III, IV**
D. I,III, IV **E. I,II, III,IV**

ANS:D, Christman's pocket companion, pg. 302-303

17. Which of the following statements correctly describe Compton scattering?
I(T) The Compton effect has to be treated relativistically
II(T) The Compton effect is significant only when the incident wavelength of the light used is comparable to the Compton wavelength of the electron
III(T) The maximum change in wavelength is given by $\Delta\lambda_{\max} = 2\lambda_C$, where λ_C is the Compton wavelength of electron
IV (F) The Compton effect is much larger for electrons bounded to atoms than for free electrons

A. II,III **B. I, II,III** **C. II, III, IV**
D. I,III, IV **E. I,II, III,IV**

ANS:B, partly Christman's pocket companion, pg. 305, partly own question

18. Which of the following statement(s) is (are) true?
I(F) The Davisson-Gremer experiment verifies the particle nature of electromagnetic wave
II(T) In the Davisson-Gremer experiment the wavelength of the electron is comparable to the interatomic spacing in the crystal
III(T) At the quantum scale particles behave like waves
IV (T) At the quantum scale waves behave like particles

A. II,III **B. I, II,III** **C. II, III, IV**
D. I,III, IV **E. I,II, III,IV**

ANS:C, My own question

19. An increase in the voltage applied to an x-ray tube causes an increase in the x-rays'
I(F) wavelength
II(F) speed
III(T) energy
IV (T) frequency

A. III,IV **B. I, II,III** **C. II, III, IV**
D. I,III, IV **E. I,II, III,IV**

ANS:A, Arthur Beiser, Modern technical physics, Q 7, pg. 801

20. The description of a particle in terms of matter waves is legitimate because
I(F) It is based on common sense
II(F) The analogy with electromagnetic waves is plausible
III(T) theory and experiment agree

A. III only **B. I, II** **C. II, III**
D. I,III **E. I,II, III**

ANS:A, Arthur Beiser, Modern technical physics, Q 9, pg. 801

Data
speed of light in free space, $c = 3.00 \times 10^8$ m s^{-1}

elementary charge, $e = 1.60 \times 10^{-19}$ C

the Planck constant, $h = 6.63 \times 10^{-34}$ J s

unified atomic mass constant, $u = 1.66 \times 10^{-27}$ kg

rest mass of electron, $m_e = 9.11 \times 10^{-31}$ kg rest mass of proton, m_p = 1.67 x 10^{-27} kg

ZCT 104/3E Modern Physics
Semester Test II, Sessi 2003/04
Duration: 1 hour

Data
speed of light in free space, c = 3.00 x 10^8 m s^{-1}
elementary charge, e = 1.60 x 10^{-19} C
the Planck constant, h = 6.63 x 10^{-34} J s
unified atomic mass constant, u = 1.66 x 10^{-27} kg
rest mass of electron, m_e = 9.11 x 10^{-31} kg
rest mass of proton, m_p = 1.67 x 10^{-27} kg

Answer all questions

1. A particle of mass m is confined to a one-dimensional box of length L. The particle's momentum is given by

   A. $h/2L$ **B.** $nh/2L$ **C.** $\hbar/2L$ **D.** $n\hbar/2L$
   E. Non of the above
   ANS: B, Ronald and William, Q10.20, pg. 92

2. The energy of the particle in Q1 is given by

   A. $n^2\dfrac{\hbar^2}{8m\pi L^2}$ **B.** $n^2\dfrac{h^2}{8mL^2}$ **C.** $n^2\dfrac{\pi^2 h^2}{2mL^2}$ **D.** $n^2\dfrac{\hbar^2}{2mL^2}$
   E. Non of the above
   ANS: B, Ronald and William, Q10.20, pg. 92

3. What is the ionisation energy of the hydrogen atom?

   A. infinity **B.** 0 **C. -**13.6 eV **D.** 13.6 eV
   E. Non of the above
   ANS: D, Modern Technical Physics, Beiser, pg. 786

4. What is the ground state energy of the hydrogen atom?

   A. infinity **B.** 0 **C. -**13.6 eV **D.** 13.6 eV
   E. Non of the above
   ANS: C, Modern Technical Physics, Beiser, pg. 786

5. An electron collides with a hydrogen atom in its ground state and excites it to a state of n =3. How much energy was given to the hydrogen atom in this collision?

   A. -12.1 eV **B.** 12.1 eV **C.** -13.6 eV **D.** 13.6 eV
   E. Non of the above
   ANS: B, Modern Technical Physics, Beiser, Example 25.6, pg. 786

6. Which of the following transitions in a hydrogen atom emits the photon of lowest frequency?

   A. n = 3 to n = 4 **B.** n = 2 to n = 1 **C.** n = 8 to n = 2 **D.** n = 6 to n = 2
   E. Non of the above
   ANS: D, Modern Technical Physics, Beiser, Q40, pg. 802, modified

7. In Bohr's model for hydrogen-like atoms, an electron (mass m) revolves in a circle around a nucleus with positive charges Ze. How is the electron's velocity related to the radius r of its orbit?

A. $v = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr}$ **B.** $v = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr^2}$ **C.** $v = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{mr^2}$ **D.** $v^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr}$
E. Non of the above
ANS: D, Schaum's series 3000 solved problems, Q39.13, pg 722 modified

8. How is the total energy of the electron in Question 7 related the radius of its orbit?

A. $E = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$ **B.** $E = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{2r}$ **C.** $E = -\frac{1}{4\pi\varepsilon_0}\frac{Ze}{2r}$
D. $E = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$
E. Non of the above
ANS: D, Schaum's 3000 solved problems, Q39.14, pg. 722

9. The quantum number n of the lowest energy state of a hydrogen atom

A. is 0 **B.** is 1 **C.** depends on the orbit size
D. depends on the electron speed **E.** Non of the above
ANS: B, Modern Technical Physics, Beiser, Q23, pg. 802

10. The electron of a ground state hydrogen atom

A. has left the atom **B.** is at rest **C.** is in its orbit of lowest energy **D.** is in its orbit of highest energy **E.** Non of the above
ANS: C, Modern Technical Physics, Beiser, Q16, pg. 801

11. A proton and an electron, far apart and at rest initially, combine to form a hydrogen atom in the ground state, A single photon is emitted in this process. What is its wavelength?

A. 13.6 nm **B.** 20 nm **C.** 91 nm **D.** infinity **E.** Non of the above
ANS: C, Modern Technical Physics, Beiser, Q30, pg. 804

12. The wave function of a particle trapped in an infinite quantum well of width L is given by $\psi_n = A_n \sin\frac{n\pi x}{L}$. Determine the normalisation constant A_n.

A. $\sqrt{\frac{L}{2}}$ **B.** $\frac{2}{L}$ **C.** $\sqrt{\frac{2n}{L}}$ **D.** $\sqrt{\frac{2}{L}}$ **E.** Non of the above
ANS: D, my own question

13. Where does the particle in Question 12 spend most of its time while in the ground state?
A. around $x = 0$ **B.** around $x = L$ **C.** around $x = L/2$ **D.** around $x = L/4$
E. Non of the above
ANS:C, My own question

SESSI 03/04/TEST2

14. How many different photons can be emitted by hydrogen atoms that undergoes transitions to the ground states from the $n = 5$ states?
A. 3 **B.** 6 **C.** 10 **D.** 15
E. Non of the above
ANS: C, Ronald and William, Q11.8, pg. 109

15. Which of the following statements are true about an electron trapped on the x-axis by infinite potential energy barriers at $x = 0$ and $x = L$?
I(T) Inside the trap the coordinate-dependent part of the wave function ψ satisfy the Schrodinger equation
II(T) ψ obeys the boundary conditions $\psi(0) = 0$ and $\psi(L) = 0$
III (F) The probability to locate the electron is everywhere the same inside the well
IV(T) Outside the trap, $\psi = 0$
A. II,III **B. I, II,III** **C. II, III, IV**
D. I, II, IV only **E. Non of the above**
ANS:D, Christman's pocket companion, Item 40.3, pg. 312

16. Which of the following statements are true?
I(T) The energy of a particle trapped inside an finite quantum well is quantised
II(T) The energy of a particle trapped inside an infinite quantum well is quantised
III (F) The lowest energy of a particle trapped in an infinite quantum well is zero
A. II,III **B. I, II,III** **C. II, III**
D. I, II **E. Non of the above**
ANS:D my own question

17. Which of the following statement(s) is (are) true?
I(T) The plum pudding model cannot explains the backscattering of alpha particles from thin gold foils
II(T) Rutherford model assumes that an atom consists of a tiny but positively charged nucleus surrounded by electrons at a relatively large distance
III(T) In the Bohr model, an electron in a stationary state emits no radiation
IV(T) In the Bohr model, electrons bound in an atom can only occupy orbits for which the angular momentum is quantised

A. III,IV **B. I, II.III** **C. I, II, III.IV** **D. I,II**
E. Non of the above
ANS:C,Giancoli, Summery on pg 972

18. Which of the following statement(s) is (are) true?
I(F) Bohr's theory worked well for one electron ions as well as for multi-electron atoms
II(F) Bohr's model is plagued by the infrared catastrophe
III(F) In the Bohr model, $n = 1$ corresponds to the first excited state
IV(T) Rutherford model cannot explain the stability of atomic orbit

A. III,IV **B. I, II.III** **C. I, II, III.IV** **D. I,II**
E. Non of the above
ANS:E, My own question

19. Which of the following statements are correct?
I(F) Balmer series corresponds to the spectral lines emitted when the electron in a hydrogen atom makes transitions from higher states to the $n = 1$ state

II(F) Lyman series corresponds to the spectral lines emitted when the electron in a hydrogen atom makes transitions from higher states to the $n = 2$ state

II(T) Paschen series corresponds to the spectral lines emitted when the electron in a hydrogen atom makes transitions from higher states to the $n = 3$ state

A. II,III **B. I, II,III** **C. II, III**
D. III only **E. Non of the above**
ANS:D, My own questions

20. Which of the following statements are correct?

I(T) Frank-Hertz experiment shows that atoms are excited to discrete energy levels
II(T) Frank-Hertz experimental result is consistent with the results suggested by the line spectra
III (T) The predictions of the quantum theory for the behaviour of any physical system must correspond to the prediction of classical physics in the limit in which the quantum number specifying the state of the system becomes very large
IV(T) The structure of atoms can be probed by using electromagnetic radiation

A. II,III **B. I, II,IV** **C. II, III, IV**
D. I,II, III, IV **E. Non of the above**

ANS:D, My own questions

UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2003/2004

February/March 2004

ZCT 104E/3 - Physics IV (Modern Physics)
[Fizik IV (Fizik Moden)]

Duration: 3 hours
[Masa: 3 jam]

Please check that the examination paper consists of **SIXTEEN** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM BELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer any **FOUR (4)** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan**: Jawab mana-mana **EMPAT** soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

Data

speed of light in free space, $c = 3.00 \times 10^{8}$ m s^{-1}

permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

elementary charge, $e = 1.60 \times 10^{-19}$ C

the Planck constant, $h = 6.63 \times 10^{-34}$ J s

unified atomic mass constant, $u = 1.66 \times 10^{-27}$ kg

rest mass of electron, $m_e = 9.11 \times 10^{-31}$ kg

rest mass of proton, $m_p = 1.67 \times 10^{-27}$ kg

molar gas constant, = 8.31 J K^{-1} mol^{-1}

the Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1}

gravitational constant, G = 6.67 x 10-11 N m^{2} kg^{-2}

acceleration of free fall, $g = 9.81$ m s^{-2}

Question 1. (25 marks)

1.1 A spaceship of proper length L_p takes t seconds to pass an Earth observer. What is its speed as measured by the Earth observer according to classical physics?
[Sebuah kapal angkasa yang panjang proper-nya L_p mengambil masa t untuk bergerak melalui seorang pemerhati di Bumi. Mengikut fizik klasik, apakah kelajuannya yang terukur oleh pemerhati di Bumi itu?]

A. L_P / t **B.** $\dfrac{cL_P / t}{\sqrt{c^2 + (L_P / t)^2}}$ **C.** c **D.** L_P

E. Non of the above
[Tiada dalam pilihan di atas]

ANS: A, Serway solution manual 2, Q9A, pg. 336

1.2 In Question 1, what is its speed as measured by the Earth observer according to special relativity?
[Dalam soalan 1, *apakah kelajuan yang terukur oleh pemerhati di Bumi mengikut teori kerelatifan khas?]*

A. L_P / t **B.** $\dfrac{cL_P / t}{\sqrt{c^2 + (L_P / t)^2}}$ **C.** c **D.** L_P

E. Non of the above
[Tiada dalam pilihan di atas]

ANS: B, Serway solution manual 2, Q9A, pg. 336

1.3 What is the momentum of a proton if its total energy is twice its rest energy?
[Apakah momentum bagi suatu proton jika jumlah tenaganya adalah dua kali tenaga rehatnya?]

A. 1620 Ns **B.** 1 MeV/c **C.** 938 MeV/c **D.** 2 MeV/c

E. 1620 MeV/c

ANS: E, Serway solution manual 2, Q21, pg. 339

1.4 The power output of the Sun is 3.8×10^{26} W. How much rest mass is converted to kinetic energy in the Sun each second?
[Output kuasa Matahari ialah 3.8×10^{26} W. *Berapakah jisim rehat yang ditukarkan kepada tenaga kinetik setiap saat di dalam Matahari?]*

A. 4.2×10^{9} kg **B.** 1.3×10^{17} kg **C.** 3.6×10^{8} kg

D. 6.6×10^{10} kg **E.** 4.2×10^{8} kg

ANS: A, Serway solution manual 2, Q37, pg. 340

1.5 What is the value of *hc/e* in unit of $\text{nm}\cdot\text{eV}$
[Apakah nilai hc/e dalam unit $\text{nm}\cdot\text{eV}$ *?]*

A. 1.240 **B.** 1240×10^{-6} **C.** 1240 **D.** 1240×10^{-9}

E. 1240×10^{-3}

ANS: C, my own question [note: typo: the quantity should read *hc* instead of *hc/e*]

1.6 By what factor is the mass of an electron accelerated to the speed of 0.999*c* larger than its rest mass?
[Berapa besarnya factor jisim satu elektron yang dipecutkan kepada kelajuan 0.999c *berbanding dengan jisim rehatnya?]*

A. 31.6 **B.** 0.03 **C.** 0.04 **D.** 22.3 **E.** 1.0

ANS: D, my own question

1.7 The rest mass of a photon
[Jisim rehat foton]

A. is zero
[ialah sifar]
B. is the same as that of an electron
[sama dengan jisim elektron]
C. depends on its frequency
[bergantung kepada frekuensinya]
D. depends on its energy
[bergantung kepada tenaganya]
E. Non of the above
[Tiada dalam pilihan di atas]

ANS: A, Modern physical technique, Beiser, MCP 6, pg. 801

1.8 Determine the vacuum wavelength corresponding to a γ-ray energy of 10^{19} eV
[Tentukan jarak gelombang vakum bagi sinar γ yang bersepadanan dengan tenaga 10^{19} eV]

A. 1.24×10^{-9} pm
B. 1.24×10^{-16} pm

SESSI 03/04/FINAL

C. 1.24×10^{-25} nm
D. 1.24×10^{-16} nm
E. 1.24×10^{-25} nm

ANS: D, Schaum's 3000 solved problems, Q38.3, pg. 708

1.9 To produce an x-ray quantum energy of 10^{-15} J electrons must be accelerated through a potential difference of about
[Untuk menghasilkan sinar-x dengan tenaga kuantum 10^{-15} J suatu elektron mesti dipecutkan melalui satu beza keupayaan yang nilainya lebih kurang]

A. 4 kV
B. 6 kV
C. 8 kV
D. 9 kV
E. 10 kV

ANS: B, OCR ADVANCED SUBSIDIARY GCE PHYSICS B (PDF), Q10, pg. 36

Question 1.10– 1.12
[Soalan 1.10-1.12]

A. 10^{-4} m
B. 10^{-7} m
C. 10^{-10} m
D. 10^{-12} m
E. 10^{-15} m

1.10 Which of the values in the list above is the best estimate of the radius of an atom?
[Nilai yang manakah dalam senarai di atas memberikan anggaran yang paling baik untuk radius satu atom?]

ANS: C, OCR ADVANCED PHYSICS B (PDF), Q1, pg. 74

1.11 Which of the values in the list above is the best estimate of the wavelength of visible light?
[Nilai yang manakan dalam senarai di atas memberikan anggaran yang paling baik untuk jarak gelombang cahaya ternampak?]

ANS: B, OCR ADVANCED PHYSICS B (PDF), Q1, pg. 74

1.12 Which of the values in the list above is the best estimate of the wavelength of a 1.5 MeV electron?

[Nilai yang manakan dalam senarai di atas memberikan anggaran yang paling baik untuk jarak gelombang bagi elektron 1.5 MeV*?]*

ANS: D, OCR ADVANCED PHYSICS B (PDF), Q1, pg. 74

1.13 What is the momentum of a single photon of red light ($\nu = 400 \times 10^{12}$ Hz) moving through free space?
[Apakah momentum foton cahaya merah ($\nu = 400 \times 10^{12}$ Hz) *yang bergerak melalui ruang bebas?]*

A. 8.8×10^{-27} kg m/s
B. 6 keV
C. 1240 eV/*c*
D. 1.65 eV/*c*
E. 2.4 eV/*c*

ANS: D, Schaum's 3000 solved problems, Q8.12, pg. 709

1.14 What potential difference must be applied to stop the fastest photoelectrons emitted by a nickel surface under the action of ultraviolet light of wavelength 2000 Å? The work function of nickel is 5.00 eV.
[Apakah beza keupayaan yang mesti dikenakan untuk menghentikan fotoelektron paling pantas yang dipancarkan dari permukaan nikel di bawah tindakan cahaya ultraungu yang jarak gelombangnya 2000 Å*? Fungsi kerja nikel ialah* 5.00 eV.*]*

A. 1.0 kV
B. 1.2 kV
C. 2.0 V
D. 1.0 V
E. 1.2 V
ANS: E, Schaum's 3000 solved problems, Q38.18, pg. 710

1.15 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I. The assumption of the Ether frame is inconsistent with the experimental observation
[Tanggapan rangka Ether adalah tidak konsisten dengan pemerhatian eksperimen]

II. The speed of light is constant
[Kelajuan cahaya adalah malar]

III. Maxwell theory of electromagnetic radiation is inconsistent with the notion of the Ether frame
[Teori sinaran keelektromagnetan Maxwell adalah tidak konsisten dengan tanggapan rangka Ether]

IV Special relativity is inconsistent with the notion of the Ether frame
[Kerelatifan Khas adalah tidak konsistent dengan tanggapan rangka Ether]

A. III,IV **B.** I, II, III **C.** I, II, III,IV

D. I, II **E.** I, II,IV

ANS: E, my own question

1.16 Which of the following statements are true about light?
[Yang manakah kenyataan berikut adalah benar berkenaan dengan cahaya?]

I. It propagates at the speed of $c = 3 \times 10^8$ m/s in all medium
[Cahaya tersebar pada kelajuan c = 3 x 10^8 m/s *dalam semua jenis medium]*

II. It's an electromagnetic wave according to the Maxwell theory
[Cahaya ialah gelombang elektromagnetik mengikut teori Maxwell]

III. It's a photon according to Einstein
[Cahaya ialah foton menurut Einstein]

IV. It always manifests both characteristics of wave and particle simultaneously in a given experiment
[Cahaya sentiasa memperlihatkan kedua-dua ciri gelombang dan kezarahan secara serentak dalam sesuatu eksperimen]

A. I,IV **B.** II, III,IV **C.** I, II, III,IV

D. I, II **E.** II,III

ANS: E, my own question

1.17 Which of the following statements are true about Lorentz transformation?
[Yang manakah kenyataan berikut adalah benar berkenaan dengan transformasi Lorentz?]

I. It relates the space-time coordinates of one inertial frame to the other
[Ia menghubung-kaitkan koordinat-koordinat ruang-masa suatu rangka inersia dengan koordinat-koordinat ruang-masa rangka inersia lain]

II. It is the generalisation of Galilean transformation
[Ia merupakan generalisasi transformasi Galilean]

III. It constitutes one of the Einstein's special relativity postulates

[Ia merupakan salah satu postulat teori kerelatifan khas Einstein]

IV. Its derivation is based on the constancy of the speed of light postulate
[Ia diterbitkan berdasarkan postulat kemalaran kelajuan cahaya]

A. I,IV **B.** I,II, IV **C.** I, II, III,IV

D. I, II **E.** II,III

ANS: B, my own question

1.18 The expression of linear momentum has to be modified in the relativistic limit in order to
[Ekspresi momentum linear kena dimodifikasikan pada limit relativistik supaya]

I. preserve the consistency between the Lorentz transformation and conservation of linear momentum
[konsistensi antara transformasi Lorentz dengan keabadian momentum linear terpelihara]

II. preserve the consistency between the Galilean transformation and conservation of linear momentum
[konsistensi antara transformasi Galilean dengan keabadian momentum linear terpelihara]

III. preserve the consistency between special relativity with Newtonian mechanics
[konsistensi antara kerelatifan khas dengan mekanik Newton terpelihara]

IV. preserve the consistency between the Lorentz transformation and Galilean transformation
[konsistensi antara transformasi Lorentz dengan transformasi Galilean terpelihara]

A. I only **B.** I,II, IV **C.** I, III,IV **D.** III,IV **E.** IV only

ANS: A, my own question

Question 2. (25 marks)
[Soalan 2 (25 markah)]

2.1 What is the kinetic energy of the fastest photoelectrons emitted by a copper surface, of work function 4.4 eV when illuminated by visible light 0f 700 nm?
[Apakah tenaga kinetik fotoelektron paling pantas yang dipancarkan oleh permukaan kuprum, yang fungsi kerjanya 4.4 eV*, semasa disinari cahaya*

SESSI 03/04/FINAL

ternampak 700 nm*?]*

A. 1.17 eV
B. 6.17 eV
C. 1.17 eV
D. 1.0 eV
E. non of the above
[Tiada dalam pilihan di atas]

ANS: E, Schaum's 3000 solved problems, Q38.21, pg. 710

2.2 Suppose that a beam of 0.2-MeV photon is scattered by the electrons in a carbon target. What is the wavelength of those photon scattered through an angle of 90^o?
[Katakan satu bim foton 0.2 MeV *diserakkan oleh elektron di dalam sasaran karbon. Apakah jarak gelombang bagi foton yang diserakkan melalui satu sudut* 90^o*?]*

A. 0.00620 nm
B. 0.00863 nm
C. 0.01106 nm
D. 0.00243 nm
E. non of the above
[Tiada dalam pilihan di atas]
ANS: B, Schaum's 3000 solved problems, Q38.31, pg. 712

2.3 Determine the cut-off wavelength of x-rays produced by 50-keV electrons in a x-ray vacuum tube?
[Tentukan jarak gelombang penggal bagi sinar-x yang dihasilkan oleh elektron 50 keV *dalam satu tiub sinar-x vakum.]*

A. 0.000248 $\mathring{A}$
B. 2.48 $\mathring{A}$
C. 248 $\mathring{A}$
D. 0.248 $\mathring{A}$

E. non of the above
[Tiada dalam pilihan di atas]
ANS: D, Schaum's 3000 solved problems, Q38.39, pg. 714

2.4 A lamp emits light of frequency 5.0×10^{15} Hz at a power of 25 W. The number of photons given off per seconds is
[Suatu lampu memancarkan cahaya berfrekuensi 5.0×10^{15} Hz *pada kuasa* 25 W. *Bilangan foton yang dihasilkan per saat ialah]*

A. 1.3×10^{-19} **B.** 8.3×10^{-17} **C.** 7.5×10^{18} **D.** 1.9×10^{50}

E. 2.9×10^{13}
ANS:C , Modern physical technique, Beiser, MCP 34, pg. 802, modified

2.5 Which of the following transitions in a hydrogen atom emits the photon of lowest frequency?
[Dalam senarai di bawah, peralihan yang manakah memancarkan foton frekuensi terendah di dalam atom hidrogen?]

A. $n = 1$ to $n = 2$ **B.** $n = 2$ to $n = 1$ **C.** $n = 2$ to $n = 6$

D. $n = 6$ to $n = 2$ **E.** n = infinitely large to $n = 1$
[n = sebesar tak terhingga ke n = 1]

ANS:D, Modern physical technique, Beiser, MCP 40, pg. 802

2.6 The speed of an electron whose de Broglie wavelength is 1.0×10^{-10} m is
[Kelajuan satu elektron yang jarak gelombang de Broglie-nya 1.0×10^{-10} m *ialah]*

A. 6.6×10^{-24} m/s **B.** 3.8×10^{3} m/s **C.** 7.3×10^{6} m/s

D. 1.0×10^{10} m/s **E.** 6.6×10^{2} m/s
ANS:C, Modern physical technique, Beiser, MCP 36, pg. 802

2.7 A large value of the probability density of an atomic electron at a certain place and time signifies that the electron
[Nilai yang besar bagi ketumpatan kebarangkalian suatu elektron atom pada sesuatu tempat dan masa menunjukkan elektron itu]

A. is likely to be found there
[agak mungkin dijumpai di sana]

B. is certain to be found there
[pasti dijumpai di sana]

C. has a great deal of energy there
[mempunyai banyak tenaga di sana]

D. has a great deal of charge
[mempunyai banyak cas]

E. is unlikely to be found there

SESSI 03/04/FINAL

[tidak berapa mungkin dijumpai di sana]

ANS:A, Modern physical technique, Beiser, MCP 25, pg. 802

2.8 Ionisation energy of hydrogen is 13.5 eV. What is the shortest wavelength in the Lyman series of hydrogen atom?
[Tenaga pengionan hidrogen ialah 13.5 eV. *Apakah jarak gelombang terpendek dalam siri Lyman hidrogen?]*

A. 364 nm **B.** 121 nm **C.** 91 nm **D.** 819 nm

E. 103 nm
ANS:C, my own question

2.9 If the momentum of a particle is doubled, its wavelength is multiplied _______ times
[Jika momentum suatu zarah digandakan dua, jarak gelombangnya digandakan _________ kali]

A. 1 **B.** 2 **C.** 1/2 **D.** 8 **E.** 0

ANS: C, Machlup, Review question 7, pg. 522, modified

2.10 A standing wave cannot have less than ___ antinode. In quantum mechanics, that fundamental mode would be called the _________.
[Suatu gelombang pegun tidak boleh mempuyai kurang daripada _____antinod. Dalam mekanik kuantum, mod asas ini dinamakan _______.]

A. 1, first excited state
[keadaan teruja pertama]
B. 1, ground state
[keadaan dasar]

C. 2, first excited state
[keadaan teruja pertama]
D. 2, ground state
[keadaan dasar]

E. 0, ground state
[keadaan dasar]
ANS: B, Machlup, Review question 9, pg. 522, modified

2.11 Assume that the uncertainty in the position of a particle is equal to its de Broglie wavelength. What is the minimal uncertainty in its velocity, v_x?
[Anggapkan bahawa ketidakpastian dalam kedudukan suatu zarah adalah sama dengan jarak gelombang de Broglie-nya. Apakah ketidakpastian minimum dalam halajunya v_x*?]*

A. $v_x/4\pi$ **B.** $v_x/2\pi$ **C.** $v_x/8\pi$ **D.** v_x

E. v_x/π

ANS: A, Schaum's 3000 solved problems, Q38.66, pg. 718

2.12 If the ionisation energy for a hydrogen atom is 13.6 eV, what is the energy of the level with quantum number $n = 3$?
[Jika tenaga pengionan satu atom hidrogen ialah 13.6 eV*, apakah tenaga untuk paras yang bernombor kuantum n = 3?]*

A. 1.51 eV **B.** 3.4 eV **C.** 12.1 eV

D. -1.51 eV **E.** -3.4 eV

ANS: D, Schaum's 3000 solved problems, Q39.6, pg. 720

2.13 What is the zero-point energy of an electron trapped in an infinite potential well of size $L = 0.5\ \mathring{A}$
[Apakah tenaga titik-sifar bagi elektron yang terperangkap di dalam suatu telaga keupayaan infinit yang saiznya $L = 0.5\ \mathring{A}$*]*

A. 7.5×10^{-9} eV **B.** 11.7×10^{-6} eV **C.** 0.30×10^{-6} eV

D. 13.6 eV **E.** 65×10^{-6} eV

ANS: 150 eV. Free marks will be given for this question since there is no correct answer in the options.

2.14 A moving body is described by the wave function ψ at a certain time and place; ψ^2 is proportional to the body's
[Suatu jasad bergerak diperihalkan oleh fungsi gelombang ψ *pada suatu masa dan tempat tertentu;* ψ^2 *adalah berkadar dengan]*

A. electric field
[medan elektrik]
B. speed
[kelajuan]
C. energy
[tenaga]
D. probability of being found
[kebarangkalian untuk dijumpai]
E. mass
[jisim]

ANS:D , Modern physical technique, Beiser, MCP 11, pg. 801

SESSI 03/04/FINAL

2.15 The continuous x-ray spectrum produced in an x-ray tube can be explained by
[Keselanjaran spektrum sinar-x yang dihasilkan dalam suatu tiub sinar-x dapat diterangkan oleh]

I. Classical Electromagnetic wave theory
[Teori klasik gelombang keelektromagnetan]
II. Pair production
[Penghalisan pasangan]
III. Bremsstrahlung
[Bremsstrahlung]
IV. Diffraction
[Belauan]

A. I,IV **B.** I,II, IV **C.** I, III,IV **D.** I, III

E. II,III
ANS: D, My own questions

2.16 Planck constant
[Pemalar Planck]

I. is a universal constant
[ialah satu pemalar universal]
II. is the same for all metals
[adalah sama bagi semua jenis logam]
III. is different for different metals
[adalah tidak sama bagi logam yang berlainan]
IV. characterises the quantum scale
[mencirikan skala kuantum]

A. I,IV **B.** I,II, IV **C.** I, III,IV **D.** I, III

E. II,III

ANS: B, Machlup, Review question 8, pg. 496, modified

2.17 A neon sign produces
[Suatu lampu neon menghasilkan]

I. a line spectrum
[suatu spektrum garis]
II. an emission spectrum
[suatu spektrum pancaran]
III. an absorption spectrum
[suatu spektrum penyerapan]
IV. photons
[foton]

A. I,IV **B.** I,II, IV **C.** I, III,IV **D.** I, III

E. II,III

ANS:B , Modern physical technique, Beiser, MCP 20, pg. 801, modified

2.18 Which of the following statements are true?
[Kenyataan berikut yang manakah benar?]

I.. the ground states are states with lowest energy
[keadaan asas adalah keadaan dengan tenaga yang paling rendah]

II. ionisation energy is the energy required to raise an electron from ground state to free state
[tenaga pengionan adalah tenaga yang diperlukan untuk menaikan suatu elektron dari keadaan asas ke keadaan bebas]

III. Balmer series is the lines in the spectrum of atomic hydrogen that corresponds to the transitions to the $n = 1$ state from higher energy states
[Balmer siri adalah garis-garis spectrum atom hidrogen yang bersepadanan dengan peralihan dari paras-paras tenaga yang lebih tinggi ke paras $n = 1$]

A. I,IV **B.** I,II, IV **C.** I, III,IV **D.** I, II

E. II,III

ANS: D, My own question
(note: this is an obvious typo error with the statement IV missing. In any case, only statement I, II are true.)

Question 3. (25 marks)
[Soalan 3. (25 markah)]

(a) Lithium, beryllium and mercury have work functions of 2.3 eV, 3.9 eV and 4.5 eV, respectively. If a 400-nm light is incident on each of these metals, determine
[Fungsi kerja Lithium, beryllium dan raksa adalah 2.3 eV, 3.9 eV dan 4.5 eV masing-masing. Jika cahaya 400 nm ditujukan ke atas setiap satu logam itu, tentukan]

(i) which metals exhibit the photoelectric effect, and
[logam yang manakah memperlihatkan kesan fotoelectrik, dan]

(ii) the maximum kinetic energy for the photoelectron in each case (in eV)
[tenaga kinetik maksimum untuk fotoelektron dalam setiap kes itu (dalam unit eV)]

Serway solution manual 2, Q21, pg. 357

(b) Molybdenum has a work function of 4.2 eV.
[Fungsi kerja Molybdenum ialah 4.2 eV.]

(i) Find the cut-off wavelength (in nm) and threshold frequency for the photoelectric effect.
[Carikan jarak gelombang penggal (dalam unit nm*) dan frekuensi ambang untuk kesan fotoelektrik]*

(ii) Calculate the stopping potential if the incident radiation has a wavelength of 180 nm.
[Hitungkan keupayaan penghenti jika sinaran tuju mempunyai jarak gelombang 180 nm.*]*

Serway solution manual 2, Q16, pg. 356

(c) A 0.0016-nm photon scatters from a free electron. For what scattering angle of the photon do the recoiling electron and the scattered photon have the same kinetic energy?
[Suatu foton 0.0016 nm diserakkan oleh elektron bebas. Apakah sudut serakan foton supaya elektron yang tersentak dan foton yand terserak itu mempunyai tenaga kinetik yang sama?]

Serway solution manual 2, Q35, pg. 358

Solution:
Q3a(i)
The energy of a 400 nm photon is $E = hc/\lambda$ = **3.11 eV**
[2 mark]

The effect will occur only in **lithium***
[2 marks, with or without explanation]

Q3a(ii)
For lithium, $K_{max} = h\nu - W_0$ = 3.11 eV – 2.30 eV = **0.81 eV***
[3 marks]

[Note*: for Q3a(i,ii), the full 2+2+3 marks only for the unique answer set {lithium, K_{max} = 0.81 eV}. Minus 2 marks for any extra answer set involving other metals]

Q3b(i)
Cut-off frequency = $\lambda_{cutoff} = hc/W_0$ = 1240 nm eV / 4.2 eV = **295 nm**
Cut-off frequency (or threshold frequency)= $\nu_{cutoff} = c/\lambda$ = **1.01 x 10^{15} Hz**
[3 + 3 marks]

Q3b(ii)
Stopping potential $V_{stop} = (hc/\lambda - W_0) / e$= (1240 nm.eV/180 nm – 4.2 eV)/e = **2. 7 V**

[3 marks]

Q3c

The energy of the incoming photon is $E_i = hc/\lambda$ = **0.775 MeV**

[3 mark]

Since the outgoing photon and the electron each have half of this energy in kinetic form,

$E_o = hc/\lambda'$ = 0.775 MeV / 2 = 0.388 MeV and

$\lambda' = hc/E_o$ = 1240 eV. nm / 0.388 MeV = 0.0032 nm

The Compton shift is $\Delta\lambda = \lambda' - \lambda$ = (0.0032 – 0.0016) nm = 0.0016 nm

[3 marks]

By $\Delta\lambda = \lambda_c (1 - \cos\theta) = h/m_ec\ (1 - \cos\theta)$

0.0016 nm = 0.00243 nm $(1 - \cos\theta)$

$\Rightarrow \theta = 70^o$

[3 marks]

Question 4. (25 marks)

[Soalan 4. (25 markah]

(a) An electron is contained in a one-dimensional box of width 0.100 nm. Using the particle-in-a-box model,
[Suatu elektron terkandung di dalam satu kotak satu dimensi yang lebarnya 0.100 nm. Dengan menggunakan model zarah-dalam-satu-kotak]

(i) Calculate the n = 1 energy level and n = 4 energy level for the electron in eV.
[Hitungkan paras tenaga n = 1 dan n = 4 untuk elektron itu dalam unit eV.]

(ii) Find the wavelength of the photon (in nm) in making transitions that will eventually get it from the the n = 4 to n = 1 state
[Hitungkan jarak gelombang foton (dalam unit nm) semasa ia membuat peralihan yang membawanya dari keadaan n = 4 ke keadaan n = 1]

Serway solution manual 2, Q33, pg. 380, modified

(b) Consider a 20-GeV electron.
[Pertimbangkan suatu elektron 20 GeV.]

(i) What is its Lorentz factor γ?
[Apakah faktor Lorentznya?]

(ii) What is its de Broglie wavelength?
[Apakah jarak gelombang de Broglie-nya?]

Serway solution manual 2, Q12, pg. 376, modified

(c) A photon is emitted as a hydrogen atom undergoes a transition from the n = 6 state to the n = 2 state. Calculate
[Suatu foton dipancarkan ketika suatu atom hidrogen melakukan satu peralihan dari keadaan n = 6 ke n = 2. Hitungkan]

(i) the energy
[tenaga]

(ii) the wavelength
[jarak gelombang]

(iii) the frequency
[frekuensi]

of the emitted photon
[foton yang dipancarkan]

Serway solution manual 2, Q47, pg. 360, modified

Solution:

Q4a(i)

In the particle-in-a-box model, standing wave is formed in the box of dimension L:

$$\lambda_n = \frac{2L}{n}$$

[1 marks]

The energy of the particle in the box is given by

$$K_n = E_n = \frac{p_n^2}{2m_e} = \frac{(h/\lambda_n)^2}{2m_e} = \frac{n^2h^2}{8m_eL^2} = \frac{n^2\pi^2\hbar^2}{2m_eL^2}$$

[2 marks]

$$E_1 = \frac{\pi^2\hbar^2}{2m_eL^2} = 37.7 \text{ eV}$$

[2 mark]

$$E_4 = 4^2 E_1 = 603 \text{ eV}$$

[2 mark]

Q4a(ii)

The wavelength of the photon going from n = 4 to n = 1 is $\lambda = hc/(E_6 - E_1)$
= 1240 eV nm/ (603 – 37.7) eV = **2.2 nm**

[2 marks]

Q4b(i)

From $E = \gamma m_ec^2$, $\gamma = E/m_ec^2$ = 20 GeV/0.51 MeV = **39216**

[4 marks]

Q4b(ii)

Momentum $p = E/c$ = 20 GeV/c (rest mass of electron ignored, $m_ec^2 << E$)
$\lambda = hc/E = hc/pc$ = 1240 eV nm / 20 GeV = **6.2 x 10^{-17} m**
[3 marks]

Q4c

For hydrogen, $E_n = -\frac{13.6}{n^2}$ eV

Q4c(i)

$$\Delta E_{6\to 2} = E_6 - E_2 = -13.6\left(\frac{1}{6^2} - \frac{1}{2^2}\right)\text{eV} = 3.02\text{eV}$$

[3 marks]

Q4c(ii)

$\lambda_{6\to 2} = hc / \Delta E_{6\to 2} = 1240\text{nm}\cdot\text{eV} / 3.02\text{eV} =$ **410 nm**
[3 marks]

Q4c(iii)

$\nu = c/\lambda$ = **7.32 x 10^{14} Hz**
[3 marks]

UNIVERSITI SAINS MALAYSIA

KSCP
Academic Session 2003/2004

April 2004

ZCT 104E/3 - Physics IV (Modern Physics)
[Fizik IV (Fizik Moden)]

Duration: 3 hours
[Masa: 3 jam]

Please check that the examination paper consists of **ELEVEN** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBLELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer all **FOUR (4)** questions.

Students are allowed to answer all questions in Bahasa Malaysia or in English.

Please answer Question 1 in the objective answer form provided. Submit the objective answer form and the answers to the structured questions (i.e. Q2 – Q4) separately.

*[**Arahan**: Jawab kesemua **EMPAT** soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris. Sila jawab Soalan 1 dalam kertas jawapan objecktif yang dibekalkan. Hantar kertas jawapan objecktif dan jawapan kepada soalan struktur (iaitu Soalan 2 – Soalan 4) berasingan.]*

Data
speed of light in free space, c = 3.00 x 10^{8} m s^{-1}
permeability of free space, μ_0 = 4π x 10^{-7} H m^{-1}
permittivity of free space, ε_0 = 8.85 x 10^{-12} F m^{-1}
elementary charge, e = 1.60 x 10^{-19} C
the Planck constant, h = 6.63 x 10^{-34} J s
unified atomic mass constant, u = 1.66 x 10^{-27} kg
rest mass of electron, m_e = 9.11 x 10^{-31} kg
rest mass of proton, m_p = 1.67 x 10^{-27} kg
molar gas constant, = 8.31 J K^{-1} mol^{-1}
the Avogadro constant, N_A = 6.02 x 10^{23} mol^{-1}
gravitational constant, G = 6.67 x 10-11 N m^{2} kg^{-2}
acceleration of free fall, g = 9.81 m s^{-2}

Q1. [25 marks]

1.1 What were the consequences of the negative result of the Michelson-Morley experiment?
[Antara berikut yang manakah akibat keputusan negatif eksperimen Michelson-Morley?]

I. It render untenable the hypothesis of the ether
[Ia menjadikan hipotesis ether tidak dapat dipertahankan]

II. It suggests the speed of light in the free space is the same everywhere, regardless of any motion of source or observer
[Ia mencadangkan bahawa laju cahaya dalam ruang bebas adalah sama di mana-mana sahaja, tidak kira sama ada punca cahaya atau pemerhati mempunyai sebarang pergerkan]

III. It implies the existence of a unique frame of reference in which the speed of light in this frame is equal to c
[Ia mengimplikasikan kewujudan suatu rangka rujukan yang laju cahaya dalam rangka tersebut adalah bersamaan dengan c]

A. III only **B. I,II** **C. I, III** **D. I, II, III**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: B
Murugeshan, S. Chand & Company, New Delhi, pg. 25, Q1.

1.2 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I. The expression for kinetic energy of a relativistic particle is given by $\frac{1}{2}mv^2$
[Ekspresi tenaga kinetic suata zarah kerelatifan ialah $\frac{1}{2}mv^2$]

II. Special theory of relativity is applicable to accelerating system
[Teori kerelatifan khas boleh dipergunakan ke atas sistem yang mengalami pecutan]

III. The maximal velocity ever attainable is that of light in free space
[Laju maksimum yang mungkin tercapai ialah laju cahaya dalam ruang bebas]

IV. The mass of a particle becomes infinite at the speed equal to c
[Jisim suatu zarah menjadi infinit pada kelajuan bersamaan dengan c]

SESSI 03/04/KSCP

A. II,III **B. I,II,III,IV** **C. I, II, III** **D. III, IV**

E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: D
Murugeshan, S. Chand & Company, New Delhi, pg. 18, Q23.(for I), pg. 26, Q5.(for II), pg. 27, Q12.(for III), pg. 27, Q14.(for IV),

1.3 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I The concept of Bohr orbit violates the uncertainty principle
[Konsep orbit Bohr melanggar prinsip ketidakpastian]

II A hydrogen atom has only a single electron
[Atom hidrogen mempunyai satu elektron tunggal sahaja]

III The spectrum of hydrogen consists of many lines even though a hydrogen atom has only a single electron
[Spektrum hidrogen terdiri daripada banyak pinggir (garisan) walaupun atom hidrogen hanya mempunyai satu elektron sahaja]

IV Most of an atom consists of empty space
[Kebanyakan daripada isipadu suatu atom terdiri daripada ruang kosong]

A. I,II **B. I,II,III,IV** **C. I, II, III** **D. III, IV**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: D
Murugeshan, S. Chand & Company, New Delhi, pg. 86, Q13.(for I), pg. 88, Q19.(for II,III), pg. 87, Q11.(for IV)

1.4 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I In the Bohr theory of the hydrogen atom, the potential energy of the orbiting electron is positive
[Dalam teori atom hidrogen Bohr, tenaga keupayaan elektron yang mengorbit ialah positif]

II In the Bohr theory of the hydrogen atom, the kinetic energy of the orbiting electron is positive
[Dalam teori atom hidrogen Bohr, tenaga kinetik elektron yang mengorbit ialah positif]

III In the Bohr theory of the hydrogen atom, the potential energy of the orbiting electron is negative

[Dalam teori atom hidrogen Bohr, tenaga keupayaan elektron yang mengorbit ialah negatif]

IV. In the Bohr theory of the hydrogen atom, the kinetic energy of the orbiting electron is negative
[Dalam teori atom hidrogen Bohr, tenaga kinetik elektron yang mengorbit ialah negatif]

A. I,II **B. III,IV** **C. I, IV** **D. II, III**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: D
Murugeshan, S. Chand & Company, New Delhi, pg. 91, Q36

Q1.5 – Q1.7 refers to the energy diagrams shown in Figure 1.
[Soalan 1.5 - Soalan 1.7 merujuk kepada gambarajah yang terpapar di Gambarajah 1.]

Some of the energy levels of the hydrogen atom are shown (not to proportion)
[Beberapa paras tenaga atom hidrogen dipaparkan seperti berikut (tidak mematuhi nisbah)]

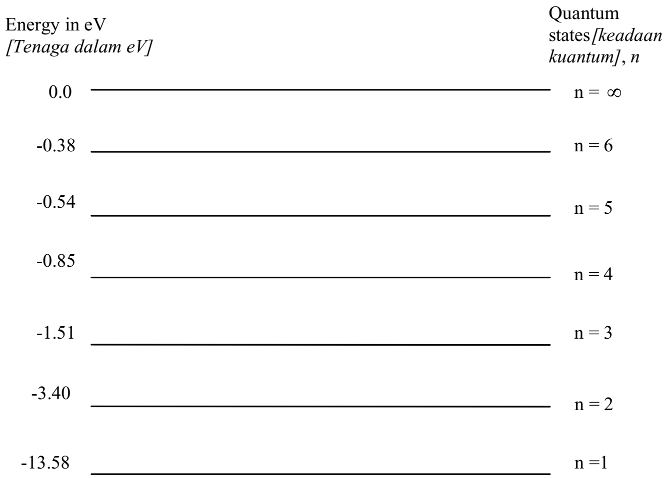


Figure 1 *[Gambarajah 1]*

1.5 How much energy in eV is required to raise an electron from the ground state to the n = 5 state? (ignore selection rules)
[Apakah tenaga (dalam unit eV) yang diperlukan untuk menaikkan suatu elektron dari keadaan bumi ke keadaan n = 5? (abaikan petua pilihan)]

SESSI 03/04/KSCP

A. 13.58 **B.** 10.18 **C.** 12.73 **D.** 13.04
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: D
Murugeshan, S. Chand & Company, New Delhi, pg. 92, Q44, modified;
Diagram adopted from Gautreau and Savin, Schaum's series, pg. 105.

1.6 What is the approximate wavelength of photon (in nm) emitted when the electron makes a transition from state n = 6 to n = 2? (ignore selection rules)
[Apakah anggaran jarak gelombang (dalam unit nm) untuk foton yang terpancar semasa elektron beralih dari keadaan n = 6 ke n = 2? (abaikan petua pilihan)]

A. 91 **B.** 122 **C.** 94 **D.** 410
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: D
My own question

1.7 How many different photons can be emitted by the hydrogen atom that undergoes transitions to the n = 4 state from the n = 6 state? (ignore selection rules)
[Terdapat berapa foton berbeza yang terpancar oleh atom hidrogen yang mengalami peralihan ke keadaan n = 6 dari keadaan n = 4? (abaikan petua pilihan)]

A.3 **B.** 4 **C.** 1 **D.** 6
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: A
Murugeshan, S. Chand & Company, New Delhi, pg. 90, Q30, modified

1.8 In relativity, which of the following observable(s) is (are) not absolute but depend on the reference frame of observer?
[Dalam teori kerelatifan, pembolehcerap yang mana adalah tidak mutlak tetapi bersandar kepada rangka rujukan pemerhati?]

I. Space
II. Time
III. Mass
IV. Energy

A. I,II **B. I,II,III,IV** **C. I, II, III** **D. III,IV**
E. Non of the above *[Tiada dalam pilihan di atas]*
Ans: B
Murugeshan, S. Chand & Company, New Delhi, pg. 28, Q23.

1.9 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I. γ-rays have much shorter wavelength than x-rays
[Jarak gelombang sinar γ adalah jauh lebih pendek daripada jarak gelombang sinar x]

II. The wavelength of x-rays in a x-ray tube can be controlled by varying the accelerating potential
[Jarak gelombang sinar x dalam suatu tiub sinar x dapat dikawal dengan menyelaraskan beza upaya pecutan]

III. x-rays are electromagnetic waves
[Sinar x ialah gelombang elektromagnetik]

IV. x-rays show diffraction pattern when passing through crystals
[Sinar x memperlihatkan corak belauan semasa ia melalui hablur]

A. I,II **B. I,II,III,IV** **C. I, II, III** **D. III.IV**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: B
Murugeshan, S. Chand & Company, New Delhi, pg. 132, Q1.(for I), pg. 132, Q3 (for II), pg. 132, Q4 (for III,IV)

1.10 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I. Photoelectric effect arises due to the absorption of electrons by photons
[Kesan fotoelektrik muncul kerana penyerapan elektron oleh foton]

II. Compton effect arises due to the scattering of photons by free electrons
[Kesan Compton muncul kerana penyerakan foton oleh elektron bebas]

III. In the photoelectric effect, only part of the energy of the incident photon is lost in the process
[Dalam kesan fotoelektrik, hanya sebahagian daripada tenaga foton tuju terlesap dalam proses tersebut]

IV. In the Compton effect, the photon completely disappears and all of its energy is given to the Compton electron
[Dalam kesan Compton, foton hilang langsung dan kesemua tenaganya diberikan kepada elektron Compton]

A. I,II **B. II,III,IV** **C. I, II, III** **D. III,IV**

Ans: E [I = false; II = true; III = false; IV = false]
Murugeshan, S. Chand & Company, New Delhi, pg. 134, Q13,

SESSI 03/04/KSCP

1.11 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I. Compton effect is experimentally observed for visible light rays
[Kesan Compton boleh dicerap secara eksperimen bagi cahaya ternampak]

II. The presence of the unmodified line in Compton scattering can be explained in terms of Rayleigh scatterings
[Kehadiran pinggir (garisan) yang tidak terubah dalam penyerakan Compton dapat diterangkan dengan penyerakan Rayleigh]

III. In Compton scattering, one neglects the effect of the nucleus on the x-rays
[Dalam penyerakan Compton, kita mengabaikan kesan ke atas sinar x oleh nucleus **]**

A. II, III **B. I, III** **C. I, II, III** **D. II only**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: A
Murugeshan, S. Chand & Company, New Delhi, pg. 134, Q14 (for I), Q15 (for II), Q16 (for III),

1.12 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I The energy of the quantum of light is proportional to the frequency of the wave model of light
[Tenaga kuantum cahaya adalah berkadar dengan frekuensi model gelombang cahaya]

II In photoelectricity, the photoelectrons has as much energy as the quantum of light which causes it to be ejected
[Dalam kesan fotoelektrik, fotoelektron mempunyai tenaga sebanyak tenaga kuantum cahaya yang menyebabkan fotoelektron terlenting]

III In photoelectricity, no time delay in the emission of photoelectrons would be expected in the quantum theory
[Dalam teori kuantum, tiada tunda masa dalam pemancaran fotoelektron dijangkakan untuk kesan fotoelektrik]

A. II, III **B. I, III** **C. I, II, III** **D. I ONLY**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: B

Murugeshan, S. Chand & Company, New Delhi, pg. 136, Q28 (for I), Q29, Q30 (for II,III)

1.13 An electron, proton and an alpha-particle have the same de Broglie wavelength. Which one moves faster?
[Elektron, proton dan zarah alpha ketiga-tiganya mempunyai jarak gelombang de Broglie yang sama. Yang manakah bergerak dengan lebih pantas?]

A. Electron **B.** Proton **C.** Alpha-particle
D. All particles move at the same speed *[kesemua zarah bergerak dengan kelajuan yang sama]*
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: A
Murugeshan, S. Chand & Company, New Delhi, pg. 163, Q3

1.14 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

I. The de Broglie wavelengths of macroscopic bodies are generally too tiny to be experimentally detected
[Jarak gelombang de Broglie jasad makroskopik secara amnya adalah terlalu kecil untuk dikesan secara eksperimen]

II. If Planck's constant were smaller than it is, quantum phenomena would be more conspicuous than they are now
[Jika nilai pemalar Planck adalah lebih kecil daripada nilainya yang sedia ada, fenomena kuantum akan menjadi lebih sedia tercerap berbanding dengan ketercerapannya yang sedia ada]

III In quantum theory, the physical variables (e.g. energy, momentum) used to describe a confined electron are discrete
[Dalam teori kuantum, pembolehubah fizikal (misalnya tenaga dan momentum) yang memerihalkan sesuatu elektron yang terkurung adalah diskrit

A. II, III **B. I ONLY** **C. I, II, III** **D. I, III**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: D
Murugeshan, S. Chand & Company, New Delhi, pg. 163, Q1 (for I), Q12 (for II), Q21 (for III)

1.15 Which of the following statement(s) is (are) true?
[Manakah kenyataan yang berikut adalah benar?]

SESSI 03/04/KSCP

I. The **e**xperimental proof for which electron posses a wavelength $\lambda = \frac{h}{p}$ was first verified by Davisson and Germer
[Pembuktian scara eksperimen bahawa elektron mempunyai jarak gelombang $\lambda = \frac{h}{p}$ *pada mula-mulanya ditentukan oleh Davisson and Germer]*

II. The experimental proof of the existence of discrete energy levels in atoms involving their excitation by collision with low-energy electron was confirmed in the Frank-Hertz experiment
[Pembuktian secara eksperimen kewujudan paras tenaga diskrit dalam atom yang melibatkan pengujaan mereka oleh perlanggaran dengan elektron bertenaga rendah telah dipastikan dalam eksperimen Frank-Hertz]

III. Compton scattering experiment establishes that light behave like particles
[Penyerakan Compton menetapkan bahawa cahaya berlagak seperti zarah]

IV. Photoelectric experiment establishes that electrons behave like wave
[Kesan fotoelektrik menetapkan bahawa elektron berlagak seperti gelombang]

A. I,II **B. I,II,III,IV** **C. I, II, III** **D. III,IV**
E. Non of the above *[Tiada dalam pilihan di atas]*

Ans: C
Serway and Moses, pg. 127 (for I), pg. 133 (for II), own options (for III,IV)

Q2. [25 marks]

(a) A man in a spaceship moving at a velocity of 0.9c with respect to the Earth shines a light beam in the same direction in which the spaceship is travelling.
[Seorang yang berada di dalam satu kapal angkasa yang bergerak pada halaju 0.9c relatif kepada Bumi menyinarkan satu bim cahaya ke arah yang mana kapal angkasa itu sedang bergerak.]

Compute the velocity of the light beam relative to Earth using
[Hitungkan halaju bim cahaya itu relatif kepada Bumi dengan menggunakan]

(i) Galilean approach *[pendekatan Galileo]*

[3 marks]

(ii) Special relativity approach *[pendekatan teori kerelatifan khas]*

[6 marks]

Please define clearly all the symbols used in your working.
[Sila nyatakan dengan jelas definasi simbol-simbol yang digunakan dalam kerja anda.]
Ans

(a) O' is the moving frame travelling at v = 0.9c with respect to the Earth. Speed of the light beam as seen in the frame O' is $u' = c$. O is the Earth frame. We wish to find the speed of the light beam as seen from frame O, u.

(i) According to Galilean transformation, $u = u' + v = c + 0.9c = 1.9c$.

(ii) Use

$$u = \frac{u'+v}{1+\left(\frac{v}{c^2}\right)u'} = \frac{c+0.9c}{1+\left(\frac{0.9c}{c^2}\right)c} = c \Rightarrow v = c$$

Acosta, Q4-7, pg. 53, modified

(b) How fast does a rocket have to go for its length to be contracted to 99% of its rest length?
[Berapa cepatkah suatu roket harus bergerak supaya panjangnya menyusut kepada 99% daripada panjang rehatnya?]

[5 marks]

Ans:

$$\frac{L}{L_0} = 0.99 = \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$\Rightarrow v = 0.141c$

Gautreau and Savin, Schaum's series modern physics, pg.21, Q. 4.1

(c) The average lifetime of μ-meson with a speed of 0.95c is measured to be 6×10^{-6} s. Compute the average lifetime of μ-meson in a frame in which they are at rest.
[Hayat purata meson-μ yang bergerak dengan kelajuan 0.95c adalah diukur sebagai 6×10^{-6} s. Hitungkan hayat purata meson-μ dalam rangka di mana mereka adalah rehat]

[5 marks]

Ans:

Lorentz factor is $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)}} = \frac{1}{\sqrt{1-(0.95)}} = 3.20$

The time measured in a frame in which the μ-mesons are at rest is the proper time, Δt_0:

$\Delta t_0 = \Delta t / \gamma = = 6\times10^{-6}\,\text{s}/3.2 = 1.87\times10^{-6}\,\text{s}$

Gautreau and Savin, Schaum's series modern physics, pg.24, Q 5.1

(d) (i) What is the rest mass of a proton in terms of MeV?
[Apakah jisim rehat satu proton dalam unit MeV?]

[2 marks]

(ii) What is the relativistic mass of a proton (in terms of MeV) whose kinetic energy is 1 GeV?
[Apakah jisim kerelatifan satu proton (dalam unit MeV) yang bertenaga kerelatifan 1 GeV?]

[4 marks]

Ans:

(i) $m_p c^2 = 1.67 \times 10^{-27}\text{kg} \times (3\times10^8\,\text{m/s})^2 = 1.503\times10^{-10}$ J = $1.503\times10^{-10}/(1.6\times10^{-19})$ eV = 939.4 MeV

(ii) $K = (\gamma-1)m_p c^2 = 1$ GeV

$(\gamma-1) = 1\,\text{GeV}/m_p c^2 = 1\,\text{GeV}/939.4\text{MeV} = 1.06$

$\gamma = 1.06 + 1 = 2.06$

$mc^2 = \gamma m_p c^2 = 2.06\times939.4$ MeV = 1939.4 MeV

Note: Due to the inconsistency between the English and Malay version of question I would also give full mark to those who used total relativisic energy $E = \gamma m_p c^2 = 1$ GeV in the calculation (instead of using $K = (\gamma - 1)m_p c^2 = 1$ GeV).

Gautreau and Savin, Schaum's series modern physics, pg.55, Q 8.34, slightly modified.

Q3. [25 marks]

(a) A proton is accelerated from rest through a potential of 1 kV. Find its de Broglie wavelength.
[Suatu proton dipecutkan dari keadaan rehat melalui satu beza keupayaan 1 keV. Hitungkan jarak gelombang de Broglienya.]
[6 marks]

Ans.

$K = \frac{p^2}{2m_p}$ = kinetic energy of the proton = 1 keV.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_p K}} = \frac{h}{\sqrt{2m_p K}} = \frac{6.63\times10^{-34}\,\mathrm{J\cdot s}}{\sqrt{2\times1.67\times10^{-27}\,\mathrm{kg}\cdot1000\times1.6\times10^{-19}\,\mathrm{J}}} =$$

$9.1\times10^{-3}\,\overset{o}{\mathrm{A}}$

Gautreau and Savin, Schaum's series modern physics, pg.97, Q. 10.38

(b) Determine the cutoff wavelength in $\overset{o}{\mathrm{A}}$ of *x*-rays produced by a 50-keV electrons in a *x*-ray tube.
[Tentukan jarak gelombang penggal (dalam unit $\overset{o}{\mathrm{A}}$) sinar x yang dihasilkan oleh elektron 50 keV dalam suatu tiub sinar x.]
[5 marks]

Ans.

$$\lambda_{cutoff} = \frac{hc}{eV} = \frac{1240\mathrm{eV\cdot nm}}{50\mathrm{keV}} = 0.0248\mathrm{nm} = 0.24\,\overset{o}{\mathrm{A}}$$

Schaum's series 3000 solved problem, pg.714, Q. 38.39

(c) Determine the photon flux (in unit of number of photons per unit time per unit area) associated with a beam of monochromatic light of wavelength 3000 $\overset{o}{\mathrm{A}}$ and intensity 3×10^{-14} W/m^2.

SESSI 03/04/KSCP

[Tentukan fluks foton (dalam unit bilangan foton per unit masa per unit luas) yang bersepadanan dengan suatu bim cahaya monokromatik berjarak gelombang 3000 $\overset{o}{\mathrm{A}}$ dan berkeamatan 3×10^{-14} W/m^2.]
[8 marks]

Ans:

$$N = I/\varepsilon = I\cdot\left(\frac{\lambda}{hc}\right)$$
$$= 3\times10^{-14}\,\mathrm{W/m^2}\times\frac{300\mathrm{nm}}{1240\mathrm{eV\cdot nm}}$$
$$= 7.26\times10^{-15}\left(\frac{\mathrm{W}}{\mathrm{eV}}\right)/\mathrm{m^2} = 7.26\times10^{-15}\left(6.25\times10^{18}/\mathrm{s}\right)/\mathrm{m^2} = 45375\mathrm{photon/m^2\cdot s}$$
$$= 4.5\mathrm{photon/cm^2\cdot s}$$

Gautreau and Savin, Schaum's series modern physics, pg.98, Q. 10.53

(d) Suppose that the *x*-component of the velocity of a 2×10^{-4} kg mass is measured to an accuracy of $\pm10^{-6}$ m/s. What is the limit of the accuracy with which we can locate the particle along the *x*-axis?
[Andaikan bahawa komponen x halaju suatu jasad berjisim 2×10^{-4} kg diukur tepat kepada kejituan $\pm10^{-6}$ m/s. Apakah limit kejituan kedudukannya yang boleh kita pastikan sepanjang paksi-x?]
[6 marks]

Ans.

$$\Delta p\Delta x \ge \frac{\hbar}{2};\ p = mv;$$
$$\Delta(mv)\Delta x = m\Delta v\Delta x \ge \frac{\hbar}{2}$$
$$\Delta x \ge \frac{\hbar}{2m\Delta v} = \frac{h}{4\pi m\Delta v} = 2.63\times10^{-25}\,\mathrm{m}$$

Gautreau and Savin, Schaum's series modern physics, pg.98, Q. 10.53

Q4. [25 makrs]

(a) Given the ground state energy of hydrogen atom -13.6 eV, estimate the ionisation energy for He^+.
[Diberi bahawa tenaga keadaan bumi atom hidrogen ialah -13.6 eV, anggarkan tenaga pengionan untuk He^+.]
[5 marks]

Ans: Generally, the energy state of an hydrogen-like atom with Z charge in its nucleus is given by $E_n = \frac{Z^2}{n^2}E_0$, E_0 = ground state energy of hydrogen atom.

Hence ionisation energy of He^+ (with $Z = 2$) =

$$E_\infty(He^+) - E_0(He^+) = 0 - \frac{2^2}{1^2}E_0 = -4(-13.6)\,\text{eV} = 54.4\text{ eV}$$

Serway solution manual 2, Q43, pg. 360, modified

(b) What are the n values in the transition that produces the third longest wavelength in the Balmer series in the hydrogen atom? (ignore selection rules)
[Apakah nilai-nilai n yang peralihannya menghasilkan jarak gelombang yang ketiga paling panjang dalam siri Balmer atom hidrogen? (abaikan petua pilihan)]

[4 marks]

Ans: **n = 5 → n = 2**
Giancoli, pg. 856, Q. 50, modified.

(c) Given the Bohr radius of the hydrogen atom $r_0 = 0.5$ Å, estimate the speed (in m/s) of the electron in the ground state orbit of the hydrogen atom.

[Diberi bahawa radius Bohr atom hidrogen ialah $r_0 = 0.5$ Å, anggarkan laju (dalam m/s) elektron dalam orbit keadaan bumi atom hidrogen.]

[8 marks]

Ans: Equating the centrepetal force required by the electron to the electrostatic force,

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \Rightarrow v_0^2 = \frac{e^2}{4\pi\varepsilon_0 m r_0} \Rightarrow v_0 = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m r_0}} = 2.25\times10^6\text{ m/s}$$

My own question

(d) Given the Rydberg constant $R = 1.0967758 \times 10^{-3}$ Å^{-1}, determine, in Å,

(i) the shortest, and
(ii) the longest

wavelengths of the Lyman series of hydrogen.

SESSI 03/04/KSCP

[Diberi bahawa pemalar Rydberg ialah $R = 1.0967758 \times 10^{-3}$ Å^{-1}. Tentukan, dalam unit Å, jarak gelombang yang

(i) paling pendek, dan
(ii) paling panjang

dalam siri Lyman hidrogen]

[4 + 4 marks]

Ans:

(i) Wavelengths in the Lyman series are given by $n_l = 1$

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right),\ n = 2,3,4...$$

(ii) The longest wavelength corresponds to $n = 2$:

$$\frac{1}{\lambda_{max}} = \left(1.097\times10^{-3}\,\text{Å}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right),\ \text{or } \lambda_{max} = 1215\,\text{Å}$$

The longest wavelength corresponds to $n \to \infty$

$$\frac{1}{\lambda_{min}} = \left(1.097\times10^{-3}\,\text{Å}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right),\ \text{or } \lambda_{max} = 912\,\text{Å}$$

Gautreau and Savin, Schaum's series modern physics, pg.107, Q. 11.1

ZCT 104/3E Modern Physics
Semester II, Sessi 2004/05
Test I (17 Dec 2004)

Data
Speed of light in free space, $c = 3.00 \times 10^8$ ms^{-1}
Elementary charge, $e = 1.60 \times 10^{-19}$ C
The Planck constant, $h = 6.63 \times 10^{-34}$ J s
Unified atomic mass constant, $u = 1.66 \times 10^{-27}$ kg
Rest mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Rest mass of proton, $m_p = 1.67 \times 10^{-27}$ kg

1. What are the major flaws in the classical model of blackbody radiation given by Rayleigh-Jeans laws?
   I (F) Molecular energy is quantized
   II (F) Molecules emit or absorb energy in discrete irreducible packets
   III(T) The intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases.
   IV (T) Energy is continuously divisible

   A. III, IV **B. I, II,III** **C. II, III, IV** **D. I, II**
   E. Non of the above

   ANS:A, Serway, questions 1, 2, page 1313

2. What are the assumptions did Planck make in dealing with the problem of radiation?
   I (T) Molecular energy is quantized
   II (T) Molecules emit or absorb energy in discrete irreducible packets
   III(F) The intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases.
   IV (F) Energy is continuously divisible

   A. III, IV **B. I, II,III** **C. II, III, IV** **D. I, II**
   E. Non of the above

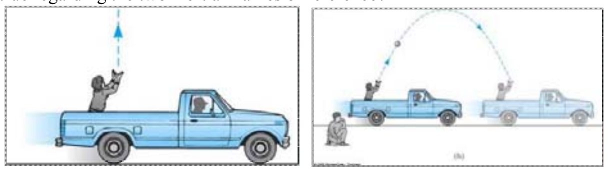
   ANS:D, Serway, questions 1, 2, page 1313

3. An unstable high-energy particle enters a detector and leaves a track of length d before it decays. Its speed relative to the detector was $v = c/2$. What is its proper lifetime? That is how long would the particle have lasted before decay had it been at rest with respect to the detector?

   A. $\frac{d}{c}$ **B.** $\frac{4d}{\sqrt{3}c}$ **C.** $\frac{2d}{\sqrt{3}c}$ **D.** $\frac{\sqrt{3}d}{c}$ **E. Non of the above**

   RHW 7th ed. P5, pg. 1050
   Solution: D

4. A ball was thrown upward by an observer in a van moving with constant speed $u << c$. He is observed by an observer in a rest frame attached to the ground, see figure below. Which of the following statement(s) is (are) true regarding the two inertial frames of reference?



   I The ball thrown follows different path

   II The kinematical laws of classical mechanics are valid only the moving frame (the van) but not to the rest frame attached to ground.
   III Classically Galilean transformation relates the trajectory of the ball in the rest frame with that in the moving frame.
   IV Since $u<<c$, Lorentz transformation will fail to relate the trajectory of the ball in the rest frame with that in the moving frame.

   A. II,III **B. I, II,III** **C. II, III, IV** **D. I Only** **E. Non of the above**
   My own question
   Solution: E (I, III are true)

5. What measurement(s) do two observers in relative motion always agree on?
   I The relativistic mass of an object
   II The relativistic momentum of an object
   III The relativistic energy of an object
   IV $E^2 - p^2$, where p is the magnitude of relativistic momentum and E the relativistic energy the object

   A. II,III **B. I, II,III** **C. II, III, IV** **D. IV Only**
   E. Non of the above

   My own question
   Solution: D
   Free marks will be given for this question due to the typo in IV. It should actually reads: "$E^2 - c^2p^2$, where p is the ".

   Actually, the original statement is dimensionally correct in the natural unit system in which the c is taken to have a value of 1. However since we are adopting S.I. unit throughout the course we will take the original statement to be 'dimensionally wrong' as far as the ZCT 104 courses is concerned.

6. Which of the following statement(s) is (are) true?
   I The upper limit of the speed of an electron is the speed of light c.
   II As more energy E is fed into an object its momentum approaches $\frac{E}{c}$.
   III There is no upper limit to the relativistic momentum of an electron.
   IV There is an upper limit to the relativistic momentum of an electron.

   A. III **B. I, II,III** **C II, IV** **D. IV Only** **E. Non of the above**
   Serway Q12, pg. 1276
   Solution: B

7. The rest energy and total energy respectively, of three particles, expressed in terms of a basic amount A are (1) A, $2A$; (2) A, $3A$; (3) $3A$, $4A$. Without written calculation, rank the particles according to their kinetic energy, greatest first.

   A. 2 > 1 = 3 **B.** 1 > 2 = 3 **C.** 2 > 1 > 3 **D.** 2 = 1 = 3
   E. 3 > 1 = 2

   RHW 7th ed. Q1, pg. 1050
   Solution: A

8. The length of a spaceship is measured to be exactly half its rest length. By what factor do the spaceship's clocks run slow relative to clocks in the observer's frame?

   A. 0.866 **B.** 0.745 **C.** 2.000 **D.** 0.366 **E.** 0.134

Solution: C

9. The length of a spaceship is measured to be exactly half its rest length. What is the speed parameter $\beta = v/c$ of the spaceship relative to the observer's frame?

A. 0.87 **B.** 2.00 **C.** 0.75 **D.** 2.73 **E.** 4.00

ANS: A

We solve $L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2} = L_0\sqrt{1-\beta} = \frac{L_0}{\gamma}$ **for** ***v*** **and then plug in:**

$$\beta = \sqrt{1-\left(\frac{L}{L_0}\right)^2} = \sqrt{1-\left(\frac{1}{2}\right)^2} = 0.866.$$

Resnick and Halliday, 7th edition, Problem 12, Pg. 1051

10. Consider a light pulse emitted from the origin, O, of a stationary frame S. The origin of a moving frame S', O', which overlaps with O at $t = t' = 0$ is moving with a constant speed u with respect to O. Which statement(s) correctly describe(s) the position of the wavefront of the light sphere as measured from the origins? r (r') is the distance of the wavefront from the origin O (O') at time t (t').

I $r = ct$ **II** $r' = ct'$ **III** $r' = r$ **IV** $r' = ut'$

A. I,II **B. I, II,III** **C. II, III, IV** **D. IV Only** **E. Non of the above**

My own question

Solution: A

11. Which of the following statement(s) is (are) true regarding Lorentz transformation (LT)?

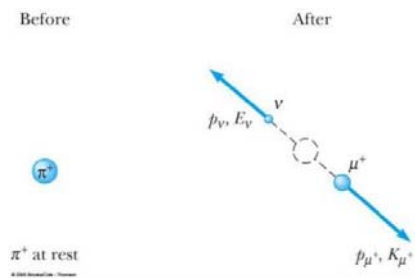
I Time dilation can be recovered from LT
II Length contraction can be recovered from LT
III Absolute simultaneity is not guaranteed by LT
IV Galilean transformation is a generalisation of LT

A. II,III **B. I, II,III** **C. II, III, IV** **D. I, II** **E. Non of the above**

My own question

Solution: B

Question 12-13 are based on the decay of a π meson into a muon and a massless neutrino shown in figure below. The mass of the muon is known to be $m_\mu = 106$ **MeV/**c^2**, and the kinetic energy of the muon is measured to be** $K_\mu = 4.6$ **MeV.** p_μ **denotes the momentum of the muon.**



12. What is the momentum of the neutrino?

A. $\sqrt{(K_\mu + m_\mu c^2)^2 - m_\mu^2 c^4}$ **B.** $(K_\mu + m_\mu c^2)$

C. $\sqrt{2m_\mu K_\mu}$ **D.** p_μ **E. Non of the above**

Serway and Mosses. pg. 53

Solution: D

13. What is the total relativistic energy of the neutrino?

A. $\sqrt{(K_\mu + m_\mu c^2)^2 - m_\mu^2 c^4}$ **B.** $(K_\mu + m_\mu c^2) + \sqrt{(K_\mu^2 + 2K_\mu m_\mu c^2)}$

C. K_μ **D.** $m_\mu c^2$ **E. Non of the above**

Serway and Mosses. pg. 52

Ans: A

Solution: $E_\nu = \sqrt{(p_\nu^2 c^2 + m_\nu^2 c^4)} = p_\nu c$ ($m_\nu c^2 = 0$). The momentum of neutrino, $p_\nu^2 = p_\mu^2$ (from Question 12 above) is related to the kinetic energy of the muon via $E_\mu = \sqrt{(p_\mu^2 c^2 + m_\mu^2 c^4)} = m_\mu c^2 + K_\mu$. Therefore the momentum of the neutrino is related to the kinetic energy of the muon via $p_\nu^2 c^2 = (m_\mu c^2 + K_\mu)^2 - m_\mu^2 c^4$. Taking the square root, we then have $E_\nu = p_\nu c = \sqrt{(K_\mu + m_\mu c^2)^2 - m_\mu^2 c^4}$.

14. Serway and Moses, Questions 12, page 37

What happens to the density of an object as its speed increases, as measured by an Earth observer?

A. Remain the same as it is when at rest
B. Increase by a factor of γ
C. Increase by a factor of γ^2
D. Increase by a factor of $1/\gamma$
E. Non of the above

ANS: **C**, my own question

15. What is the upper limit of the momentum of an electron?

A. $m_e c$ **B.** c **C.** 0 **D.** Infinity **E. Non of the above**

Serway, Q12, pg. 1276

Solution: D

16. Which of the following statement(s) is (are) true?

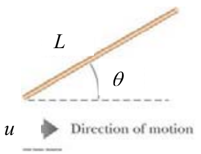
I Only massless particle can travel at the speed of c.
II Not all massless particle can travel at the speed of c.
III It is not necessary that a massless particle must travel at the speed of c.
IV All particles which are not massless must travel at the speed lower than c.

A. II,III **B. I, II,III** **C. I, III, IV** **D. I, IV** **E. Non of the above**

My own question

Solution: D

17. A moving rod is observed to have a length of L and to be orientated at an angle of $\theta = 45°$ with respect to the direction of motion, as shown in the figure below. The rod has a speed of $u = \frac{c}{\sqrt{2}}$.



What is the proper length of the rod?

A. $\frac{3}{2}L$ **B.** L **C.** $\sqrt{\frac{3}{2}}L$ **D.** $\frac{\sqrt{3}L}{2}$ **E. Non of the above**

Serway, P23, page 1279
Solution: C

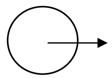
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}} = \sqrt{2}$$

We are also given L and θ (both measured in a reference frame moving relative to the rod).

Thus, $L_x = L\cos\theta = \frac{L}{\sqrt{2}}; L_y = L\sin\theta = \frac{L}{\sqrt{2}}$. L'_x is a proper length, related to L_x by $L_x = \frac{L'_x}{\gamma}$.

Therefore, $L'_x = \gamma L_x = \sqrt{2}\frac{L}{\sqrt{2}} = L$, and $L'_y = L_y = \frac{L}{\sqrt{2}}$. (Lengths perpendicular to the motion are unchanged). $\Rightarrow (L')^2 = (L'_y)^2 + (L'_x)^2 = \frac{L^2}{2} + L^2 = \frac{3L^2}{2} \Rightarrow L' = \sqrt{\frac{3}{2}}L$

18. A spaceship in the shape of a sphere moves past and observer on Earth with a speed of $v = 0.5c$ in the direction as indicated by the arrow. What shape will the observer see as the spaceship move past?



E. Non of the above
Solution: A

19. What is the speed of an object having relativistic momentum of magnitude p and rest mass m?

A. $\frac{p}{m}$ **B.** $\frac{c}{\sqrt{1 + (mc/p)^2}}$ **C.** $\frac{mc^2}{u}$ **D.** $\frac{mu^2}{c}$ **E. Non of the above**

Serway, P32, page 1280
Solution: B

20. An electron with rest mass m_e moves with a speed of $\frac{\sqrt{3}}{2}c$. What is the work required to increase its speed to $\frac{2\sqrt{2}}{3}c$?

A. $m_e c^2$ **B.** $0.511 m_e c^2$ **C.** $\frac{5}{36}m_e c^2$ **D.** $\frac{\sqrt{5}}{6}m_e c^2$ **E. Non of the above**

SESSI 04/05/TEST1

Serway, P35, page 1280
Solution: A

ZCT 104/3E Modern Physics
Semester II, Sessi 2004/05
Test II (18 Feb 200b)

1. Which statements is (are) TRUE about photoelectricity according to classical physics? (ANS: D)

I) Light beam of higher intensity is expected to eject electrons with higher kinetic energy from the metal surface (T)
II) In photoelectric experiment the energy carried by a beam of light is considered to be continuous (T)
III) Light is wave and not comprised of quantum of energy (T)
IV) When light is irradiated on the metal surface, some time lag is expected before photoelectrons are ejected from the surface (T)

A. I, II **B. II, III**
C. III **D. I, II, III, IV**
E. Non of A, B, C, D

2. Let a given metal surface is irradiated with monochromatic light of intensity I_1. Then the same surface is irradiated by monochromatic light with intensity I_2 (where $I_2 > I_1$) but with a longer wavelength. Which of the following statement(s) is (are) true? (ANS: E)

I) The energy of the photon in the beam with intensity I_2 is larger than that in the beam with intensity I_1. (F)
II) The saturated photocurrents will remain unchanged. (F)
III) The maximum kinetic energy of the photoelectron will increase for the beam with intensity I_2 (F)
IV) The different intensity of light will alter the work function of the metal surface (F)

A. I, II **B. II, III**
C. III **D. III, IV**
E. Non of A, B, C, D

3. Which of the following statements is (are) correct about Bohr's atom and a quantum particle trapped inside a simple infinite quantum well of width d? (ANS: A)

I) The gap separating energy levels of higher quantum number becomes closer and closer in the Bohr's hydrogen atom, whereas in the case of particle in a box the gap becomes larger and larger at higher quantum levels. (T)
II) The electron in the Bohr's atom is subjected to a non-zero potential due to Coulomb's attraction, whereas in the box the particle is subjected to zero potential. (T)
III) The energy levels in the Bohr's atom are negative whereas they are positive for the particle in the well. (T)
IV) In both cases the particles involved form standing waves (T)

A. I, II, III, IV **B. II, III**
C. III **D. III, IV**
E. Non of A, B, C, D

4. Which of the following statements is (are) true? (ANS: C)

I) A particle has a de Broglie wavelength that is related to its linear momentum (T)
II) A particle's momentum must be quantised in all systems, bounded or unbounded (F)
III) A particle's kinetic energy must be quantised in all systems, bounded or unbounded (F)
IV) A particle's kinetic energy is only quantised in bounded system (T)

A. I, II, IV **B. I, II, III**
C. I, IV **D. II, III**
E. Non of A, B, C, D

5. In order to have photoelectrons ejected from a metal surface in a typical photoelectric effect experiment, (ANS: C)

I) the frequency of the light used must be larger than a certain cut-off value (T)
II) the intensity of the light used must be larger than a certain cut-off value (F)
III) the wavelength of the light used must be larger than a certain cut-off value (F)
IV) the saturated photocurrent must be larger than a certain cut-off value (F)

A. I, II, IV **B. I, III**
C. I **D. II, III, IV**
E. Non of A, B, C, D

6. What of the following statements are TRUE regarding photoelectric effect (PE) and Compton effect (CE)? (ANS: D)

I) In PE light behaves like particle, whereas in CE light behave like wave (F)
II) In PE light behaves like wave, whereas in CE light behave like particle (F)
III) In PE only part of the photon's energy is lost to the atom, whereas in CE all of the photon's energy is lost to the free electron (F)
IV) In PE all of the photon's energy is lost to the atom, whereas in CE only part of the photon's energy is lost to the free electron (T)

A. I, III **B. II, III**
C. II, IV **D. IV**
E. Non of A, B, C, D

7. Which statements is (are) TRUE about photoelectric and Compton effects? (ANS: E)

I) Compton effect experiment confirms that the energy of the quantum of light is proportional to the frequency of the wave model of light (F)
II) Compton effect experiment confirms that the radiant energy of light is quantized into concentrated bundle (F)
III) Photoelectric effect infers that the radiant energy of light is quantized into concentrated bundle (T)
IV) Both Compton effect and photoelectric effect confirm that EM radiation has both wave and particle properties (F)

A. I, III **B. II, III**
C. II, IV **D. IV**
E. Non of A, B, C, D

8. Which of the following is (are) the correct statement(s) about X-ray production in a conventional X-ray tube? (ANS: B)

I) Part or all of the kinetic energy of the moving electron is converted into X rays photon (T)
II) X-rays is emitted when the bombarding electrons undergo Compton scattering (F)
III) The production of x-rays can be considered as a photoelectric process (F)
IV) The shortest wavelength in the x-rays spectrum is the same for different material (T)

A. II, III **B. I, IV**
C. II, IV **D. IV**
E. Non of A, B, C, D

9. Which of these statements is (are) true about blackbody radiation? (ANS: B)

I) Rayleigh-Jeans law is behaving in a physically acceptable manner at short wavelengths (F)
II) Rigel (the blue star) is hotter than Betelguese (red star) because of the position of the peak wavelength in their black body spectrum (T)
III) According to Rayleigh-Jeans law the average energy of the oscillators is given by the equipartition theorem (T)
IV) The spectral distribution of radiation from a blackbody can only be explained in terms of quantised energy levels of the oscillators (T)

A. I, II, III, IV **B. II, III, IV**
C. II, IV **D. III, IV**
E. Non of A, B, C, D

10. Which of these statements are correct? (ANS: E)

I) We conclude that light behave like wave when we find that the light from the sun arrives to the Earth after 8 minutes it was emitted. (F)
II) When we consider light to behave like a particle we expect some detectable time lag for the electron to be emitted from the surface of the metal in a PE experiment. (F)
III) When we consider light to behave like wave we expect some detectable time lag for the electron to be emitted from the surface of the metal in a PE experiment. (T)
IV) Photoelectric effect occurs at the same energy scale as that of the x-rays production because x-rays production is the inverse of the photoelectric process. (F)

A. I, II, III, IV **B. II, III, IV**
C. II, IV **D. III, IV**
E. III

11. Which of the following statements is (are) TRUE? (ANS: E)

I) The energy levels of the atomic orbit is quantized (T)
II) The energy associated with the orbits of the electron in a hydrogen atom is negative because it is not a bounded system (F)
III) E = 0 means the electron is free from the bondage of the nucleus' potential field. (T)
IV) Electron at very large quantum number n is tightly bounded to the nucleus by the EM force. (F)

A. I, II, III, IV **B. II, III, IV**
C. II, IV **D. III, IV**
E. I, III

12. Which of the following statements is (are) TRUE about the Bohr's model of hydrogen-like atom? (ANS: C)

I) It applies the Newton's second law for the atom's mechanical stability (T)
II) The angular momentum is postulated to be quantised via $L = nh/2\pi$ (T)
III) It assumes the validity of classical electromagnetic theory for the orbiting electron (F)
IV) The only stable orbits of radius r are those that can fit in a multiple number of standing wave of the electron, i.e $2\pi r = n\lambda$ (T)

A. I, II, III, IV **B. II, III, IV**
C. I, II, IV **D. III,IV**
E. Non of A, B, C, D

13. Which of the following statements is (are) true? (ANS C)

I) Thompson suggestion of the Plum Pudding Model is falsified by Rutherford's alpha particle experiment (T)
II) Rutherford suggested the planetary model of atoms. (T)
III) de Broglie is the first to experimentally confirm that electron manifests wave nature. (F)
IV) Frank-Hertz experiment confirms the existence of discrete energy levels in mercury atom (T)

A. I, II, III, IV **B. II, III, IV**
C. I, II, IV **D. III,IV**
E. Non of A, B, C, D

14. Which of the following statement is (are) true about the Plum-pudding model by Thompson and Rutherford's experiment? (ANS A)

I) Plum-pudding model fails to explain the emission & absorption line spectrum from atoms because it predicts only a single unique emission frequency. (T)
II) Plum-pudding model cannot explain the 180 degree back-scattering of alpha particle seen in Rutherford's scattering experiment. (T)
III) The planetary model of atoms is plagued by infrared catastrophe (T)
IV) In the Rutherford's alpha particle scattering experiment, the large deflection of alpha particle is caused by a close encounter between alpha particle and the diffused distribution of the positive charge of an atom. (F)

A. I, II, III **B. II, III, IV**
C. I, II, IV **D. III, IV**
E. Non of A, B, C, D

15. Which of the following statements is (are) true regarding the basic properties of atoms? (ANS: A)

I) Atoms are of microscopic size, ~10^{-10} m (T)
II) Atoms are stable (T)
III) Atoms contain negatively charges, electrons, but are electrically neutral. (T)
IV) Atoms never emit and absorb EM radiation. (*F*)

A. I, II, III **B. II, III, IV**
C. I, II, IV **D. III, IV**
E. Non of A, B, C, D

16. Which of the following statements is (are) true about Bohr's hydrogen-like atom? (ANS C)

I) The increase in the quantum number n means an increase in the energy of the atomic states. (T)

SESSI 04/05/TEST2

II) When n approach infinity, the energy states become infinity. (F)
III) Free electron is the electron which has the smallest quantum number n (F)
IV) The zero point energy is the energy of the lowest possible quantum level (T)

A. I, II, III **B. II, III, IV**
C. I, IV **D. III, IV**
E. Non of A, B, C, D

17. Heisenberg's uncertainty principle is a consequence of (ANS: A)

A. the intrinsic wave nature of particle
B. the intrinsic particle nature of wave
C. the indivisible nature of particle
D. the divisible nature of particle
E. probabilistic interpretation of the wave function

18. Which of the following statements is (are) true about the spectrum from hydrogen atom? (ANS: A)

I) Balmer series involve transitions of electron from higher orbits to the $n = 2$ orbit
II) Balmer series is the first spectral series of hydrogen atom observed
III) When electron in higher orbit is de-excited to lower orbit, photons of discrete frequency are emitted from the atom, as seen in the emission spectrum
IV) When electron in lower orbit is excited to higher orbit, photons of discrete frequency are absorbed by the atom, as seen in the absorption spectrum

A. I, II, III, IV **B. II, III, IV**
C. I, IV **D. III, IV**
E. Non of A, B, C, D

19. Which of the following statements is (are) true regarding a quantum particle trapped inside an infinite well of width L? (ANS B)

I) It forms stationary (standing) wave inside the well (T)
II) The linear momentum of the particle becomes quantised (T)
III) The minimum energy of the particle inside the well is given by $h^2/8mL^2$ (T)
IV) The energy of the particle inside the well can take on negative value (F)

A. I, II, III, IV **B. I, II, III**
C. I, IV **D. III, IV**
E. Non of A, B, C, D

20. Which of the following statements is (are) true regarding pair production and pair annihilation of electron-positron pair? (ANS D)

I) Pair annihilation occurs only above the threshold energy of $2m_ec^2$ (F)
II) Pair production occurs only above the threshold energy of $2m_ec^2$ (T)
III) Energy is always conserved in both processes of pair production and pair annihilation (T)
IV) Momentum is always conserved in both processes of pair production and pair annihilation (T)

A. I, II, III, IV **B. I, II, III**
C. I, IV **D. II, III, IV**
E. Non of A, B, C, D

UNIVERSITI SAINS MALAYSIA

Final Exam
Academic Session 2004/2005
March 2005

ZCT 104E/3 - Physics IV (Modern Physics)
[Fizik IV (Fizik Moden)]

Duration: 3 hours
[Masa: 3 jam]

Please check that the examination paper consists of **XXX** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **XXX** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

<u>Instruction</u>: Answer all questions. Please answer the objective questions from Part A in the objective answer sheet provided. Answer both structured questions from Part B. Please submit the objective answer sheet and the answers to the structured questions separately.

Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**<u>Arahan</u>**: Jawab **SEMUA** soalan. Sila jawab soalan-soalan objektif daripada bahagian A dalam kertas jawapan objektif yang dibekalkan. Jawab kedua-dua soalan struktur daripada Bahagian B. Hantar kertas jawapan objektif dan jawapan kepada soalan struktur berasingan.]*

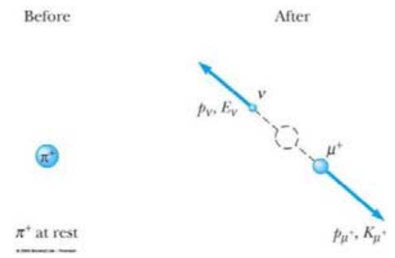
[Pelajar dibenarkan untuk menjawab samada dalam bahasa Malaysia atau bahasa Inggeris.]

Data
speed of light in free space, $c = 3.00 \times 10^8$ m s^{-1}
permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
elementary charge, $e = 1.60 \times 10^{-19}$ C
Planck constant, $h = 6.63 \times 10^{-34}$ J s
unified atomic mass constant, $u = 1.66 \times 10^{-27}$ kg
rest mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton, $m_p = 1.67 \times 10^{-27}$ kg
molar gas constant, = 8.31 J K^{-1} mol^{-1}
Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1}
gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
acceleration of free fall, $g = 9.81$ m s^{-2}

Part A: Objective
Instruction: Answer all 40 objective questions in this Part.
[Bahagian A: Objektif.]
[Arahan: Jawab kesemua 40 soalan objektif dalam Bahagian ini.]

Question 1 - 3 are based on the decay of a π meson into a muon and a massless neutrino shown in the figure below. The mass of the muon is known to be m_μ = 106 MeV/c^2, and the kinetic energy of the muon is measured to be K_μ = 4.6 MeV. p_μ denotes the momentum of the muon.

[Soalan 1-3 adalah berdasarkan pereputan satu meson π kepada satu muon dan satu neutrino tanpa jisim, sepertimana ditunjukkan dalam gambarajah di bawah. Diketahui jisim muon ialah m_μ = 106 MeV/c^2, dan tenaga kinetik muon yang terukur ialah K_μ = 46 MeV. p_μ menandakan momentum muon.]



1. How is the momentum of the muon, p_μ related to the kinetic energy of the muon? E_μ denotes the total relativistic energy of muon.
[Bagaimanakah momentum muon p_μ dikaitkan dengan tenaga kinetik muon? E_μ menandakan tenaga keretatifan muon]
A. $p_\mu c = \sqrt{\left(K_\mu + m_\mu c^2\right)^2 - m_\mu^2 c^4}$ **B.** $p_\mu = \sqrt{\left(K_\mu + m_\mu c^2\right)^2 - m_\mu^2 c^4}$
C. $p_\mu = \sqrt{2 m_\mu K_\mu}$ **D.** $p_\mu c = \sqrt{\left(E_\mu^2 + m_\mu c^2\right)^2}$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS:A, Inspired by Serway and Mosses 2005 edition, pg. 52-53.

2. What is the rest energy of the π meson?
[Apakah tenaga rehat meson π ?]
A. $K_\mu + m_\mu c^2$ **B.** $\left(K_\mu + m_\mu c^2\right) + \sqrt{\left(K_\mu^2 + 2K_\mu m_\mu c^2\right)}$
C. K_μ **D.** $m_\mu c^2$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS:B, Inspired by Serway and Mosses 2005 edition, pg. 52-53.

3. What is the kinetic energy of the neutrino?
[Apakah tenaga kinetik neutrino?]
A. $\sqrt{\left(K_\mu + m_\mu c^2\right)^2 - m_\mu^2 c^4}$ **B.** $\left(K_\mu + m_\mu c^2\right) + \sqrt{\left(K_\mu^2 + 2K_\mu m_\mu c^2\right)}$

C. K_μ **D.** $m_\mu c^2$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS:A, Inspired by Serway and Mosses 2005 edition, pg. 52-53.

4. Which of the following statement(s) is (are) true? *[Manakah kenyataan yang berikut adalah benar?]*

I (T) All inertial frames are equivalent *[Semua rangka inersia adalah setara]*

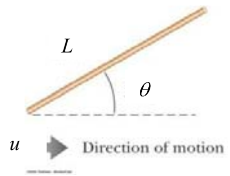
II (T) If light obeys Galilean transformation, light waves would appear stationary in an inertial frame that moves with the same speed with that of the light. *[Jika cahaya mematuhi transformasi Galilean, gelombang cahaya akan kelihatan pegun dalam satu rangka inersia yang kelajuannya sama dengan kelajuan cahaya]*

III(F) In an inertial frame moving approximately with the speed of light, light waves would appear stationary according to the postulates of special theory of relativity *[Dalam satu rangka inersia yang bergerak dengan kelajuan hampir dengan kelajuan cahaya, gelombang cahaya akan kelihatan pegun mengikut postulat teori kerelatifan khas.]*

IV (F) It is experimentally verified that electromagnetic waves propagate through a medium called Ether *[Telah disahkan secara eksperimen bahawa gelombang elektromagnetik merambat melalui satu jenis medium digelar Ether.]*

A. II,III **B. I, II,III** **C. II, III, IV** **D. I, II**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS:**D**, my own question

5. A moving rod is observed to have a length of L and to be orientated at an angle of $\theta = 45°$ with respect to the direction of motion, as shown in the figure below. The rod has a speed of $u = \frac{c}{\sqrt{2}}$.
[Suatu rod bergerak diperhatikan mempunyai panjang L dan diorientasikan pada suatu sudut $\theta = 45°$ merujuk kepada arah gerakannya sepertimana ditunjukkan dalam gambarajah di bawah. Kelajuan rod ialah $u = \frac{c}{\sqrt{2}}$.]



Serway, page 1279, question 23 (modified)

What is the tangent of the angle in the proper frame (in terms of $\tan\theta$)?
[Apakah tangen sudutnya (dinyatakan dalam sebutan $\tan\theta$) dalam rangka `proper']

A. $\tan\theta$ **B.** $\frac{\tan\theta}{\sqrt{2}}$ **C.** $\sqrt{2}\tan\theta$ **D.** $2\tan\theta$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS:B

6. What measurement(s) do two observers in relative motion always agree on?
[Apakah ukuran(-ukuran) yang sentiasa disetujui oleh dua orang pemerhati yang berada dalam pergerakan relatif]
I The speed of light *c* in vacuum *[Laju cahaya c dalam vakum]*
II The speed *v* of their relative motion *[Laju relatif v di antara mereka]*
III The momentum of an object *[Momentum suatu objek]*
IV The rest mass of an object *[Jisim rehat suatu objek]*

A. II, III **B. I, II, IV** **C. II, III, IV** **D. I, II**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
Serway Q1, pg. 1276
Solution: B

7. Given {*x,t*}, {*x',t'*} are two sets of coordinates used by two reference frames which are moving with a constant relative velocity, which statement(s) correctly describe(s) the transformation between them?
[Diberi {x,t}, {x',t'} merupakan dua set koordinat yang digunakan oleh dua rangka rujukan yang bergerak dengan halaju relatif mantap, kenyataan yang manakah memerihalkan transformasi di antara dua set koordinat tersebut dengan betul?]

I {*x,t*} is related to {*x',t'*} by Galilean transformation at $u << c$
[{x,t} dikaitkan dengan {x',t'} oleh transformasi Galilean pada u << c]

II {*x,t*} is related to {*x',t'*} by Galilean transformation at $u \rightarrow c$
[{x,t} dikaitkan dengan {x',t'} oleh transformasi Galilean pada u → c]

III {*x,t*} is related to {*x',t'*} by Lorentz transformation at $u << c$
[{x,t} dikaitkan dengan {x',t'} oleh transformasi Lorentz pada u << c]

IV {*x,t*} is related to {*x',t'*} by Lorentz transformation at $u \rightarrow c$
[{x,t} dikaitkan dengan {x',t'} oleh transformasi Lorentz pada u → c]

A. I,II **B. I, III,IV** **C. II, III, IV** **D. I, IV Only**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
My own question
Solution: B

8. What is the upper limit of the speed of an electron?
[Apakah limit atas bagi laju suatu elektron?]
A. $m_e c$ **B.** c **C.** 0 **D.** Infinity
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
Serway, Q12, pg. 1276
Solution: B

9. The units of the Planck constant *h* are those of:
[Unit bagi pemalar Planck h adalah sama dengan unit bagi …]

A. energy **B.** power **C.** momentum **D.** angular momentum
E. frequency
Solution: D, Chap 38, Q1, RHW 7th ed testbank,

10. Rank following electromagnetic radiations according to the energies of their photons, from least to greatest.
[Menyusun sinaran elektromagnetik berikut mengikut tenaga foton mereka, daripada yang paling lemah kepada yang paling besar]

1. blue light 2. yellow light 3. x-rays 4. radio waves

A. 1, 2, 3, 4 **B.** 4, 2, 1, 3 **C.** 4, 1, 2, 3 **D.** 3, 2, 1, 4 **E.** 3, 1, 2, 4
Solution: B, Chap 38, Q9, RHW 7th ed testbank,

11. In a photoelectric effect experiment the stopping potential is:
[Dalam eksperimen kesan fotoelektrik keupayaan penghenti adalah]

A. the energy required to remove an electron from the sample
[tenaga yang diperlukan untuk menyingkirkan satu elektron daripada sampel]

B. the kinetic energy of the most energetic electron ejected
[tenaga kenetik bagi elektron terlenting yang paling bertenaga]

C. the potential energy of the most energetic electron ejected
[tenaga keupayaan bagi elektron terlenting yang paling bertenaga]

D. the photon energy *[tenaga foton]*

E. the electric potential that causes the electron current to vanish
[keupayaan elektrik yang menyebabkan arus elektron hilang]
Solution: E, Chap 38, Q13, RHW 7th ed testbank,

12. In a photoelectric effect experiment no electrons are ejected if the frequency of the incident light is less than A/h, where h is the Planck constant and A is:
[Dalam eksperimen kesan fotoelektrik tiada elektron akan terlenting jika frekuensi cahaya tuju adalah kurang daripada A/h, di mana h ialah pamalar Planck dan A ialah:]

A. the maximum energy needed to eject the least energetic electron
[tenaga maksimum yang diperlukan untuk melentingkan elektron yang paling kurang bertenaga]

B. the minimum energy needed to eject the least energetic electron

SESSI 04/05/FINAL

[tenaga miminum yang diperlukan untuk melentingkan elektron yang paling kurang bertenaga]

C. the maximum energy needed to eject the most energetic electron
[tenaga maksimum yang diperlukan untuk melentingkan elektron yang paling bertenaga]

D. the minimum energy needed to eject the most energetic electron
[tenaga minimum yang diperlukan untuk melentingkan elektron yang paling bertenaga]

E. the intensity of the incident light *[keamatan cahaya tuju]*

Solution: D, Chap 38, Q16, RHW 7th ed testbank,

13. Consider the following: *[Pertimbangkan yang berikut]*

I. A photoelectric process in which some emitted electrons have kinetic energy greater than *hf,* where *f* is the frequency of the incident light.
[Satu proses fotoelektrik di mana sebahagian elektron terlenting mempunyai tenaga kinetik yang lebih besar daripada hf, di mana f ialah frekuensi cahaya tuju]

II. A photoelectric process in which all emitted electrons have energy less than *hf.*
[Satu proses fotoelektrik di mana kesemua elektron terlenting mempunyai tenaga kurang daripada hf]

III. Compton scattering from stationary electrons for which the emitted light has a frequency that is greater than that of the incident light.
[Penyerakan Compton daripada elektron-elektron rehat yang mana cahaya tertenting mempunyai frekuensi yang lebih besar daripada frekuensi cahaya tuju]

IV. Compton scattering from stationary electrons for which the emitted light has a frequency that is less than that of the incident light.
[Penyerakan Compton daripada elektron-elektron rehat yang mana cahaya tertenting mempunyai frekuensi yang lebih kecil daripada frekuensi cahaya tuju]

The only possible processe(s) is (are) *[Proses(-proses) yang mungkin ialah]*:

A. I **B. III** **C. I** and **III** **D. I** and **IV** **E. II** and **IV**
Solution: E, Chap 38, Q29, RHW 7th ed testbank (model answer in the testbank is incorrect)

14. In Compton scattering from stationary electrons the largest change in wavelength that can occur is:
[Dalam penyerakan Compton daripada elektron-elektron rehat, perubahan paling besar yang mungkin dalam jarak gelombang adalah]

A. 2.43×10^{-15} m **B.** 2.43×10^{-12} m **C.** 4.9×10^{-12} m
D. dependent on the frequency of the incident light *[bergantung kepada frekuensi cahaya tuju]*
E. dependent on the work function *[bergantung kepada fungsi kerja]*
Solution: C, Chap 38, Q25, RHW 7th ed testbank (model answer in the testbank is incorrect)

15. Of the following, Compton scattering from electrons is most easily observed for:
[Daripada yang berikut, penyerakan Compton daripada elektron-elektron adalah paling mudah dicerap dalam]

A. microwaves **B.** infrared light **C.** visible light
D. ultraviolet light **E.** x rays
Solution: E, Chap 38, Q22, RHW 7th ed testbank,

16. In Compton scattering from stationary particles the maximum change in wavelength can be made larger by using:
[Dalam penyerkan Compton daripada zarah-zarah rehat, perubahan maksimum dalam jarak gelombang boleh dijadikan lebih besar dengan menggunakan]

A. higher frequency radiation *[sinaran yang berfrekuensi lebih tinggi]*
B. lower frequency radiation *[sinaran yang berfrekuensi lebih rendah]*
C. more massive particles *[zarah yang berjisim lebih besar]*
D. less massive particles *[zarah yang berjisim lebih kecil]*
E. particles with greater charge *[zarah yang casnya lebih besar]*

Solution: D, Chap 38, Q21, RHW 7th ed testbank (modified)

17. Evidence for the wave nature of matter is: *[Bukti untuk sifat gelombang bagi jasad ialah]*

A. Electron diffraction experiments of Davisson and Germer
[eksperimen belauan elektron oleh Davisson dan Germer]

B. Photoelectric effect *[kesan fotoelektrik]*

C. Young's double slit experiment *[eksperimen dwi-celah Young]*

D. the Compton effect *[kesan Compton]*

E. Frank-Hertz experiment *[eksperimen Frank-Hertz]*

Solution: A, Chap 38, Q31, RHW 7th ed testbank,

18. Monoenergetic electrons are incident on a single slit barrier. If the energy of each incident electron is increased the central maximum of the diffraction pattern:
[Elektron monotenaga ditujukan pada satu sawar celah tunggal. Jika tenaga setiap elektron tuju dinaikkan, maka maksimum pusat corak belauan]

A. widens *[dilebarkan]* **B.** narrows *[disempitkan]*

C. stays the same width *[kelebaran tetap tak berubah]*

D. widens for slow electrons and narrows for fast electrons
[dilebarkan untuk elektron yang lambat dan disempitkan untuk elektron yang pantas]

E. narrows for slow electrons and widens for fast electrons

SESSI 04/05/FINAL

[disempitkan untuk elektron yang lambat dan dilebarkan untuk elektron yang pantas]

Solution: B, Chap 38, Q34, RHW 7th ed testbank,

19. Which of the following statement(s) is (are) true? *[Manakah kenyataan yang berikut adalah benar?]*

I (T) An ideal blackbody absorbs all of the light that is incident on it. *[Jasad hitam yang ideal menyerap kesemua cahaya yang tertuju padanya]*

II (F) The distribution of energy in the blackbody radiation depends upon the material from which the blackbody is constructed.
[Taburan tenaga dalam pancaran jasad hitam bergantung kepada jenis bahan yang membentuk dinding jasad hitam]

III(T) A blackbody is a perfect emitter of the radiation it generates. *[Jasad hitam adalah pemancar pancaran yang sempurna.]*

IV (T) The energy of an ultraviolet photon is more than the energy of an infrared photon. *[Tenaga suatu foton ultraungu adalah lebih tinggi daripada tenaga bagi suatu foton inframerah]*

A. III, IV **B. I, II, III** **C. I, III, IV** **D. I, III**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

Solution: C
I: testgen Physics 2 by Walker, Q1, Walker Chap 30
II: testgen Physics 2 by Walker, Q2, Walker Chap 30
III: testgen Physics 2 by Walker, Q11, Walker Chap 30
IV: testgen Physics 2 by Walker, Q12, Walker Chap 30

20. If the wavelength of a photon is doubled, what happens to its energy?
[Jika jarak gelombang digandakan dua kali, apa yang akan berlaku ke atas tenaganya?]

A. It is halved. *[ia diseparuhkan]*
B. It stays the same. *[tetap tak berubah]*
C. It is doubled. *[ia digandaduakan]*
D. It is quadrupled. *[ia digandakan 4 kali]*
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: A, testgen Physics 2 by Walker, Q24, Walker Chap 30

21. Light of a given wavelength is used to illuminate the surface of a metal, however, no photoelectrons are emitted. In order to cause electrons to be ejected from the surface of this metal you should
[Cahaya dengan jarak gelombang tertentu digunakan untuk memancari permukaan satu logam, tapi tiada fotoelektron yang terlentingkan. Unutk menlentingkan elektron daripada permukaan logam tersebut anda kena]

A. use light of a longer wavelength.

[menggunakan cahaya yang berjarak gelombang lebih panjang]

B. use light of a shorter wavelength.
[menggunakan cahaya yang berjarak gelombang lebih pendek]
C. use light of the same wavelength but increase its intensity.
[menggunakan cahaya yang berjarak gelombang sama tapi menambahkan keamatannya]

D. use light of the same wavelength but decrease its intensity.
[menggunakan cahaya yang berjarak gelombang sama tapi mengurangkan keamatannya]

E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS: B, testgen Physics 2 by Walker, Q35, Walker Chap 30

22. Protons are being accelerated in a particle accelerator at sub-relativistic energies. When the energy of the protons is doubled, their de Broglie wavelength will
[Proton dipecutkan dalam satu pemecut zarah pada tenaga sub-kerelatifan. Bila tenaga proton digandaduakan, jarak gelombang de Broglienya akan]

A. increase by a factor of 2. *[bertambah dengan satu factor 2]*
B. decrease by a factor of 2. *[berkurang dengan satu factor 2]*
C. increase by a factor of $\sqrt{2}$. *[bertambah dengan satu factor $\sqrt{2}$]*
D. decrease by a factor of $\sqrt{2}$. *[berkurang dengan satu factor $\sqrt{2}$]*
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: D, testgen Physics 2 by Walker, Q64, Walker Chap 30

23. A proton and an electron are both accelerated to the same final speed. If λ_p is the de Broglie wavelength of the proton and λ_e is the de Broglie wavelength of the electron, then
[Kedua-dua proton dan elektron dipecutkan kepada laju akhir yang sama. Jika λ_p ialah jarak gelombang de Broglie proton dan λ_e ialah jarak gelombang de Broglie elektron maka]

A. $\lambda_p > \lambda_e$. **B.** $\lambda_p = \lambda_e$. **C.** $\lambda_p < \lambda_e$.
D. Not enough data to answer this question. *[tak cukup data untuk menjawab soalan ini]*
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
ANS: C, testgen Physics 2 by Walker, Q67, Walker Chap 30

24. If the position of an electron is measured very precisely there is an uncertainty in measuring its
[Jika kedudukan suatu elektron diukur dengan sangat tepat maka akan wujud ketidakpastian dalam pengukuran …nya]

A. rest mass. **B.** momentum. **C.** potential energy.
D. charge. **E.** Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: B, testgen Physics 2 by Walker, Q71, Walker Chap 30

25. Which of the following statement(s) is (are) true? *[Manakah kenyataan yang berikut adalah benar?]*

I (T) A zero value for the Planck's constant would mean that the laws of classical physics would apply to quantum physics.
[Jika pemalar Planck bernilai sifar ini bermakna hukum-hukum fizik klasik akan teraplikasikan dalam fizik kuantum]

II (T) In quantum tunneling, electrons and other quantum particles can tunnel through a region of space that would be forbidden to them if they were classical particles.
[Dalam penerowongan kuantum, elektron dan zarah-zarah kuantum lain boleh menerowongi satu rantau yang terlarang bagi mereka yang merupakan zarah-zarah klasikal.]

III(F) A large value for the Planck's constant would mean that the laws of classical physics would apply to quantum physics.
[Jika pemalar Planck bernilai besar ini bermakna hukum-hukum fizik klasik akan teraplikasikan dalam fizik kuantum]

A. III **B. II, III** **C. I** **D. I, II**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
Solution: D
I,II: testgen Physics 2 by Walker, Q72, Walker
II: testgen Physics 2 by Walker, Q73, Walker

26. A major advantage of an electron microscope over a visible light microscope is that the electron microscope
[Manfaat yang major bagi satu mikroskop elektron berbanding dengan mikroskop cahaya nampak ialah bahawa mikroskop elektron]

A. has much greater magnification. *[memberikan pembesaran yang lebih tinggi]*
B. operates with much lower intensity. *[beroperasi pada keataman yang lebih rendah]*
C. can penetrate opaque samples. *[boleh menembusi sampel legap]*
D. can have much better resolution.*[memberikan leraian yang lebih baik]*
E. requires no lenses for its operation. *[tidak memerlukan kanta-kanta dalam operasinya]*

ANS: D, testgen Physics 2 by Young and Freeman , Q27, Chap 39

27. An important observation that led Bohr to formulate his model of the hydrogen atom was the fact that
[Salah satu pencerapan yang merangsangkan Bohr memformulasikan model atom hidrogennya ialah fakta bahawa]

A. a low density gas emitted a series of sharp spectral lines.
[gas berketumpatan rendah memancarkan pinggir-pinggir spectrum yang tajam]

B. neutrons formed a diffraction pattern when scattered from a nickel crystal.
[neutron membentuk corak belauan bila diserakkan daripada hablur nickel]

C. electrons were found to have a wave nature.
[elektron didapti mempunyai sifat gelombang]

D. the peak of the blackbody radiation moved to shorter wavelengths as the temperature was increased.
[puncak jasad hitam bergerak menghampiri jarak gelombang yang lebih pendek bila suhu bertambah]

E. the emission of light by an atom does not appear to conserve energy.
[pancanran cahaya oleh atom tidak mengabadikan tenaga]
ANS: A, testgen Physics 2 by Young and Freeman , Q40, Chap 39

28. The particle nature of light is best illustrated by which of the following?
[Sifat zarah cayaha adalah paling baik diilustrasikan oleh yang mana berikut?]

A. The scattering of alpha particles from gold foil. *[Serakan zarah alfa daripada foil emas]*

B. The fact that hot objects emit electromagnetic radiation.
[Fakta bahawa objek panas memancarkan pancaran elektromagnetik]

C. The diffraction pattern observed when a beam of electrons is scattered by a crystal
[Corak belauan yang dicerap bila satu bim elektron diserakkan oleh satu hablur]

D. The fact that a rainbow consists of a continuous spectrum of colors
[Fakta bahawa pelangi mengandungi satu spektrum warna yang selanjar]

E. The ejection of electrons from a metal surface illuminated by light.
[Pelentingan elektron daripada permukaan logam yang disinari cahaya]
ANS: E, testgen Physics 2 by Young and Freeman , Q18, Chap 38

29. A wave function is given by
[Satu fungsi gelombang diberikan oleh]
$\Psi(x) = 0$ for $x < 0$
$\Psi(x) = Ax$ for $0 \leqslant x \leqslant L$
$\Psi(x) = 0$ for $x > L$

The product of the normalization constant A and the quantity $L^{3/2}$ is equal to:
[Hasildarab pemalar normalisasi A dengan kuantiti $L^{3/2}$ bersamaan dengan]

A. $\sqrt{12}$ **B.** $\sqrt{15}$ **C.** $\sqrt{20}$ **D.** $\sqrt{24}$ **E.** $\sqrt{3}$

ANS: E, testgen Physics 2 by Young and Freeman , Q1, Chap 40, modified

30. If a wave function ψ for a particle moving along the x axis is "normalized" then:
[Jika satu funsi gelombang ψ untuk satu zarah yang bergerak sepanjang paksi x adalah ternormalisasikan, maka

A. $\int |\psi|^2 dt = 1$ **B.** $\int |\psi|^2 dx = 1$ **C.** $\partial\psi/\partial x = 1$ **D.** $\partial\psi/\partial t = 1$
E. $|\psi|^2 = 1$

Solution: B, Chap 39, Q1, RHW 7th ed testbank,

SESSI 04/05/FINAL

31. The energy of an electron in a hydrogen atom that is about to get ionised is
[Tenaga elektron dalam atom hidrogen yang hampir-hampir diionkan adalah]

A. -13.6 eV **B.** -3.4 eV **C.** -10.2 eV
D. -1.0 eV **E.** 0 eV

Solution: E, Chap 39, Q26, RHW 7th ed testbank, modified.

32. According to the Bohr model of hydrogen atom, the energy E_n of a hydrogen atom of a state with quantum number n is proportional to:
[Mengikut model hidrogen Bohr tenaga E_n suatu atom hidrogen pada keadaan dengan nombor kuantum n adalah berkadaran dengan]

A. n **B.** n^2 **C.** $1/n$ **D.** $1/n^2$

E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

Solution: D, Chap 39, Q25, RHW 7th ed testbank,

33. The series limit for the Balmer series represents a transition $m \rightarrow n$, where (m, n) is
[Limit siri bagi siri Balmer mewakili satu peralihan m $\rightarrow$ n, di mana (m, n) ialah]

A. (2,1) **B.** (3,2) **C.** (∞,0) **D.** (∞,1) **E.** (∞,2)

Solution: E, Chap 39, Q33, RHW 7th ed testbank,

34. The location of a particle is measured and specified as being exactly at $x = 0$, with *zero* uncertainty in the x direction. How does this affect the uncertainty of its velocity component in the y direction?
[Lokasi suatu zarah adalah diukur dan dispesifikasikan sebagai tepat-tapat pada x = 0 dengan ketidakpastian sifar dalam arah x. Bagaimanakah keadaan ini mempengaruhi ketidakpastian komponen halajunya dalam arah y?]

A. It does not affect it. *[Keadaan ini tidak mempengaruhinya]*
B. It makes it infinite. *[Keadaan ini menjadikannya infinit]*
C. It makes it zero. *[Keadaan ini menjadikannya sifar]*
D. It makes it negative *[Keadaan ini menjadikannya negatif]*
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*
Ans: A, QQ serway 40.10

35. The Balmer series of hydrogen is important because it:
[Siri Balmer bagi hidrogen adalah penting kerana ia]

A. is the only one for which the Bohr theory can be used
[merupakan satu-satunya siri yang dapat diaplikasikan oleh teori Bohr]

B. is the only series which occurs for hydrogen
[merupakan satu-satunya siri yang berlaku dalam hidrogen]

C. is in the visible region
[berada dalam rantau nampak]

D. involves the lowest possible quantum number *n*
[melibatkan numbor kuantum yang terendah mungkin]

E. involves the highest possible quantum number *n*
[melibatkan numbor kuantum yang tertinggi mungkin]

Solution: C, Chap 39, Q34, RHW 7th ed testbank,

36. The quantization of energy, $E = nhf$, is not important for an ordinary pendulum because:
[Pengkuantuman tenaga, E = nhf, adalah tidak penting bagi suatu bandul kerana]

A. the formula applies only to mass-spring oscillators
[formular hanya teraplikasikan ke atas pengayun jisim-spring]

B. the allowed energy levels are too closely spaced
[selang paras tenaga diizinkan adalah terlalu padat]

C. the allowed energy levels are too widely spaced
[selang paras tenaga diizinkan adalah terlalu lebar]

D. the formula applies only to atoms
[formular hanya teraplikasikan ke atas atom]

E. the value of *h* for a pendulum is too large
[nilai h bagi bandul terlalu besar]
Solution: B, Chap 38, Q3, RHW 7th ed testbank,

37. A hydrogen atom is in its ground state. Incident on the atom are many photons each having an energy of 5 eV. The result is that
[Suatu atom hidrogen berada dalam keadaan buminya. Foton-foton bertenaga 5 eV setiap satu ditujukan pada atom itu. Hasilnya ialah]

A. the atom is excited to a higher allowed state
[atom teruja kepada keadaan dizinkan yang lebih tinggi]

B. the atom is ionized
[atom diionkan]

C. the photons pass by the atom without interaction
[foton merentasi atom tanpa berinteraksi]

D. the photons are ionised

SESSI 04/05/FINAL

[foton diionkan]

E. the atom is de-excited to a lower quantum state
[atom ternyah-uja kepada keadaan dizinkan yang lebih rendah]
ANS (C), Serway, qq 42.1, pg. 1360. Because the energy of 5 eV does not correspond to raising the atom from the ground state to an allowed excited state, there is no interaction between the photon and the atom (modified)

38. A hydrogen atom makes a transition from the $n = 3$ level to the $n = 2$ level. It then makes a transition from the $n = 2$ level to the $n = 1$ level. Which transition results in emission of the longest-wavelength photon?
[Satu atom hidrogen melakukan peralihan dari paras n=3 ke paras n=2. Kemudiannya ia melakukan satu peralihan dari paras n=2 ke paras n=1. Peralihan yang manakan menghasilkan pancaran foton berjarak gelombang paling panjang?]

A. the first transition *[peralihan pertama]*
B. the second transition *[peralihan kedua]*

C. neither, because the wavelengths are the same for both transitions.
[bukan A ataupun B kerana jarak gelombang kedua-dua kes adalah sama]

D. one cannot determine the answer because data provided is not sufficient.
[jawapan tidak boleh ditentukan kerana data yang diberikan tak cukup]

E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS (A), Serway, qq 42.3, pg. 1360. The longest-wavelength photon is associated with the lowest energy transition, which is $n = 3$ to $n = 2$.

39. An electron and a proton are accelerated to a common relativistic energy (i.e. $E >> m_e c^2, m_p c^2$), where m_e and m_p denote the masses of the electron and proton respectively. Determine the ratio of the de Broglie wavelength of the electron to that of the proton.
[Satu elektron dan proton dipecutkan kepada satu tenaga kerelatifan E yang sama, (iaitu E >>$m_e c^2$, $m_p c^2$), di mana m_e dan m_p menandakan jisim elektron dan proton masing-masing. Tentukan nisbah jarak gelombang de Broglie elektron kepada proton.]

(A) $\frac{m_p}{m_e}$ (B) $\sqrt{\frac{m_p}{m_e}}$ (C) $\sqrt{\frac{m_e}{m_p}}$ (D) $\frac{m_p}{m_e}$
(E) 1

ANS (E), My own question, pg. 897.

40. How is the empirical Ryberg constant, R_H, be related to the other constants of nature in the Bohr model of hydrogen atom?
[Bagaimanakah pemalar empirikal Ryberg R_H dikaitkan kepada pemalar-pemalar alam yang lain mengikut model Bohr atom hidrogen?]

A. $R_H = \frac{2\pi^2 m_e e^4}{h^2 c}\left(\frac{1}{4\pi\varepsilon_0}\right)^2$ **B.** $R_H = \frac{2\pi^2 m_e e^4}{h^3 c}\left(\frac{1}{4\pi\varepsilon_0}\right)^2$

C. $R_H = \frac{2\pi^2 m_e e^4}{h^3 c}\left(\frac{1}{4\pi\varepsilon_0}\right)$ **D.** $R_H = \frac{2\pi^2 m_e e^4}{h^3 c^3}\left(\frac{1}{4\pi\varepsilon_0}\right)^2$

(E) Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

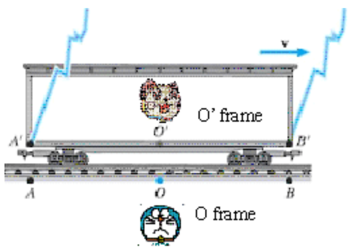
ANS (B), Cutnell and Johnson, pg. 910.

Part B: Structured Questions [60 marks]
Instruction: Answer both questions 1 and 2 in this Part.

[Bahagian B: Soalan Struktur. 60 markah]
[Arahan: Jawab kedua-dua soalan 1 dan 2 dalam Bahagian ini.]

1(a) Consider the Gedanken experiment of a moving train (the O' frame) passing by an observer called Doraemon on the ground (the O frame) with a speed of v, see figure below. The length of the train, as measured by Doraemon, is L. Another observer, Doraemiyan is seen by Doraemon to sit at the middle of the train, $L/2$, when Doraemiyan passes by Doraemon at time t=0. At that instance, two lightning bolts strike points A and B at the edges of the train such that both events appear to occur simultaneously according to Doraemon. What is the time lag between the lights from event A and event B arriving at Doraemiyan, $t_A - t_B$, as seen by Doraemon, where both t's are measured in Doraemon's frame. Express your answer in terms of v, L, and the speed of light c. [*Hint: Do you think you should apply time-dilation or length contraction formulae here?*]
[Pertimbangkan eksperimen Gedanken di mana satu tren (rangka O') bergerak melepasi seorang pemerhati Doraemon yang berada di atas bumi (rangka O) dengan laju v, rujuk gambarajah di bawah. Panjang tren sebagaimana yang diukur oleh Doeaemon ialah L. Seorang lagi pemerhati, Doraemiyan diperhatikan oleh Doraemon sebagai duduk di tengah-tengah tren, L/2, bila Doraemiyan bergerak melepasi Doraemon pada masa t =0. Pada ketika itu, dua petir menyambar titik-titik A dan B pada pinggir tren sedemikian rupa supaya kedua-dua peristiwa itu kelihatan berlaku secara serentak kepada Doraemon. Apakah masa susulan di antara cahaya dari peristiwa A dan peristiwa B yang sampai kepada Doraemiyan, $t_A - t_B$*, mengikut Doraemon? Kedua-dua masa* t_A*,* t_B *adalah diukur dalam rangka Doraemon. Nyatakan jawapan anda dalam sebutan v, L dan laju cahaya c. [Hint: Adakah anda perlu mengaplikasikan formular-formular pendilatan-masa dan susutan panjang?]*

[10 marks]



Solution

SESSI 04/05/FINAL

By the time t_B, light from event B hits Doramiyan. Since then she has moved for a distance of vt_B to the right from Doramon. Hence, light from B fulfils the relation $ct_B = L/2 - vt_B$.

Likewise, by the time t_A (> t_B) light from A hits Doramiyan. Since then she has moved for a distance of vt_A to the right from Doramon. Hence, light from A fulfils the relation $ct_A = L/2 + vt_A$.

$t_B = L/2(c+v)$; $t_A = L/2(c-v)$
$\Rightarrow t_A - t_B = L/2(c-v) - L/2(c+v) = (uL)/(c^2-v^2)$

[10 marks]

1(b) When a photoelectric surface is illuminated with light of wavelength 437 nm, the stopping potential is 1.67 V.
[Bila satu permukaan fotoelektrik disinari cahaya berjarak gelombang 437 nm, keupayaan penghenti ialah]

[5 + 5 marks]

(*i*) What is the work function of the metal in eV?
[Apakah fungsi kerja logam tersebut dalam unit eV?]

(*ii*) What is the maximum speed of the ejected electrons?
[Apakah laju maksimum elektron terlenting?]

Solution:

(*i*) $W_0 = hc/\lambda - K_{max}$ = 1240 nm.eV/437 nm -1.67 eV = 1.17 eV
(*ii*) $K_{max} = mv^2/2 \Rightarrow v^2 = (2K_{max}/m)^{1/2} = (2 \times 1.67e\text{ J} / 9.11 \times 10^{-31}\text{ kg})^{1/2} = 7.66 \times 10^5$ m/s

ANS: testgen Physics 2 by Young and Freeman , Q2.4, Chap 38

1(c) An electron has a speed of 0.95c. What is the the magnitude of its momentum? **[5 marks]**
[Suatu elektron berlaju 0.95c. Apakah magnitud momentumnya?]

Solution:
$\gamma = 1/\sqrt{1-0.95^2} = 3.20$
$p = m\gamma u = 9.1\times10^{-31}\times3.2\times(0.95\times3\times10^8)\text{Ns}=8.3\times10^{-22}\text{ Ns}$

Chap 37, Q54, RHW 7th ed testbank,

1(d) A 29.0 pm photon is Compton scattered by a stationary electron. What is the maximum energy loss of the photon?
[Satu foton 29.0 pm diserak Compton oleh satu elektron pegun. Apakah kehilangan tenaga foton yang maksimum?]

[5 marks]

Solution:
Maximal kinetic energy loss of the photon occurs when

$$\Delta\lambda = \Delta\lambda_{\max} = 2\lambda_c = \frac{2hc}{m_e c^2} = \frac{2\times(1240\text{keV}\cdot\text{pm})}{522\text{keV}} = 4.75\text{pm}$$

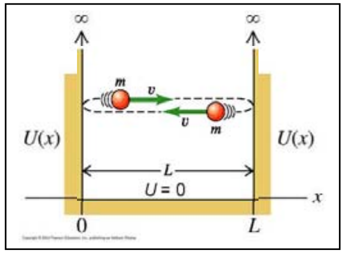
$$\Delta E_{\max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{\max}} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda + \Delta\lambda_{\max}}\right)$$

$$= (1240\text{keV}\cdot\text{pm})\left(\frac{1}{29\text{pm}} - \frac{1}{29\text{pm}+4.75\text{pm}}\right) = 6.01\ \text{keV}$$

ANS: testgen Physics 2 by Young and Freeman , Q1.12, Chap 38 (Model answer may be incorrect)

2(a) Consider a quantum particle trapped in an infinite quantum well (with width L) given by
[Pertimbangkan satu zarah kuantum yang terperangkap dalam satu telaga kuantum infinit (dengan lebar L) yang diberikan oleh]

$$U(x) = \begin{cases} \infty, & x \le 0, x \ge L \\ 0, & 0 < x < L \end{cases}$$



The behaviour of a particle inside the infinite well [i.e. the region where $U(x) = 0$ for $0 < x < L$] is governed by the 1-D time-independent Schrodinger equation $\frac{\partial^2\psi(x)}{\partial x^2} = -B^2\psi(x)$, where $B^2 = \frac{2mE}{\hbar^2}$. E is the energy of the particle.
[Kelakuan zarah dalam telaga infinit (iaitu dalam rantau $U(x) = 0$ for $0 < x < L$) diperintah oleh persamaan merdeka-masa Schrodinger 1-D $\frac{\partial^2\psi(x)}{\partial x^2} = -B^2\psi(x)$, di mana $B^2 = \frac{2mE}{\hbar^2}$. E ialah tenaga zarah.]

(*i*) Show that $\psi(x) = A\sin Bx + C\cos Bx$ is a solution to the Schrodinger equation for the particle inside the well, where A, C are some constants.
[Tunjukkan bahawa $\psi(x) = A\sin Bx + C\cos Bx$ merupakan penyelesaian kepada persamaan Schrodinger untuk zarah dalam telaga, di mana A dan C adalah pemalar.]

[5 marks]

Solution: Plug $\psi(x) = A\sin Bx + C\cos Bx$ into the LHS of $\frac{\partial^2\psi(x)}{\partial x^2} = -B^2\psi(x)$:

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left[A\sin Bx + C\cos Bx\right] = \frac{\partial}{\partial x}\left[BA\cos Bx - BC\sin Bx\right]$$

$$= -B^2A\sin Bx - B^2C\cos Bx = -B^2\left[A\sin Bx + C\cos Bx\right]$$

$$= -B^2\psi(x) = \text{RHS of the Schroginger equation}$$

(*ii*) Determine the values of C and B by applying boundary conditions that must be fulfilled by the Schrodinger equation governing the particle.
[Tentukan nilai-nilai C dan B dengan mengaplikasikan syarat-syarat sempadan yang mesti dipenuhi oleh persamaan Schrodinger yang memerintah zarah itu.]

[4 + 6 marks]

Solution:
Boundary condition (1)
Plug $\psi(x = 0) = 0$ into $\psi = A\sin Bx + C\cos Bx$, we obtain
$\psi(x=0) = 0 = A\sin 0 + C\cos 0 = C$, ie, $C = 0$

[4 marks]

Hence the solution is reduced to $\psi = A\sin B$
Next we apply the second boundary condition: $\psi(x = L) = 0 = A\sin(BL)$
Only either A or $\sin(BL)$ must be zero but not both; A cannot be zero
This means it must be $\sin BL = 0$, or in other words $B = n\pi/L \equiv B_n$, $n = 1,2,3,\cdots$

[6 marks]

(*iii*) Hence show that the energy of the particle in the infinite well is quantized.
[Seterusnya tunjukkan bahawa tenaga zarah dalam telaga infinit adalah terkuantumkan]

[5 marks]

Solution

$$B_n^2 = \frac{2mE_n}{\hbar^2} = \frac{n^2\pi^2}{L^2} \Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}, n = 1,2,3...$$ **[5 marks]**

2(b) What is the kinetic energy of an electron at the ground state of the hydrogen atom, given that the ground state energy of the hydrogen atom is -13.6 eV? Give your answer in unit of eV.
[Apakah tenaga kinetik elektron pada keadaan bumi atom hidrogen? Diberitahu tenaga bumi atom hidrogen ialah -13.6 eV. Berikan jawapan anda dalam unit eV.]

[10 marks]

Solution: Serway and Moses, Problem 22

From the requirement that the centripetal force comes from the electrostatic force $\frac{mv_0^2}{r_0} = \frac{ke^2}{r_0^2}$,

[1 marks]

the kinetic energy of the ground state electron can be written as $K_0 = \frac{mv_0^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r_0}$.

[2 marks]

Potential energy of the electron at ground state is $U_0 = -\frac{ke^2}{r_0}$.

[1 marks]

Hence ground state energy is $E_0 = K_0 + U_0 = \left(\frac{1}{2}\right)\frac{ke^2}{r_0} - \frac{ke^2}{r_0} = -\frac{ke^2}{2r_0} = -13.6\text{ eV}$.

[3 marks]

This gives $K_0 = \frac{ke^2}{2r_0} = 13.6\text{ eV}$

[3 marks]

UNIVERSITI SAINS MALAYSIA

KSCP
Academic Session 2004/2005
APRIL 2005

ZCT 104E/3 - Physics IV (Modern Physics)
[Fizik IV (Fizik Moden)]

Duration: 3 hours
[Masa: 3 jam]

Please check that the examination paper consists of **XXX** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **XXX** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer all questions. Please answer the objective questions from Part A in the objective answer sheet provided. Answer ALL structured questions from Part B. Please submit the objective answer sheet and the answers to the structured questions separately.

Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan**: Jawab **SEMUA** soalan. Sila jawab soalan-soalan objektif daripada bahagian A dalam kertas jawapan objektif yang dibekalkan. Jawab kesemua soalan struktur daripada Bahagian B. Hantar kertas jawapan objektif dan jawapan kepada soalan struktur berasingan.]*

[Pelajar dibenarkan untuk menjawab samada dalam bahasa Malaysia atau bahasa Inggeris.]

Data
speed of light in free space, $c = 3.00 \times 10^{8}$ m s^{-1}
permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
elementary charge, $e = 1.60 \times 10^{-19}$ C
Planck constant, $h = 6.63 \times 10^{-34}$ J s
unified atomic mass constant, $u = 1.66 \times 10^{-27}$ kg
rest mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton, $m_p = 1.67 \times 10^{-27}$ kg
molar gas constant, = 8.31 J K^{-1} mol^{-1}
Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1}
gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
acceleration of free fall, $g = 9.81$ m s^{-2}

Part A: Objective 25 marks
Instruction: Answer all 25 objective questions in this Part.
[Bahagian A: Objektif.]
[Arahan: Jawab kesemua 25 soalan objektif dalam Bahagian ini.]

ANS: A, Young and Freeman study guide, pg 271

1. A massive particle has a speed of 0.95c. Can its energy and speed be increased by more than 500%?
[Laju suatu zarah yang berjisim ialah 0.95c. Bolehkah tenaga dan lajunya bertambah sebanyak 500%?]

A. The energy can but not the speed
B. The speed can but not the energy
C. Both the energy and speed can be increased by this amount
D. Both the energy and speed cannot be increased by this amount
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: A, Modified from Young and Freeman study guide, pg 271

2. Consider a photon travelling in vacuum. Can its energy and speed be increased by more than 500%?
[Pertimbangkan suatu foton yang bergerak di dalam vakuum. Bolehkah tenaga dan lajunya bertambah sebanyak 500%?]
A. The energy can but not the speed
B. The speed can but not the energy
C. Both the energy and speed can be increased by this amount
D. Both the energy and speed cannot be increased by this amount
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: B, Modified from Young and Freeman study guide, pg 271, Example 1

3. Constancy of the speed of light in all inertial reference systems implies that
[Kemantapan laju cahaya dalam semua rangka rujukan inersia mengimplikasikan]
A $x^2 + y^2 + z^2 + c^2t^2 = x'^2 + y'^2 + z'^2 + c^2t'^2$
B. $x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$
C. $x + y + z - ct = x' + y' + z' - ct'$
D. $x + y + z + ct = x' + y' + z' + ct'$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: C, Modified from Young and Freeman study guide, pg 280, Example 9

4. If a neutron spontaneously decays into a proton, an electron and a neutrino (which is massless), the decay products are observed to have a total kinetic energy of E_k. If the proton mass is M_P and the electron mass is m_e how large is the neutron mass?
[Jika suatu neutron mereput kepada satu proton, satu elektron dan satu neutrino (yang tak berjisim) secara spontan, jumlah tenaga kinetik hasil reputannya dicerap sebagai E_k. Jika jisim proton ialah M_P dan jisim elektron ialah m_e apakah jisim neutron?]

A. $(M_P + m_e) - \frac{E_k}{c^2}$ B. $\frac{E_k}{c^2} - (M_P + m_e)$ **C.** $M_P + m_e + \frac{E_k}{c^2}$

SESSI 04/05/KSCP

D. $\sqrt{(M_P + m_e)^2 + \left(\frac{E_k}{c^2}\right)^2}$ **E.** Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: B, Cutnell, page 1271, QQ 39.10

5. The following pairs of energies represent the rest energy and total energy of three different particles: particle 1: E, 2E; particle 2: E, 3E; particle 3: 2E, 4E. Rank the particles according to their speed.
[Pasangan tenaga berikut mewakili tenaga rehat dan jumlah tenaga bagi tiga zarah yang berbeza: zarah 1: E, 2E; zarah 2: E, 3E; zarah 3: 2E, 4E. Aturkan zarah-zarah tersebut mengikut laju mereka.]
A. $v_3 > v_2 = v_1$ **B.** $v_2 > v_3 = v_1$ **C.** $v_1 > v_2 = v_3$
D. $v_3 > v_2 > v_1$ **E.** Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: A, Modified from Walker Test Item, pg 629, Q28

6. Observer A sees a pendulum oscillating back and forth in a relativistic train and measures its period to be T_A. Observer B moves together with the train and measures the period of the pendulum to be T_B. These two results will be such that
[Tempoh suatu bandul yang mengayun berulang-alik di dalam suatu keretapi kerelatifan diukur sebagai T_A oleh pemerhati A. Manakala pemerhati B yang gerak bersama dengan keretapi tersebut mengukur tempoh bandul tersebut sebagai T_B. Keputusan pengukuran tempoh-tempoh tersebut adalah]

A. $T_A > T_B$ **B.** $T_A = T_B$ **C.** $T_A < T_B$
D. T_A could be greater or smaller than T_B depending on the direction of the motion
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: D, Walker Test Item, pg 642, Q1,Q2,Q4, Tutorial 2 Problems 1.

7. Which of the following statements are (is) correct? *[Pilih kenyataan(-kenyataan) yang benar daripada yang berikut]*

I(T) An ideal blackbody absorbs all of the light that is incident on it. *[Jasad hitam yang ideal menyerap kesemua cahaya yang jatuh ke atasnya]*
II(T) The distribution of energy in the blackbody radiation does not depends upon the material from which the blackbody is constructed. *[Taburan tenaga dalam pancaran jasad hitam tidak bergantung kepada jenis bahan yang membentuk jasad hitam itu.]*
III(F) The correct expression for the energy of a photon is $E = h\lambda$ *[Ekspresi yang betul bagi tenaga suatu foton ialah $E = h\lambda$]*
IV(T) For a blackbody, the total intensity of energy radiated over all wavelengths increases as the forth power of the temperature.
[Bagi satu jasad hitam, keamatan tenaga yang dipancarkan bila sumbangan kesemua jarak gelombang dijumlahkan bertambah mengikut kuasa empat suhunya.]

A. I,II,III **B. I,II** **C. II, III, IV** **D. I,II,IV**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: E, Young and Freeman study guide, page 286, Question

8. Which of the following statements are (is) correct?
[Pilih kenyataan(-kenyataan) yang benar daripada yang berikut]

I(T) In the Compton Effect, there is a zero wavelength shift for forward scattering ($\theta = 0°$) *[Dalam kesan Compton, anjakan jarak gelombang sifar berlaku dalam serakan ke depan ($\theta = 0°$)]*

II (T) In the Compton Effect, no energy or momentum is transferred to the electron in the forward scattering. *[Dalam kesan Compton, tiada tenaga atau momentum dipindahkan kepada elektron dalam serakan ke depan.]*

III(T) In the Compton Effect, conservation of momentum and energy must be simultaneously satisfied. *[Dalam kesan Compton, keabadian tenaga dan momentum mesti dipatuhi secara serentak.]*

IV(T) In the Compton Effect, energy and momentum are transferred to the scattered electron when θ is non zero. *[Dalam kesan Compton, tenaga dan momentum dipindahkan kepada elektron terserakkan jika sudut θ bukan sifar.]*

A. I,II,III **B. I,II** **C. II, III, IV** **D. I,II,IV**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: B, Walker Test Item, page 646, Q25, own suggested options

9. Which of the following statements are (is) correct?
[Pilih kenyataan(-kenyataan) yang benar daripada yang berikut]

I(F) A photon is a particle with positive charge *[Foton adalah zarah yang bercas positif]*
II (F) A photon's mass is not necessarily zero *[Jisim foton tidak semestinya sifar]*
III(F) Photon always move with a speed of *c* irrespective of the medium through which it is moving *[Tidak kisah medium apa yang dilaluinya, foton sentiasa bergerak dengan laju c]*
IV(T) The number of photons per unit cross sectional area in a beam of light is proportional to the intensity of the light beam. *[Nombor foton per unit keratan rentas dalam satu alur cahaya adalah berkadaran dengan keamatan alur cahaya itu.]*

A. I,II,III **B. IV** **C. II, III, IV** **D. I,II,IV**
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: A, Walker Test Item, page 648, Q30

10. In photoelectric effect, which one of the following is the correct expression for the cut-off frequency of the metal in terms of its work function, W_0?
[Dalam kesan fotoelectric, kenyataan yang mana satukah adalah ekspresi yang betul yang menyatakan frekuensi penggal sesuatu logam dalam sebutan fungsi kerjanya?]

A. W_0 / h **B.** W_0 / c **C.** h / W_0 **D.** $(h/c)W_0$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

SESSI 04/05/KSCP

ANS: B, Cutnel, page 889, CYU 2

11. In Compton effect, an incident X-ray photon of wavelength λ is scattered by an electron, the scattered photon having a wavelength of λ'. Suppose that the incident photon is scattered by a proton instead of an electron. For a given scattering angle θ, the change $\lambda' - \lambda$ in the wavelength of the photon scattered by the proton
[Dalam kesan Compton, suatu foton sinar-X tuju dengan jarak gelombang λ diserakkan oleh suatu elektron manakala jarak gelombang bagi foton terserak ialah λ'. Katakan foton tuju diserakkan oleh suatu proton yang manggantikan elektron. Untuk suatu sudut serakan θ yang diberikan, perubahan $\lambda' - \lambda$ dalam jarak gelombang foton terserak oleh proton adalah]

A. is greater than that scattered by the electron
B. is less than that scattered by the electron
C. is same as that scattered by the electron
D. cannot be determined
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS: A, Own question

12. In an electron-positron pair production by an energetic photon in the vicinity of a nucleus, the frequency of the photon λ must be
[Dalam penghasilan pasangan elektron-positron oleh suatu foton bertenaga tinggi di persekitaran suatu nucleus, frekuensi foton λ semestinya]

A. $\lambda \le h / 2m_e c$ **B.** $\lambda \ge h / 2m_e c$ **C.** $\lambda \le h / m_e c$ **D.** $\lambda \ge h / m_e c$
E. $\lambda \le h / 2m_e$

13. ANS C: Young and Freeman test bank, pg. 414, Q14

In an important experiment in 1927 a beam of electrons was scattered off a crystal of nickel. The intensity of the scattered beam varied with the angles of scattering, and analysis of these results lead to confirmation of
[Dalam suatu eksperimen yang dilakukan dalam tahun 1927, suatu alur elektron diserakkan oleh suatu hablur nikel. Keamatan alur yang terserak berubah-ubah mengikut sudut ia diserakkan, dan analisis keputusan itu membawa kepada pengesahan]

A. the particle nature of light
B. the Bohr model of atom
C. the wave nature of electrons
D. the Rutherford model of the nucleus
E. the quantisation of energy levels

ANS A: Young and Freeman test bank, pg. 425, Q2

14. Consider a particle in a box of width L and infinite height. Let the particle be in a state $n = 11$. What is the first value of x ($0 \le x \le L$), where the probability of finding the particle is highest?
[Pertimbangkan suatu zarah dalam kotak dengan lebarL dan ketinggian infini. Biar ia berada dalam keadaan n = 11. Apakah nilai x ($0 \le x \le L$) yang pertama di mana keberangkalian menjumpai zarah terserbut adalah paling tinggi?]

A. $L/22$ **B.** $L/11$ **C.** L **D.** $L/10$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS B: Walker test item, pg. 654, Q65

15. Protons are being accelerated in a particle accelerator. When the speed of the proton is doubled, their de Broglie wavelength will
[Proton sedang dipecutkan oleh pemecut zarah. Bila laju proton digandakan dua kali, jarak gelombang de Broglie mereka akan]

A. increase by a factor of 2
B. decrease by a factor of 2
C. increase by a factor of $\sqrt{2}$
D. decrease by a factor of $\sqrt{2}$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS B: Walker student guide, pg. 506, quiz 9

16. If the minimum uncertainty in an object's position is decreased by half, what can we say about the uncertainty in its momentum?
[Jika ketidakpastian minimum bagi kedudukan suatu objek dikurangkan separuh, apa yang boleh dikatakan ke atas ketidakpastian dalam momentumnya?]

A. The uncertainty in momentum is at most half of what it was before the change
B. The uncertainty in momentum is at least twice what it was before the change
C. The uncertainty in momentum does not change
D. The minimum uncertainty in momentum is precisely half of what it was before the change
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS A: Walker student guide, pg. 657, Q6

17. To which of the following values of *n* does the longest wavelength in the Balmer series correspond?
[Nilai n yang manakah bersepadanan dengan jarak gelombang paling panjang dalam siri Balmer?]

A. 3 **B.** 5 **C.** 1 **D.** infiniti
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS D: Young and Freeman test bank, pg. 418, Q36

18. In order for an atom to emit light, it
[Untuk memancarkan cahaya, sesuatu atom kena]

A. must be in the gaseous state *[berada dalam keadaan gas]*
B. must be stimulated by external radiation *[dirangsang oleh pancaran luar]*
C. must be in the ground state *[berada dalam keadaan bumi]*
D. must be in an excited state *[berada dalam keadaan teruja]*
E. must be fluorescent *[berpendarfluor]*

ANS C: Young and Freeman test bank, pg. 660, Q18,19,20

19. Which of the following statements are (is) correct?
[Pilih kenyataan(-kenyataan) yang benar daripada yang berikut]

SESSI 04/05/KSCP

A. Einstein proposed the model of the atomic structure that provides the best explanation of the observation that each atom in the periodic table has a unique sets of spectral lines.
[Einstein menyarankan model struktur atom yang membekalkan penjelasan paling baik ke atas pencerapan hahawa setiap atom di dalam jadual berkala mempunyai satu set garisan spektrum yang unik.]
B. According to one of the assumptions of the Bohr model, the electron in a hydrogen atom moves in an elliptical orbit about the nucleus
[Menurut salah satu anggapan model Bohr, elektron di dalam atom hidrogen berkisar di dalam orbit elips yang mengelilingi nucleus.]
C. Bohr's model of an atom includes idea from both classical and quantum physics.
[Model atom Bohr mengandungi idea-idea daripada kedua-dua bidang fizik klasik dan fizik kuantum.]
D. The plum-pudding model of atom by Thomson was verified by Rutherford's alpha scattering experiment
[Model atom 'plum-pudding' oleh Thomson telah diverifikasikan oleh eksperimen penyerakan alfa Rutherford.]
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS A: Serway. 1333, Quiz 41.5

20. Consider an electron, a proton and an alpha particle each trapped separately in identical infinite square wells. Which particle corresponds to the highest ground-state energy?
[Pertimbangkan suatu elektron, suatu proton dan suatu zarah alfa yang masing-masing diperangkapkan secara berasingan di dalam telaga segiempat infinit yang identikal. Zarah yang manakan bersepadanan dengan paras tenaga bumi yang paling tinggi?]

A. the electron **B.** the proton **C.** the alpha particle
D. The ground state energy is the same in all three cases
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS D: Serway. 1333, Quiz 41.6

21. Consider the three particles in Question 20 again. Which particle has the longest wavelength when the system is in the ground state?
[Pertimbangkan semula zarah-zarah dalam Soalan 20. Zarah yang manakan mempunyai jarak gelombang yang paling panjang bila sistem berada dalam keadaan bumi?]
A. the electron **B.** the proton **C.** the alpha particle
D. All three particles have the same wavelength
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS D: Young and Freeman test bank, pg. 663, Q22,34,40,44

22. Which of the following statements are (is) correct?
[Pilih kenyataan(-kenyataan) yang benar daripada yang berikut]
A. The kinetic energy of the electron in the first Bohr orbit of hydrogen is -13.6 eV.
[Tenaga kinetik elektron dalam orbit Bohr pertama ialah -13.6 eV]
B. The electron in a doubly ionised lithium atom experiences a weaker attractive force than the single electron in a hydrogen atom.
[Elektron dalam atom lithium yang dua kali terionkan mengalami daya tarikan yang lebih lemah berbanding dengan elektron tunggal dalam atom hidrogen]
C. In a hydrogen atom, the difference in the energy between adjacent orbit radii increases with the increasing value of *n*

[Dalam atom hidrogen, perbezaan tenaga di antara dua radius orbit yang berjiranan bertambah bila nilai n bertambah]

D. The Bohr model correctly predicts the energy for the ground state of the hydrogen atom. *[Model Bohr meramal dengan tepatnya tenaga keadaan bumi atom hidrogen]*

E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS B: Walker test item, pg. 664, Q36

23. Hydrogen atoms can emit four lines with visible colours from red to violet. These four visible lines emitted by hydrogen atoms are produced by electrons
[Atom hidrogen boleh memancarkan empat garis warna nampak daripada merah ke ungu. Empat garis nampak yang dipancarkan oleh atom hidrogen ini adalah dihasilkan oleh elektron]

A. that starts in the $n = 2$ level.
B. that end up in the $n = 2$ level.
C. that end up in the ground state.
D. that start in the ground state.
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS D: Cutnel page 911

24. An electron in the hydrogen atom is in the $n = 4$ energy level. When this electron makes a transition to a lower level, the wavelength of the photon emitted is in the
[Suatu elektron dalam atom hidrogen berada dalam paras n = 4. Bila elektron tersebut melakukan peralihan kepada paras tenaga yang lebih rendah, jarak gelombang foton yang terpancarkan berada dalam]

I. the Lyman series **II.** the Blamer siries
III. the Pashech series **IV.** the Pfund series

A. I **B.** II **C.** III **D.** I,II,III
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

ANS A: Cutnel page 934, Q 7

25. What is the longest radiation wavelength that can be used to ionized the ground-state hydrogen atom?
[Apakah jarak gelombang pancaran yang paling panjang yang boleh digunakan untuk mengiokan atom hidrogen pada keadaan bumi?]

A. $hc/(13.6\text{ eV})$
B. $2hc/(13.6\text{ eV})$
C. $13.6\ hc$
D. $(13.6\text{ eV})/hc$
E. Non of A, B, C, D *[Jawapan tiada dalam A, B, C, D]*

SESSI 04/05/KSCP

Part B: Structured Questions [75 marks]
Instruction: Answer ALL questions in this Part.
[Bahagian B: Soalan Struktur. 75 markah]
[Arahan: Jawab KESEMUA soalan dalam Bahagian ini.]

1. **(a)** Based on the physics constants data sheet provided (first page), calculate the ratio of the mass of proton to that of the electron.
[Berdasarkan lampiran data (dalam m/s pertama) pemalar-pemalar fizik yang dibekalkan, hitungkan nisbah antara jisim proton kepada jisim elektron.]

[5 marks]

Solution: $\dfrac{M_P}{m_e} = \dfrac{1.67\times10^{-27}}{9.11\times10^{-31}} = 1833.2$

(b) Calculate the kinetic energy of the electrons in a beam, in units of electron rest energy m_ec^2, such that the relativistic mass of the electrons in the beam is as large is that of the proton.
[Hitungkan tenaga kinetik bagi elektron-elektron dalam satu alur elektron, dalam unit tenaga rehat elektron m_ec^2, sedemikian rupa supaya jisim kerelatifan elektron dalam alur tersebut bersamaan dengan jisim proton.)]

[5 marks]

Solution: Young and Freeman study guide, pg 281, Quiz 2,3

$$E = m'_e c^2 = m_e c^2 + K$$
$$\text{set } m'_e c^2 = M_P c^2 = (1833.2)m_e c^2$$
$$\Rightarrow K = (1833.2-1)m_e c^2 = (1832.2)m_e c^2$$

(c) What is the electric potential (in unit of Volt) that is required to accelerate the electron in (b) (from rest) ?
[Apakah beza keupayaan elektrik (dalam unit Volt) yang diperlukan untuk memecutkan elektron dalam (b) di atas (dari keadaan rehat)?]

[5 marks]

Solution: Young and Freeman study guide, pg 281, Quiz 2,3
$$eV = K = (1832.2)m_e c^2 \Rightarrow V = (1832.2)m_e c^2/e = 938.9\text{ MV}$$

(d) If a 'moving clock' runs slower, what will the age difference between two twins if one stays on the Earth while the second makes a round trip to a point in space ten light years from Earth at a speed of $0.95c$?
[Jika masa bagi 'jam yang bergerak' mengalir lebih perlahan', apakah perbezaaan umur di antara dua orang anak kembar jika salah satu daripada mereka tinggal di Bumi manakala yang seorang lagi menjalani satu penggembaraan dengan laju 0.95c ke satu tempat sejauh 10 tahun-cahaya daripada Bumi dan kembali ke Bumi selepas penjelajahan tersebut?]

[10 marks]

Solution: Young and Freeman study guide, pg 278, Example 7

$$\gamma = \frac{1}{\sqrt{1-(0.95)^2}} = 3.2$$

Time taken for the round trip, according to the twin on Earth, is

$T_E = D/v = 20\ c\cdot\text{yr}/0.95c = 21.05$ yr.

Time taken for the round trip, according to the twin on ship, is
$T_S = D'/v = D/(\gamma v) = 20\ c\cdot\text{yr}/(3.2\cdot 0.95c) =$ 6.58 yr, where $D' = 20\ \text{ly}/\gamma$ due to length contraction.

$\Rightarrow T_E - T_S = (21.05 - 6.58)$ yr = 14.47 yr

2. **(a)** A 60-W bulb is at an efficiency of 6.20%. What is the number of photons per second given off by the bulb assuming the wavelength of light to be 580 nm?

[6 marks]

Solution: Walker Test Item, page 642, Q5:

$0.062\times 60\text{Watt} = 2.325\times10^{19}\text{eV/s}$

$\text{energy of 1 photon} = \frac{hc}{\lambda} = \frac{1240}{580}\text{eV} = 2.13\text{eV}$

Let number of photon per second = N

$\text{therefore } N\frac{hc}{\lambda} = 2.325\times10^{19}\text{eV/s}$

$N = \frac{2.325\times10^{19}\text{eV/s}}{2.13\text{eV}} = 1.09\times10^{19}/\text{s}$

(b) The work functions of several metals are listed below.

Metal	ϕ (in eV)
W	4.5
Ag	4.8
Cs	1.8
Cs on W	1.36

(*i*) Which metals yield photoelectrons when bombarded with light of wavelength 500 nm?
(*ii*) For those surfaces where photoemission occurs with the above light source, calculate the stopping potential in volts.
(*iii*) For the metal tungsten calculate the threshold wavelength which would just start producing photoelectrons.

[3 + (2+2) + 2 = 9 marks]

Solution: Young and Freeman study guide, pg 287, Example 2

(*i*) $E = hf = hc/\lambda = 2.48$ eV; Cs and Cs on W yields photoelectrons
(*ii*) For Cs: stopping potential is $(2.48\text{ eV} - 1.8\text{ eV})/e = 0.68$ V
For Cs on W: stopping potential is $(2.48\text{ eV} - 1.36\text{ eV})/e = 1.12$ V
(*iii*) $\lambda_t = hc/\phi = 1240\text{ eV}\cdot\text{nm} / 4.5\text{ eV} = 276$ nm

(c) A large number if 30.0 pm photons are scattered twice by stationary electrons. Find the **RANGE** of wavelength of the scattered photon in pm.
[Sejumlah besar foton-foton yang berjarak gelombang 30.0 pm diserakkan dua kali oleh satu elektron rehat. Hitungkan julat bagi jarak gelombang foton yang terserakkan dalam unit pm.]

[10 marks]

Solution: Young and Freeman test bank, pg 409, Q14:

When bombarded once, the maximal increase in the photon wavelength is given by $\Delta\lambda_{max} = \frac{2h}{m_e c} = 2\times 2.43\text{pm} = 4.86\text{pm}$ when the scattering angle $\theta = 180°$. When the once-scattered photon is scattered again, the maximum shift in wavelength suffered by that photon is also $\Delta\lambda_{max}$, making the maximal total shift in wavelength = $2\Delta\lambda_{max}$ = 2 x 4.86 pm = 9.72 pm. Hence the range of scattered photon lies between λ_0 to $\lambda_0 + 2\Delta\lambda_{max}$, i.e. 30.0 pm – 39.72 pm.

3. **(a)** Find the frequency of revolution of electron in $n = 1$ and $n = 2$ Bohr orbits. What is the frequency of the photon emitted when an electron in the $n = 2$ orbit drops to $n = 1$ orbit?
[Hitungkan frekuensi kisaran bagi elektron dalam orbit-orbit Bohr n = 1 dan n = 2. Apakah frekuensi foton yang dipancarkan bila suatu elektron dalam orbit n=2 jatuh ke orbit n = 1?]

[3 + 2 + 2 + 3 marks]

Solution: Bieser, pg 137/tutorial 5

From Bohr's postulate of quantisation of angular momentum, $L = (mv)r = nh/2\pi$, the velocity is related to the radius as $v = nh/2mr\pi$. Furthermore, the quantised radius is given in terms of Bohr's radius as $r_n = n^2 r_0$. Hence, $v = h/2\pi mnr_0$. The frequency of revolution $f = 1/T$ (where T is the period of revolution) can be obtained from $v = 2\pi r/T = 2\pi n^2 r_0 f$. Hence, $f = v/2\pi r = (h/2\pi mnr_0)/2\pi r = h/4\pi^2 mn^3 (r_0)^2$.

For $n = 1$, $f_1 = h/4\pi^2 m(r_0)^2 = 6.56\times10^{15}$ Hz.
For $n = 2$, $f_2 = h/4\pi^2 m(2)^3(r_0)^2 = 6.56\times10^{15}/8$ Hz $= 8.2\times10^{14}$.

Photon's frequency = $\Delta E/h = 13.6\ (1/1^2 - 1/2^2)$ eV / h = 2.46 x 0^{15} Hz

(b) Consider the case of 'particle in a box' (infinite square well). The lowest energy level of a particle (call it particle A) confined to a 1-D region of space with fixed dimension L is E_0. If an identical particle (call it particle B) is confined to a similar region with fixed distance $L/4$, what is the energy of the lowest energy level of the particle B? Express your answer in terms of E_0.
[Pertimbangkan kes 'zarah di dalam kotak' (telaga segiempat infinit). Tenaga paling rendah bagi satu zarah (label ia zarah A) terkongkong di dalam satu ruang 1-D dengan dimensi L yang tetap ialah E_0. Jika suatu zarah lain (zarah B) yang identical dengan zarah A dikongkongkan di dalam satu ruang yang serupa tapi dengan jarak tetap L/4, apakah tenaga bagi paras tenaga yang terendah bagi zarah B? Nyatakan jawapan anda dalam sebutan E .]

[5 marks]

Solution: Young and Freeman test bank, pg 425, Short Questions 1: $16E_0$

$$E_0 = \frac{h^2}{8mL^2}$$

$$E'_0 = \frac{h^2}{8mL'^2} = \frac{h^2}{8m(L/4)^2} = 16\frac{h^2}{8mL^2} = 16E_0$$

(c) Estimate the kinetic energy (in eV) should electrons have if they are to be diffracted from crystal with interatomic distance of the order of a few Å.

[Anggarkan tenaga kinetik (dalam unit eV) yang harus diperolehi oleh elektron-elektron jika mereka hendak dibelaukan oleh hablur yang berjarak antara-atom dalam tertib beberapa Å]

[5 marks]

Solution
Serway, Mosses and Mayer, page 150, Example 4.3

For diffraction to happen, we require $\lambda \sim$ interactomic distance $\sim$ a few Å

$$p = \frac{hc}{\lambda c} \sim \frac{1240\text{eV}\cdot\text{nm}}{\text{few}\left(0.1\text{nm}\right)\times c} = \frac{0.01124\text{MeV}}{\text{few}\times c}$$

$$\Rightarrow K = \frac{p^2}{2m_e} \sim \left(\frac{0.0124\text{MeV}/c}{\text{few}}\right)^2 \frac{1}{2\times 0.5\text{MeV}/c^2} = \frac{1.5\times 10^{-4}}{\text{few}^2}\text{MeV}$$

(d) What is the frequency of the de Broglie waves associated with a body of rest mass m_0 moving with velocity v?
[Apakah frekuensi bagi gelombang de Broglie yang dikaitkan dengan jasad yang jisim rehatnya m_0 dan bergerak dengan laju v?]

[5 marks]

Solution: Arthur Beiser 5th edition, page 99

$$E = hf = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow f = \frac{m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$