ZCT 104/3E Modern Physics
Semester II, Sessi 2006/07
Open Book Quiz IV
Duration: 30 min

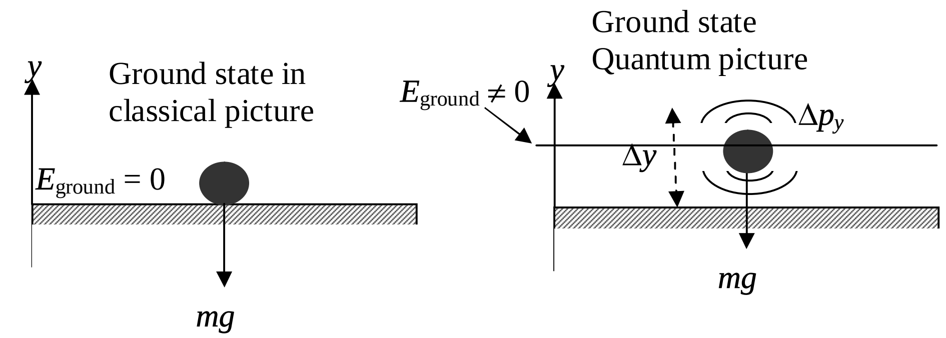
Name:

Matrics No:

INSTRUCTION: Answer the following question. **[10 marks]**

A particle under gravity

A particle falls under gravity towards an impenetrable floor.



According to classical mechanics, the ground state (the state of least energy) is one in which the particle is at rest on the floor. Let us measure the distance vertically from the floor and call it y. Thus, we know the position of the particle in its ground state ($y = 0$) and also its momentum ($p_y = 0$). This contradicts the uncertainty principle. In the quantum picture, we know that a particle cannot rest on the floor even under the pull of gravity. Even in the lowest energy state, the particle bounces up and down with a range Δy and Δp_y according to the Uncertainty principle. See the picture above.

(i) Write down the uncertainty relation that relates Δy and Δp_y.

[2 marks]

The potential energy of the particle is

$$V(y) = mgy \text{ if } y > 0,$$
$$= +\infty \text{ if } y < 0.$$

(ii) What is the approximate energy of the particle at the ground state? (*Hint: The ground state energy is the sum of the potential energy and the kinetic energy. You should express the energy estimate in terms of* Δy.)

[4 marks]

(iii) Estimate the order of Δy in terms of m (*Hint: To obtain the estimate of* Δy*, you simply minimise the answer obtained in (ii) with respect to* Δy)

[4 marks]

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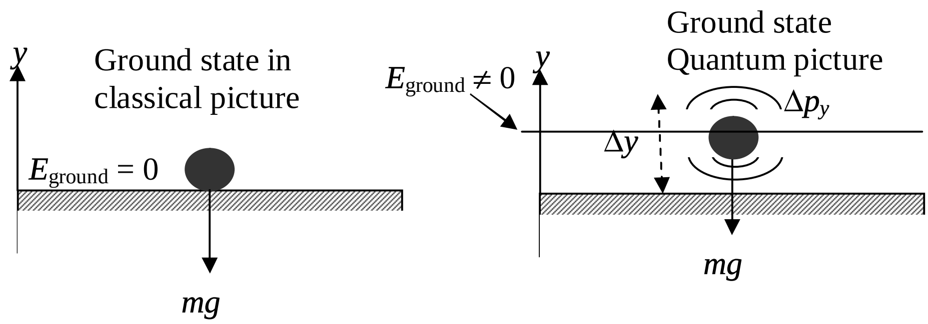

According to classical mechanics, the ground state (the state of least energy) is one in which the particle is at rest on the floor. Let us measure the distance vertically from the floor and call it y. Thus, we know the position of the particle in its ground state ($y = 0$) and also its momentum ($p_y = 0$). This contradicts the uncertainty principle. In the quantum picture, we know that a particle cannot rest on the floor even under the pull of gravity. Even in the lowest energy state, the particle bounces up and down with a range Δy and Δp_y according to the Uncertainty principle. See the picture above.

(i) Write down the uncertainty relation that relates Δy and Δp_y. [2 marks]

ANS: The ground state will differ from the classical solution by having an uncertainty in position of Δy and momentum Δp_y where

$$\Delta p_y \sim \hbar / (2\Delta y).\ (\mathbf{2\ marks})$$

(Note: never mind if the factor 2 is missing since this is an estimate anyway)

(ii) Then the energy is approximately

$$E \sim \frac{(\Delta p_y)^2}{2m} + mg\Delta y = \frac{\hbar^2}{8m(\Delta y)^2} + mg\Delta y$$

[4 marks]

Note: Deduct half the marks if relativistic form of kinetic energy $K = pc$ is used instead of non-relativistic one (i.e. $K = \frac{p_y^2}{2m} \sim \frac{(\Delta p_y)^2}{2m}$). The question is obviously to be treated non-relativistically as we are talking about a 'rest' particle, of which motion is not expected to be fluctuating violently. (The violent the motion is the more relativistic it will be.)

(iii) Minimizing the energy with respect to Δy,

$$\frac{dE}{d(\Delta y)} = -\frac{\hbar^2}{8m}(\Delta y)^{-3} + mg = 0$$

$$\Rightarrow (\Delta y) = \left(\frac{\hbar^2}{8m^2 g}\right)^{1/3} = \frac{1}{2}\left(\frac{\hbar^2}{m^2 g}\right)^{1/3} \sim \left(\frac{\hbar^2}{m^2 g}\right)^{1/3} \sim 10^{-23}\left(\frac{kg}{m}\right)^{2/3} \text{ m}$$ [4 mark]

Note: IF the candidate uses relativistic expression for the energy in (ii), i.e. $E = p_y c + mg\Delta y \sim \frac{\hbar c}{2\Delta y} + mg\Delta y$, and minimise it as per

$$\frac{d}{d(\Delta y)}\left[\frac{\hbar c}{2\Delta y} + mg\Delta y\right] = -\frac{\hbar c}{2(\Delta y)^2} + mg = 0$$

to get

$$\frac{\hbar c}{2(\Delta y)^2} = mg \Rightarrow \Delta y = \left(\frac{\hbar c}{2mg}\right)^{1/2} \sim \left(\frac{\hbar c}{g}\right)^{1/2}\left(\frac{\text{kg}}{m}\right)^{1/2} \text{m} \sim 10^{-27}\left(\frac{\text{kg}}{m}\right)^{1/2} \text{m},$$

only half of 4 marks shall be given.