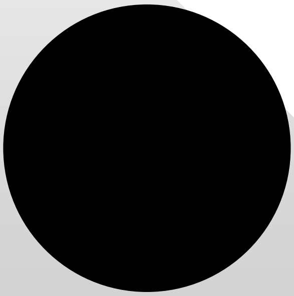
CHAPTER 2

- PROPERTIES OF WAVES AND MATTER
- BLACK BODY RADIATION

Matter, energy and interactions

- One can think that our universe is like a stage existing in the form of space-time as a background
- All existence in our universe is in the form of either matter or energy (Recall that matter and energy are \`equivalent′ as per the equation $E = mc^2$)



matter

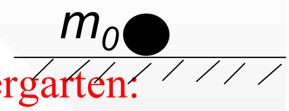
energy

Interactions

- Matter and energy exist in various forms, but they constantly transform from one to another according to the law of physics
- we call the process of transformation from one form of energy/matter to another energy/matter as ‘interactions’
- Physics attempts to elucidate the interactions between them
- But before we can study the basic physics of the matter-energy interactions, we must first have some general idea to differentiate between the two different modes of physical existence: matter and wave
- This is the main purpose of this lecture

Matter (particles)

- **Consider a particles with mass:**



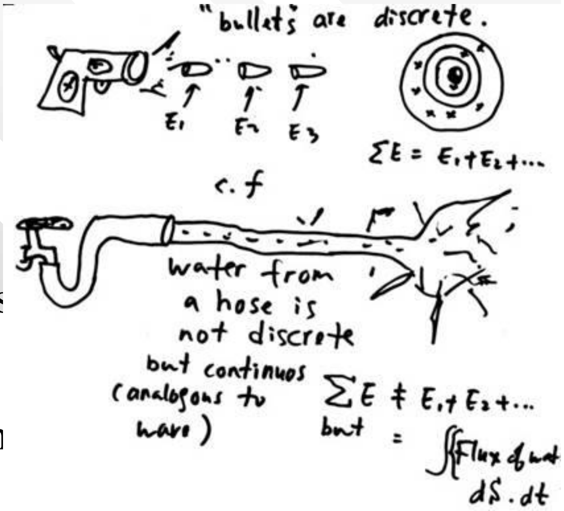
- you should know the following facts since kindergarten:
- A particle is discrete, or in another words, corpuscular, in nature.
- a particle can be localized completely, has mass and electric charge that can be determined with infinite precision (at least in principle)
- So is its momentum
- These are all implicitly assumed in Newtonian mechanics
- This is to be contrasted with energy exists in the forms of wave which is not corpuscular in nature (discuss later)

Energy in particle is corpuscular (discrete) i.e. not spread out all over the place like a continuum

- The energy carried by a particle is given by

$$E^2 = m_0^2 c^4 + p^2 c^2$$

- The energy of a particles is concentrated within the boundary of a particle (e,g. in the bullet)
- Hence we say "energy of a particle is corpuscular"
- This is in contrast to the energy carried by the water from the host, in which the energy is distributed spread all over the space in a continuous manner



Example of particles

- Example of `particles': bullet, billiard ball, you and me, stars, sands, etc...
- Atoms, electrons, molecules (or are they?)

What is not a `particle′?

- Waves - electromagnetic radiation (light is a form of electromagnetic radiation), mechanical waves and matter waves is classically thought to not have attributes of particles as mentioned

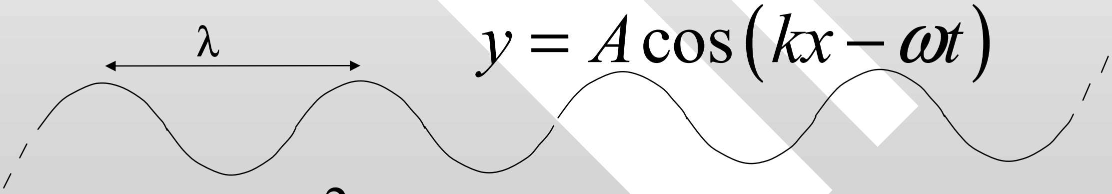
Analogy

- Imagine energy is like water
- A cup containing water is like a particle that carries some energy within it
- Water is contained within the cup as in energy is contained in a particle.
- The water is not to be found outside the cup because they are all retained inside it. Energy of a particle is corpuscular in the similar sense that they are all inside the carrier which size is a finite volume.
- In contrast, water that is not contained by any container will spill all over the place (such as water in the great ocean). This is the case of the energy carried by wave where energy is not concentrated within a finite volume but is spread throughout the space

Wave

- Three kinds of wave in Nature: mechanical, electromagnetical and matter waves
- The simplest type of wave is strictly sinusoidal and is characterised by a `sharp′ frequency ν (= $1/T$, T = the period of the wave), wavelength λ and its travelling speed c

$$y = A\cos\left(kx - \omega t\right)$$



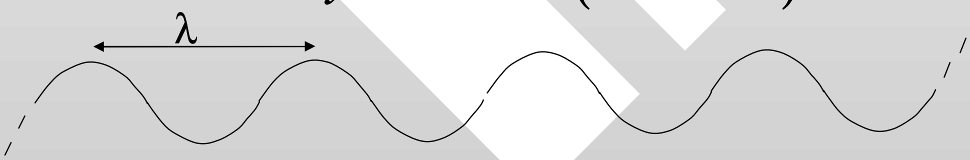
$$C = \lambda\nu; k = \frac{2\pi}{\lambda}$$

A `pure’ (or ‘plain’) wave which has `sharp’ wavelength and frequency

Quantities that characterise a pure wave

- The quantities that quantify a pure (or called a plane) wave:
  λ, wave length, equivalent to $k = 2\pi/\lambda$, the wave number
  $\nu = 1/T$, frequency, equivalent angular frequency,
  $\omega = 2\pi\nu$
- c speed of wave, related to the above quantities via
- $c = \lambda\nu = \omega/k$

$$y = A\cos\left(kx - \omega t\right)$$



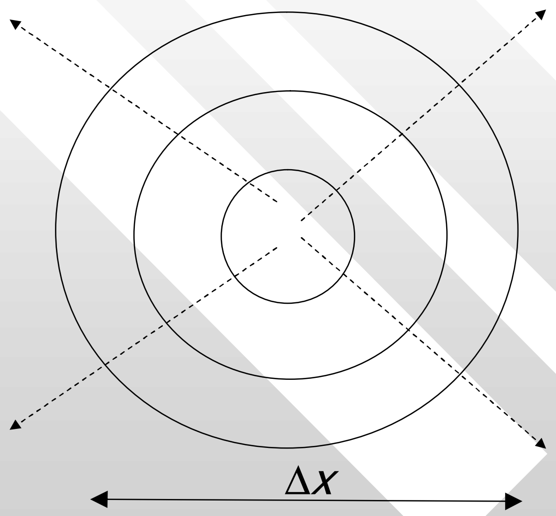
$$C = \lambda\nu;$$

Where is the wave?

- For the case of a particle we can locate its location and momentum precisely
- But how do we 'locate' a wave?
- Wave spreads out in a region of space and is not located in any specific point in space like the case of a particle
- To be more precise we says that a plain wave exists within some region in space, Δx
- For a particle, Δx is just the 'size' of its dimension, e.g. Δx for an apple is 5 cm, located exactly in the middle of a square table, x = 0.5 m from the edges. In principle, we can determine the position of x to infinity
- But for a wave, Δx could be infinity

In fact, for the `pure' (or 'plain') wave which has `sharp' wavelength and frequency mentioned in previous slide, the Δx is infinity

For example, a ripple



the ripple exists within the region Δx

A pure wave has $\Delta x \rightarrow$ infinity

- If we know the wavelength and frequency of a pure wave with infinite precision (= the statement that the wave number and frequency are 'sharp'), one can shows that :
- The wave cannot be confined to any restricted region of space but must have an infinite extension along the direction in which it is propagates
- In other words, the wave is 'everywhere' when its wavelength is 'sharp'
- This is what it means by the mathematical statement that "Δx is infinity"

More quantitatively,

$$\Delta x \Delta \lambda \geq \lambda^2$$

- This is the uncertainty relationships for classical waves

  $\Delta\lambda$ is the uncertainty in the wavelength.
- When the wavelength `sharp' (that we knows its value precisely), this would mean $\Delta\lambda = 0$.
- In other words, $\Delta\lambda \rightarrow$ infinity means we are totally ignorant of what the value of the wavelength of the wave is.

Δx is the uncertainty in the location of the wave (or equivalently, the region where the wave exists)

- $\Delta x = 0$ means that we know exactly where the wave is located, whereas $\Delta x \rightarrow$ infinity means the wave is spread to all the region and we cannot tell where is it's `location'

$\Delta\lambda\ \Delta x \geqslant \lambda^2$ means the more we knows about x, the less we knows about λ as Δx is inversely proportional to $\Delta\lambda$

Other equivalent form

- $\Delta x \Delta \lambda \geq \lambda^2$ can also be expressed in an equivalence form

$$\Delta t \Delta \nu \geq 1$$

via the relationship $c = \nu\lambda$ and $\Delta x = c\Delta t$

- Where Δt is the time required to measure the frequency of the wave
- The more we know about the value of the frequency of the wave, the longer the time taken to measure it
- If u want to know exactly the precise value of the frequency, the required time is Δt = infinity
- We will encounter more of this when we study the Heisenberg uncertainty relation in quantum physics

- The classical wave uncertain relationship

$$\Delta x \Delta \lambda \geq \lambda^2$$

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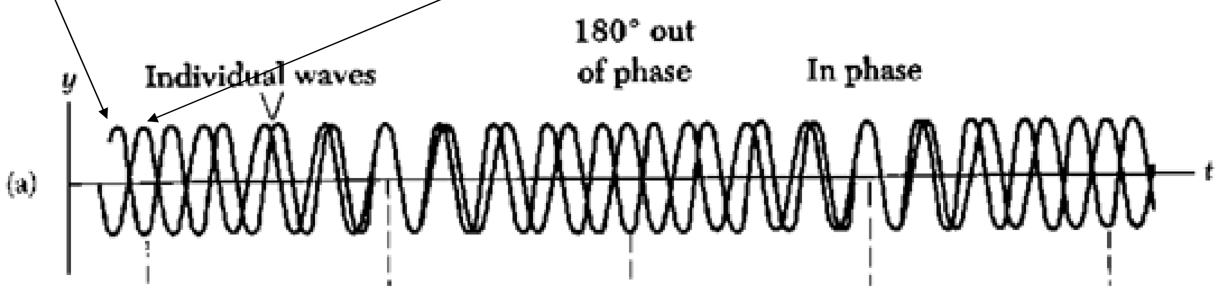
Wave can be made more ``localised″

- We have already shown that the 1-D plain wave is infinite in extent and can′t be properly localised (because for this wave, $\Delta x \rightarrow$infinity)
- However, we can construct a relatively localised wave (i.e., with smaller Δx) by :
- adding up two plain waves of slightly different wavelengths (or equivalently, frequencies)
- Consider the `beat phenomena′

Constructing wave groups

- Two pure waves with slight difference in frequency and wave number $\Delta\omega = \omega_1 - \omega_2$, $\Delta k = k_1 - k_2$, are superimposed

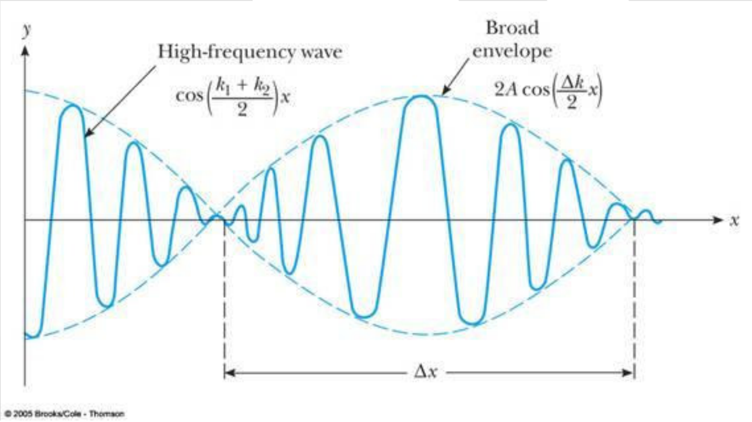
$$y_1 = A\cos(k_1 x - \omega_1 t); \quad y_2 = A\cos(k_2 x - \omega_2 t)$$



Envelop wave and phase wave

The resultant wave is a 'wave group' comprise of an `envelop' (or the group wave) and a phase waves

$$y = y_1 + y_2$$
$$= 2A\cos\frac{1}{2}\left(\{k_2 - k_1\}x - \{\omega_2 - \omega_1\}t\right)\cdot\cos\left\{\left(\frac{k_2 + k_1}{2}\right)x - \left(\frac{\omega_2 + \omega_1}{2}\right)t\right\}$$



- As a comparison to a plain waves, a group wave is more 'localised' (due to the existence of the wave envelop. In comparison, a plain wave has no `envelop' but only 'phase wave')

- It comprises of the slow envelop wave

$$2A\cos\frac{1}{2}\left(\{k_2-k_1\}x-\{\omega_2-\omega_1\}t\right)=2A\cos\frac{1}{2}(\Delta kx-\Delta\omega t)$$

that moves at group velocity $v_g = \Delta\omega/\Delta k$

- and the phase waves (individual waves oscillating inside the envelop)

$$\cos\left\{\left(\frac{k_2+k_1}{2}\right)x-\left(\frac{\omega_2+\omega_1}{2}\right)t\right\}=\cos\{k_p x-\omega_p t\}$$

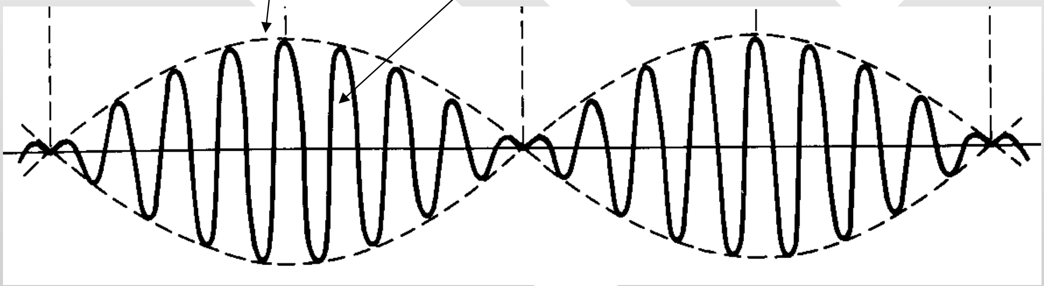
moving at phase velocity $v_p = \omega_p/k_p$

In general, $v_g = \Delta\omega/\Delta k << v_p = (\omega_1+\omega_2)/(k_1 + k_2)$ because $\omega_2 \approx \omega_1$, $k_1 \approx k_2$

$$y=y_1+y_2=\left\{2A\cos\frac{1}{2}(\Delta kx-\Delta\omega t)\right\}\cdot\cos\{k_p x-\omega_p t\}$$

`envelop' (group waves). Sometimes it's called 'modulation'

Phase waves



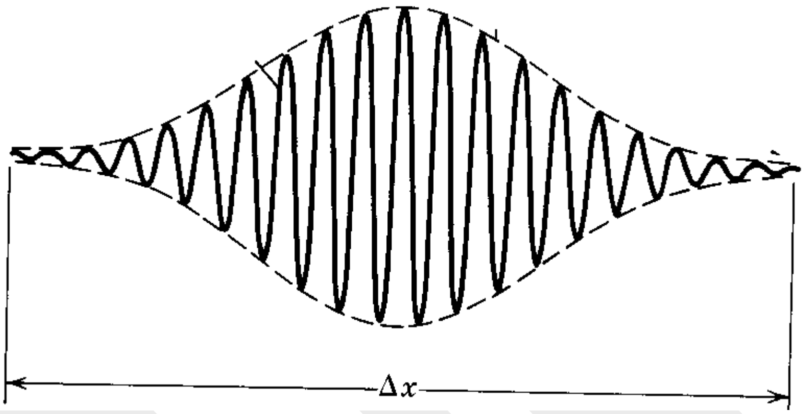
Energy is carried at the speed of the group wave

- The energy carried by the group wave is concentrated in regions in which the amplitude of the envelope is large
- The speed with which the waves' energy is transported through the medium is the speed with which the envelope advances, not the phase wave
- In this sense, the envelop wave is of more 'physical' relevance in comparison to the individual phase waves (as far as energy transportation is concerned)

Wave pulse – an even more `localised' wave

- In the previous example, we add up only two slightly different wave to form a train of wave group
- An even more `localised' group wave – what we call a "*wavepulse*" can be constructed by adding more sine waves of different numbers k_i *and* possibly different amplitudes so that they interfere constructively over a small region Δx and outside this region they interfere destructively so that the resultant field approach zero
- Mathematically,

$$y_{\text{wave pulse}} = \sum_i^{\infty} A_i \cos(k_i x - \omega_i t)$$



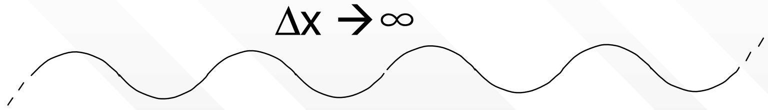
A wavepulse – the wave is well localised within Δx. This is done by adding a lot of waves with with their wave parameters $\{A_i, k_i, \omega_i\}$ slightly differ from each other (i = 1, 2, 3….as many as it can)

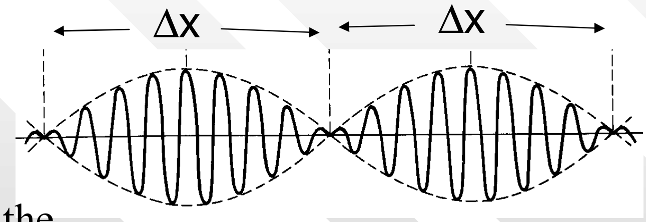
such a wavepulse will move with a velocity

$$v_g = \left.\frac{d\omega}{dk}\right]_{k_0}$$

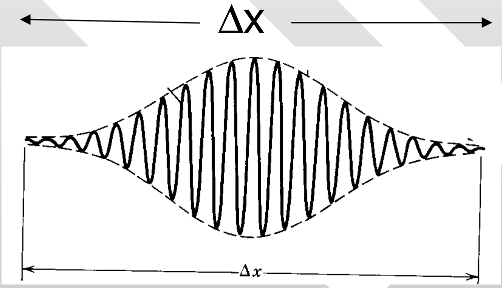
(c.f the group velocity considered earlier $v_g = \Delta\omega/\Delta k$)

Comparing the three kinds of wave





Which wave is the most localised?

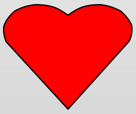


Why are waves and particles so important in physics?

- Waves and particles are important in physics because they represent the only modes of energy transport (interaction) between two points.
- E.g we signal another person with a thrown rock (a particle), a shout (sound waves), a gesture (light waves), a telephone call (electric waves in conductors), or a radio message (electromagnetic waves in space).

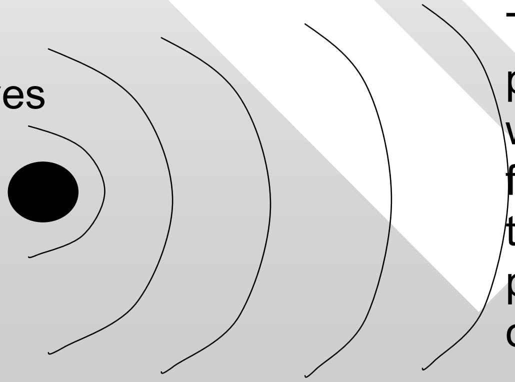
Interactions take place between

(i) particles and particles (e.g. in particle-particle collision, a girl bangs into a guy) or



(ii) waves and particle, in which a particle gives up all or part of its energy to generate a wave, or when all or part of the energy carried by a wave is absorbed/dissipated by a nearby particle (e.g. a wood chip dropped into water, or an electric charge under acceleration, generates EM wave)

Oscillating electron gives off energy

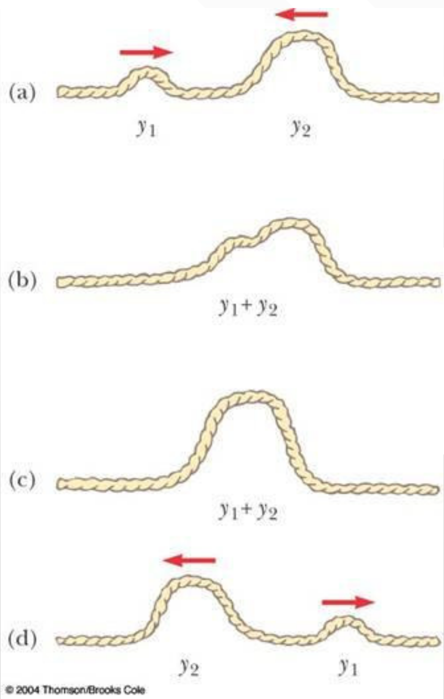


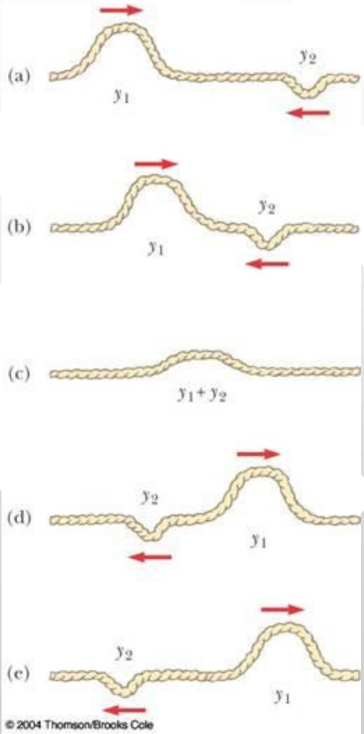
This is an example where particle is interacting with wave; energy transform from the electron's K.E. to the energy propagating in the form of EM wave wave

Waves superimpose, not collide

- In contrast, two waves do not interact in the manner as particle-particle or particle-wave do
- Wave and wave simply "**superimpose**": they pass through each other essentially unchanged, and their respective effects at every point in space simply add together according to the principle of superposition to form a resultant at that point -- a sharp contrast with that of two small, impenetrable particles

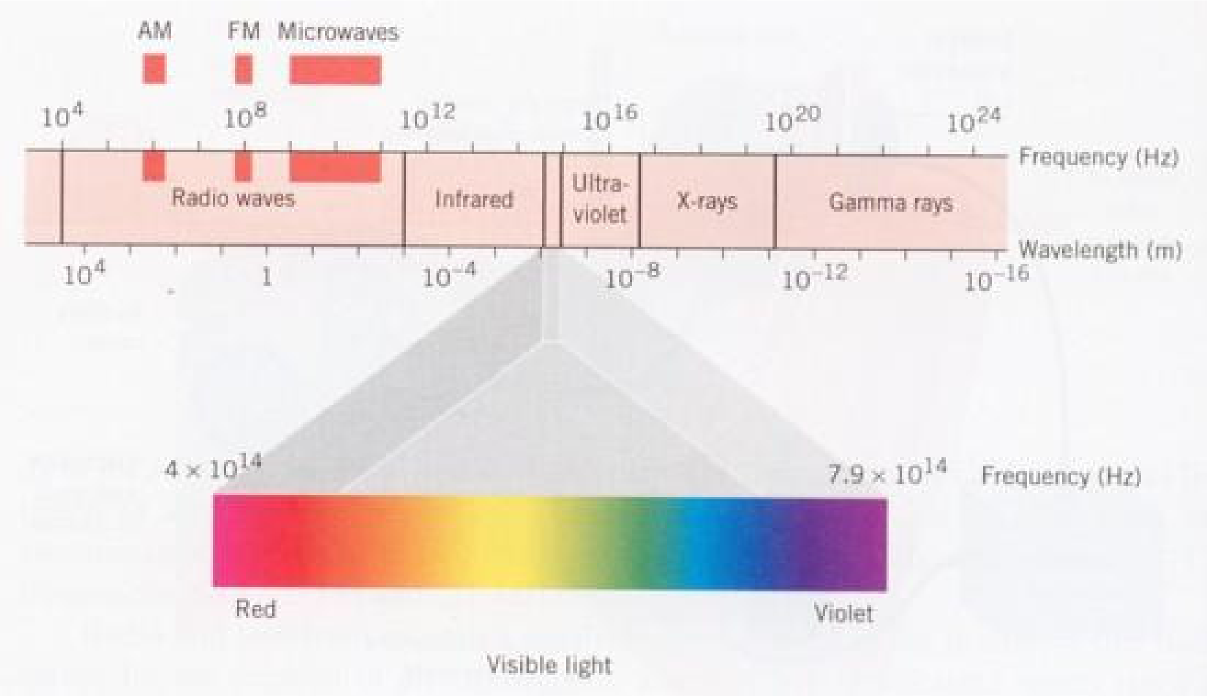
Superposition of waves



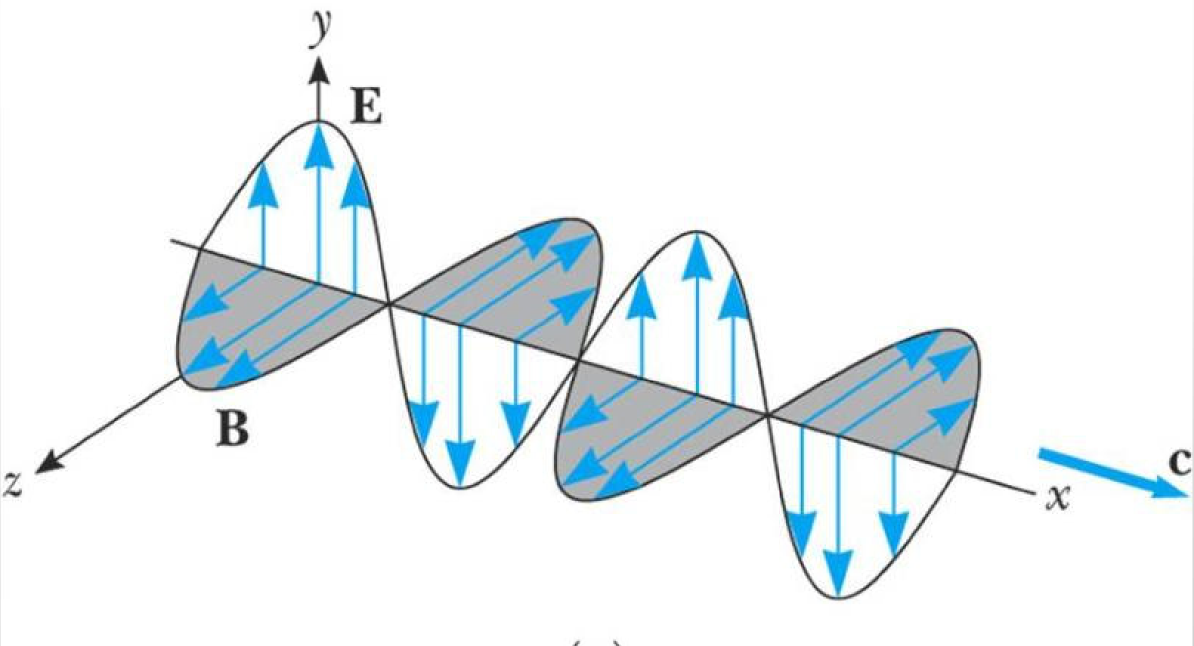


A pure EM wave

- According to Maxwell theory, light is a form of energy that propagates in the form of electromagnetic wave
- In Maxwell theory light is synonym to electromagnetic radiation is synonym to electromagnetic wave
- Other forms of EM radiation include heat in the form of infra red radiation, visible light, gamma rays, radio waves, microwaves, x-rays



A pure EM wave



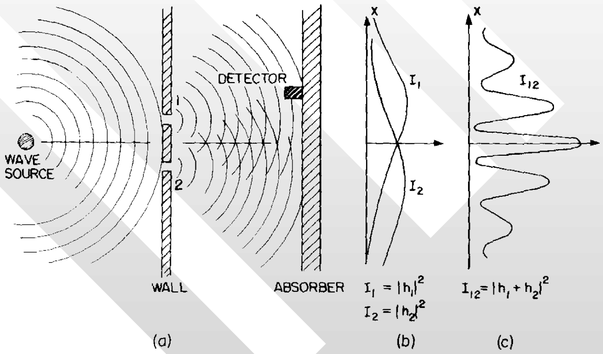
(a)

Heinrich Hertz (1857-1894), German, Established experimentally that light is EM wave

Interference experiment with water waves

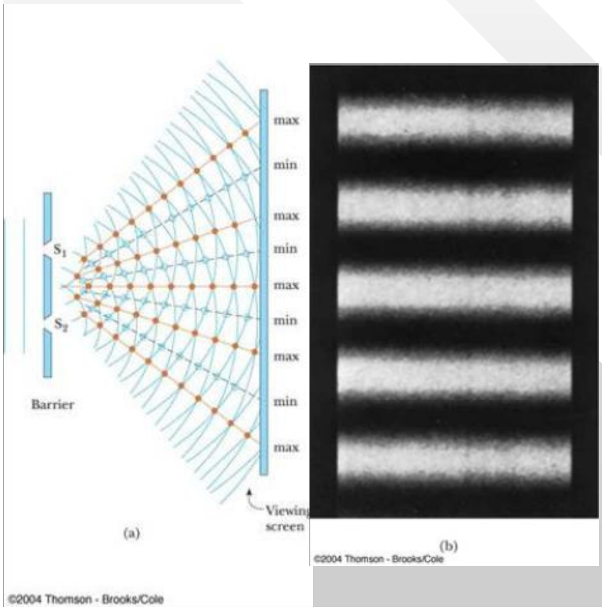
- If hole 1 (2) is block, intensity distribution of (I_1) I_2 is observed
- However, if both holes are opened, the intensity of I_{12} is such that $I_{12} \neq I_1 + I_2$
- Due to the wave nature, the intensities do not simply add
- In addition, and interference term exist,

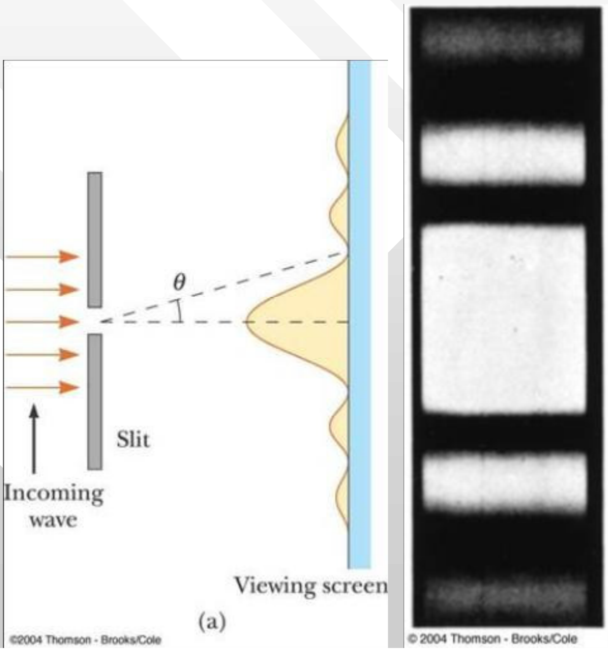
  $I_{12} = I_1 + I_2 + 2\cos\delta\,(I_1 + I_2)$
- "waves interfere"



Pictures of interference and diffraction pattern in waves

- Interference
- diffraction



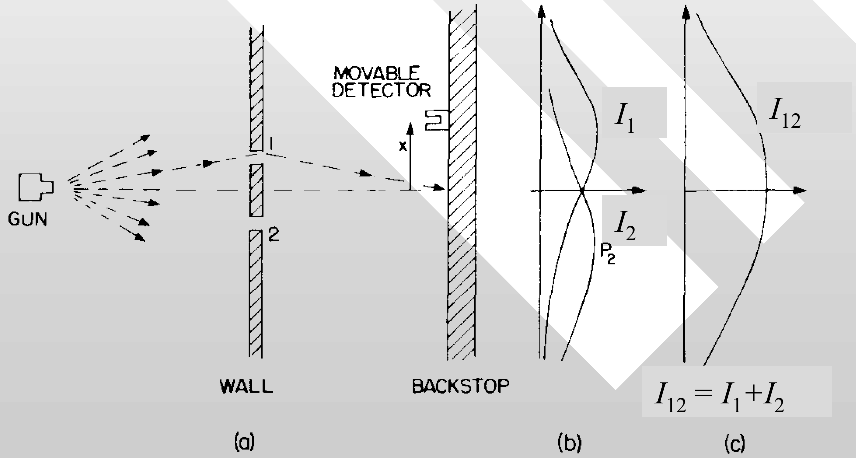


Since light display interference and diffraction pattern, it is wave

- Furthermore, Maxwell theory tell us what kind of wave light is
- It is electromagnetic wave
- (In other words it is not mechanical wave)

Interference experiment with bullets (particles)

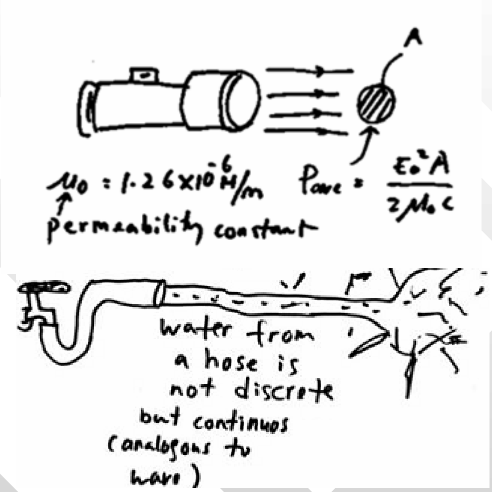
- I_2, I_1 are distribution of intensity of bullet detected with either one hole covered. I_{12} the distribution of bullets detected when both holes opened
- Experimentally, $I_{12} = I_1 + I_2$ (the individual intensity simply adds when both holes opened)
- Bullets always arrive in identical lump (corpuscular) and display no interference

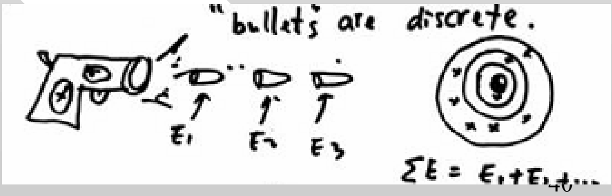


EM radiation transports energy in flux, not in bundles of particles

- The way how wave carries energy is described in terms of 'energy flux', in unit of energy per unit area per unit time
- Think of the continuous energy transported by a stream of water in a hose

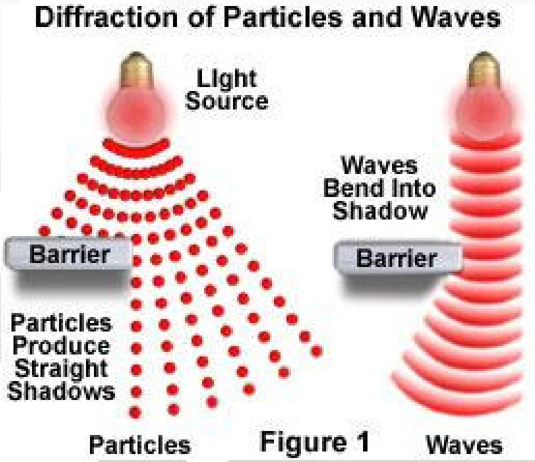
This is in contrast to a stream of 'bullet' from a machine gun where the energy transported by such a steam is discrete in nature





Essentially,

- Particles and wave are disparately distinct phenomena and are fundamentally different in their physical behaviour
- Free particles only travel in straight line and they don't bend when passing by a corner
- However, for light, it does
- Light, according to Maxwell's EM theory, is EM wave
- It display wave phenomena such as diffraction and interference that is not possible for particles
- Energy of the EM wave is transported in terms of energy flux



BLACK BODY RADIATION

- Object that is HOT (anything > 0 K is considered "hot") emits EM radiation
- For example, an incandescent lamp is red HOT because it emits a lot of EM wave, especially in the IR region

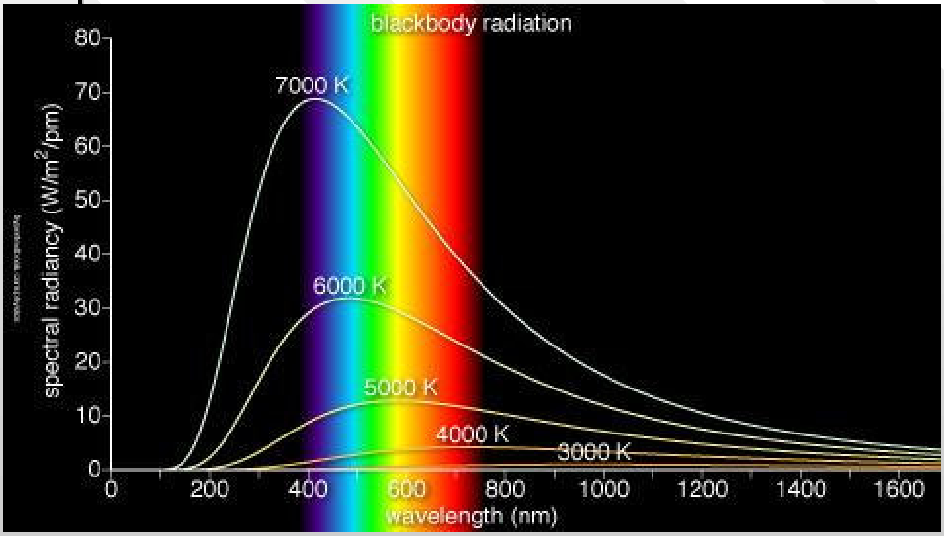
Attempt to understand the origin of radiation from hot bodies from classical theories

- In the early years, around 1888 – 1900, light is understood to be EM radiation
- Since hot body radiate EM radiation, hence physicists at that time naturally attempted to understand the origin of hot body in terms of classical EM theory and thermodynamics (which has been well established at that time)

- All hot object radiate EM wave of all wavelengths
- However, the energy intensities of the wavelengths differ continuously from wavelength to wavelength (or equivalently, frequency)
- Hence the term: the spectral distribution of energy as a function of wavelength

Spectral distribution of energy in radiation depends only on temperature

- The distribution of intensity of the emitted radiation from a hot body at a given wavelength depends on the temperature



Radiance

- In the measurement of the distribution of intensity of the emitted radiation from a hot body, one measures dI where dI is the intensity of EM radiation emitted between λ and λ +dλ about a particular wavelength λ.
- Intensity = power per unit area, in unit if Watt per m^2.
- **Radiance** $R(\lambda, T)$ is defined as per d$I = R(\lambda, T)\, d\lambda$
- $R(\lambda, T)$ is the power radiated per unit area (*intensity*) per unit wavelength interval at a given wavelength λ and a given temperature T.
- It's unit could be in Watt per meter square per m or
- W per meter square per nm.

Total radiated power per unit area

- The total power radiated per unit area (intensity) of the BB is given by the integral

$$I(T) = \int_0^\infty R(\lambda, T)\, \mathrm{d}\lambda$$

- For a blackbody with a total area of *A*, its total power emitted at temperature *T* is

$$P(T) = AI(T)$$

- Note: The SI unit for *P* is Watt, SI unit for *I* is Watt per meter square; for *A*, the SI unit is meter square

Introducing idealised black body

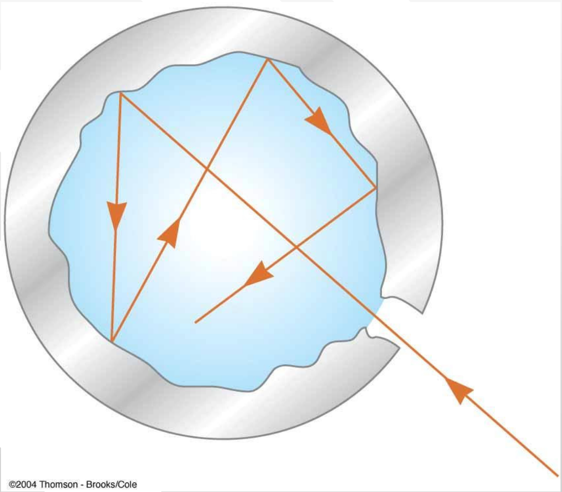
- In reality the spectral distribution of intensity of radiation of a given body could depend on the type of the surface which may differ in absorption and radiation efficiency (i.e. frequency-dependent)
- This renders the study of the origin of radiation by hot bodies case-dependent (which means no good because the conclusions made based on one body cannot be applicable to other bodies that have different surface absorption characteristics)
- E.g. At the same temperature, the spectral distribution by the exhaust pipe from a Proton GEN2 and a Toyota Altis is different

Emmissivity, *e*

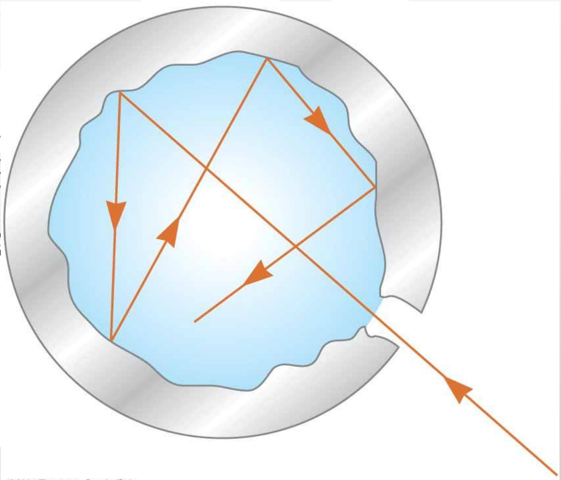
- As a strategy to overcome this non-generality, we introduce an idealised black body which, by definition, absorbs all radiation incident upon it, regardless of frequency
- Such idealised body is universal and allows one to disregard the precise nature of whatever is radiating, since all BB behave identically
- All real surfaces could be approximate to the behavior of a black body via a parameter EMMISSIVITY *e* (*e*=1 means ideally approximated, *e* << 1 means poorly approximated)

Blackbody Approximation

- A good approximation of a black body is a small hole leading to the inside of a hollow object
- The HOLE acts as a perfect absorber
- The Black Body is the HOLE

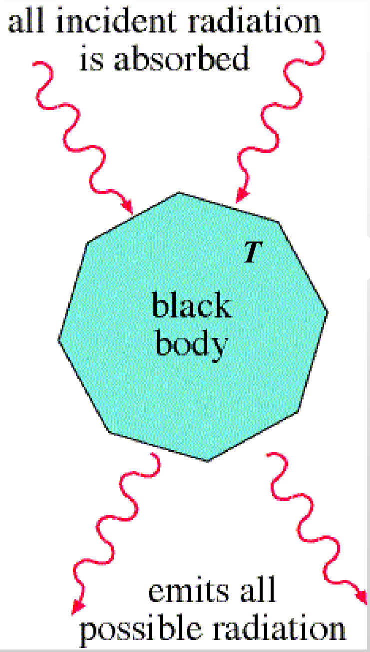


- Any radiation striking the HOLE enters the cavity, trapped by reflection until is absorbed by the inner walls
- The walls are constantly absorbin and emitting energy at thermal EI
- The nature of the radiation leavin the cavity through the hole depends only on the temperature of the cavity and not the detail of the surfaces nor frequency of the radiation



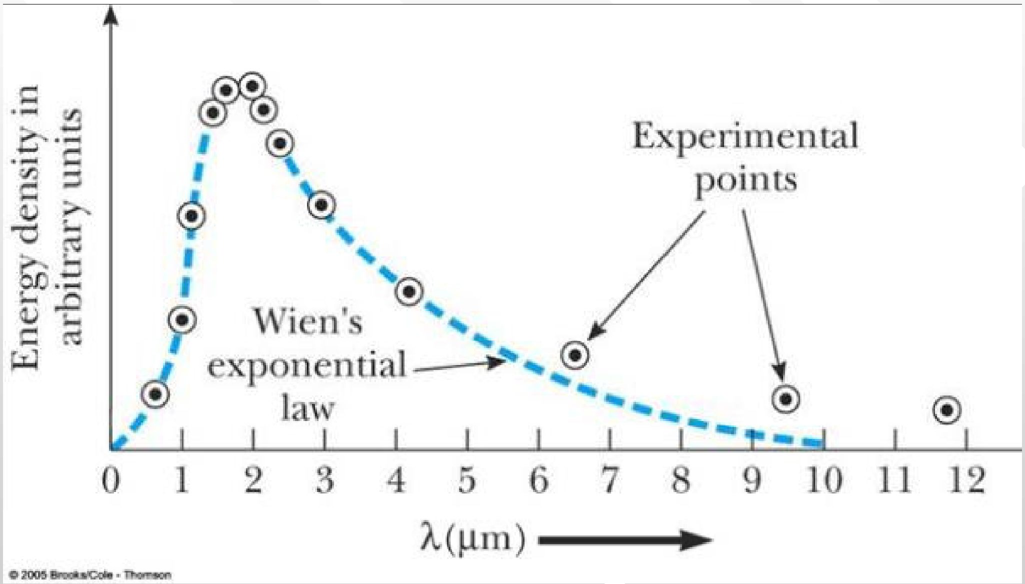
Essentially

- A black body in thermal EB absorbs and emits radiation at the same rate
- The HOLE effectively behave like a Black Body because it effectively absorbs all radiation fall upon it
- And at the same time, it also emits all the absorbed radiations at the same rate as the radiations are absorbed
- The measured spectral distribution of black bodies is universal and depends only on temperature.
- In other words: THE SPECTRAL DISTRIBUTION OF EMISSION DEPENDS SOLELY ON THE TEMPERATURE AND NOT OTHER DETAILS.



BB at thermodynamic equilibrium at a fixed temperature

Experimentally measured curve of a BB



Stefan's Law

- $P = \sigma A e T^4$
  - P total power output of a BB
  - A total surface area of a BB
  - σ Stefan-Boltzmann constant

    $\sigma = 5.670 \times 10^{-8}\ \mathrm{W / m^2 \cdot K^4}$
- Stefan's law can be written in terms of intensity
  - $I = P/A = \sigma T^4$
    - For a blackbody, where $e = 1$

Wien's Displacement Law

- $\lambda_{max} T = 2.898 \times 10^{-3}$ m·K
  - λ_{max} is the wavelength at which the curve peaks
  - T is the absolute temperature
- The wavelength at which the intensity peaks, λ_{max}, is inversely proportional to the absolute temperature
  - As the temperature increases, the peak wavelength λ_{max} is "displaced" to shorter wavelengths.

Example

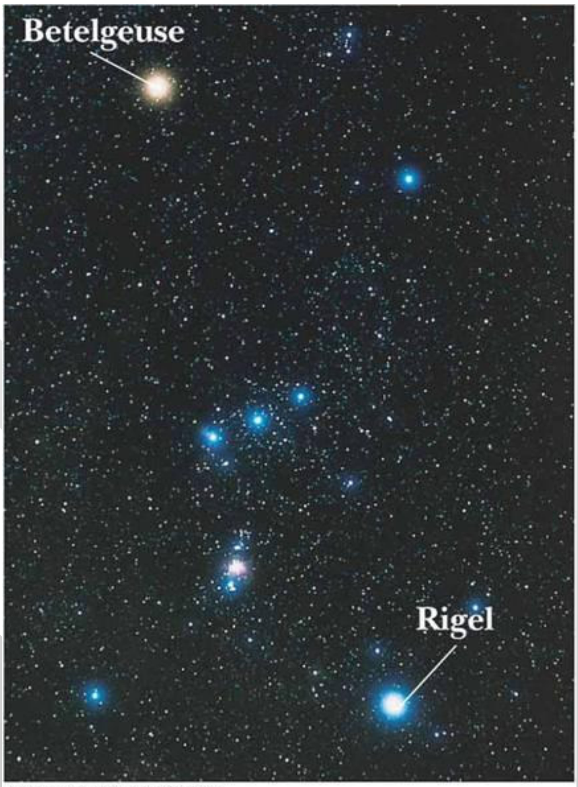
This figure shows two stars in the constellation Orion. Betelgeuse appears to glow red, while Rigel looks blue in color. Which star has a higher surface temperature?

(a) Betelgeuse

(b) Rigel

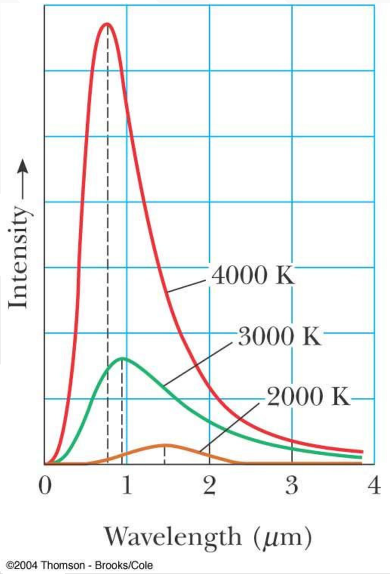
(c) They both have the same surface temperature.

(d) Impossible to determine.



Intensity of Blackbody Radiation, Summary

- The intensity increases with increasing temperature
- The amount of radiation emitted increases with increasing temperature
  - The area under the curve
- The peak wavelength decreases with increasing temperature



Example

- Find the peak wavelength of the blackbody radiation emitted by
- (A) the Sun (2000 K)
- (B) the tungsten of a light bulb at 3000 K

Solutions

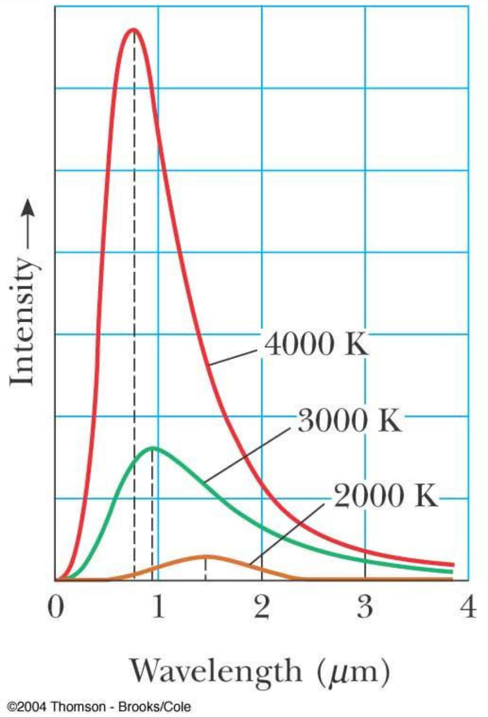
- (A) the sun (2000 K)
- By Wein's displacement law,

$$\lambda_{\max} = 2.898\times10^{-3}\,\mathrm{m\cdot K} / 2000\mathrm{K}$$
$$= 1.4\,\mu\mathrm{m}$$

- (infrared)
- (B) the tungsten of a lightbulb at 3000 K

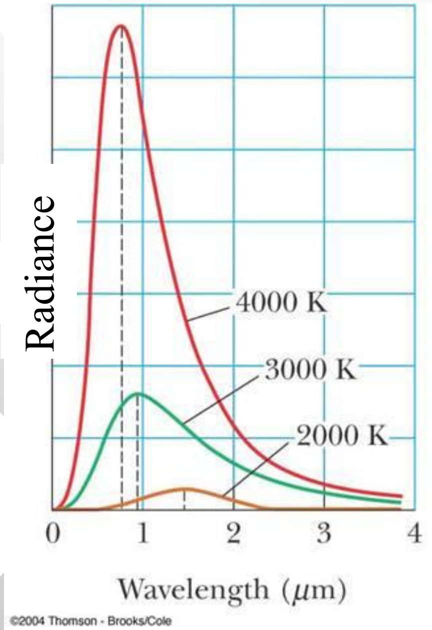
$$\lambda_{\max} = 2.898\times10^{-3}\,\mathrm{m\cdot K} / 5800\mathrm{K}$$
$$= 0.5\,\mu\mathrm{m}$$

- Yellow-green



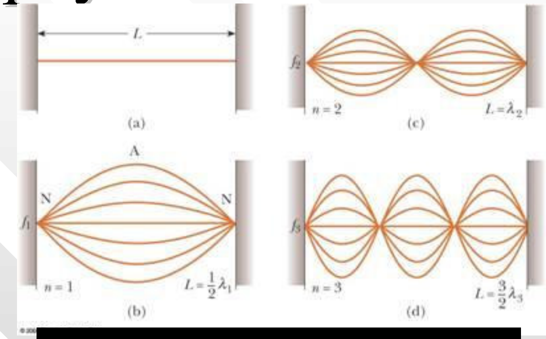
Why does the spectral distribution of black bodies have the shape as measured?

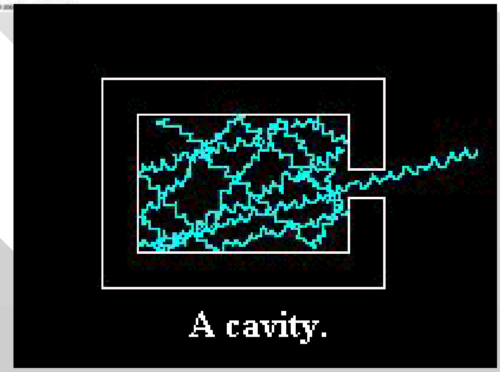
- Lord Rayleigh and James Jeans at 1890's try to theoretically derive the distribution based on statistical mechanics (some kind of generalised thermodynamics) and classical Maxwell theory
- (Details omitted, u will learn this when u study statistical mechanics later)



RJ's model of BB radiation with classical EM theory and statistical physics

- Consider a cavity at temperature *T* whose walls are considered as perfect reflectors
- The cavity supports many modes of oscillation of the EM field caused by accelerated charges in the cavity walls, resulting in the emission of EM waves at all wavelength
- These EM waves inside the cavity are the BB radiation
- They are considered to be a series of standing EM wave set up within the cavity





Number density of EM standing wave modes in the cavity

- The number of independent standing waves $G(\nu)d\nu$ in the frequency interval between ν and $\nu+d\nu$ per unit volume in the cavity is (by applying statistical mechanics)

$$G(\nu)\,d\nu = \frac{8\pi\nu^2 d\nu}{c^3}$$

- The next step is to find the average energy per standing wave

The average energy per standing wave, $\langle \varepsilon \rangle$

- Theorem of equipartition of energy (a mainstay theorem from statistical mechanics) says that the average energy per standing wave is
- $\langle \varepsilon \rangle = kT$

  $k = 1.38 \times 10^{-23}$ J/K, Boltzmann constant
- In classical physics, $\langle e \rangle$ can take any value CONTINOUSLY and there is not reason to limit it to take only discrete values
- (this is because the temperature T is continuous and not discrete, hence e must also be continuous)

Energy density in the BB cavity

- Energy density of the radiation inside the BB cavity in the frequency interval between ν and ν + dν, $u(\nu, T)\mathrm{d}\nu$
- = the total energy per unit volume in the cavity in the frequency interval between ν and ν + dν
- = the number of independent standing waves in the frequency interval between ν and $\nu + \mathrm{d}\nu$ per unit volume, $G(\nu)\mathrm{d}\nu$, × the average energy per standing wave.

$$\Rightarrow \quad u(\nu, T)\mathrm{d}\nu = G(\nu)\mathrm{d}\nu \times \langle \varepsilon \rangle = \frac{8\pi \nu^2 kT d\nu}{c^3}$$

Energy density in terms of radiance

- The energy density in the cavity in the frequency interval between ν and $\nu + d\nu$ can be easily expressed in terms of wavelength, λ via $c = \nu\lambda$

$$u(v,T) = \frac{8\pi v^2 kTdv}{c^3} \rightarrow u(\lambda,T) = \frac{8\pi kT}{\lambda^4} d\lambda$$

- In experiment we measure the BB in terms of radiance $R(\lambda,T)$ which is related to the energy density via a factor of $c/4$:
- $R(\lambda,T) = (c/4)u(\lambda, T) = \dfrac{2\pi ckT}{\lambda^4}$

Rayleigh-Jeans Law

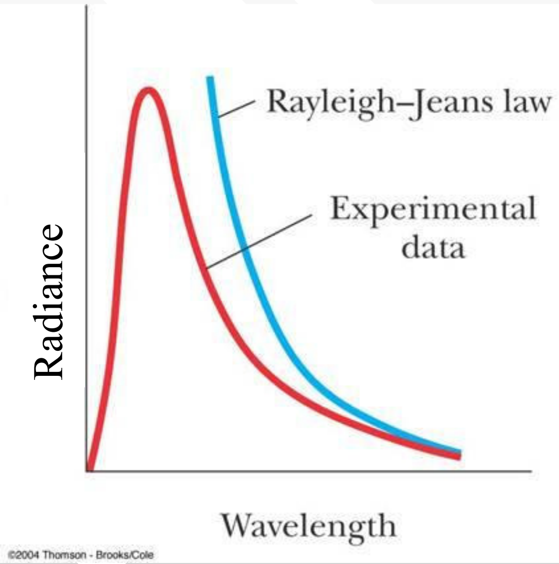
- **Rayleigh-Jeans law for the radiance (based on classical physics):**

$$R(\lambda,T) = \frac{2\pi ckT}{\lambda^4}$$

- At long wavelengths, the law matched experimental results fairly well

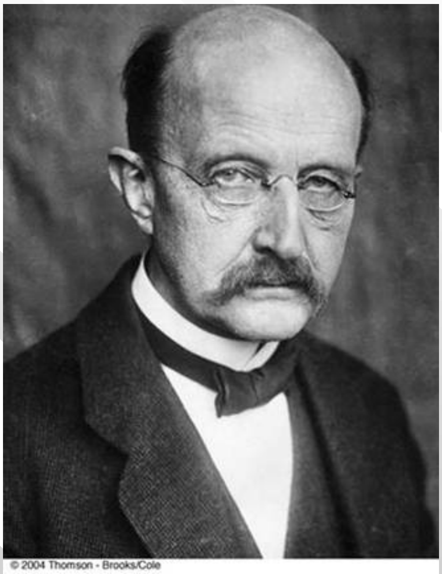
Rayleigh-Jeans Law, cont.

- At short wavelengths, there was a major disagreement between the Rayleigh-Jeans law and experiment
- This mismatch became known as the *ultraviolet catastrophe*
  - You would have infinite energy as the wavelength approaches zero



Max Planck

- Introduced the concept of "quantum of action"
- In 1918 he was awarded the Nobel Prize for the discovery of the quantized nature of energy



Planck's Theory of Blackbody Radiation

- In 1900 Planck developed a theory of blackbody radiation that leads to an equation for the intensity of the radiation
- This equation is in complete agreement with experimental observations

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Planck's Wavelength Distribution Function

- Planck generated a theoretical expression for the wavelength distribution (radiance)

$$R(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$$

  - $h = 6.626 \times 10^{-34}$ J·s
  - h is a fundamental constant of nature

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Planck's Wavelength Distribution Function, cont.

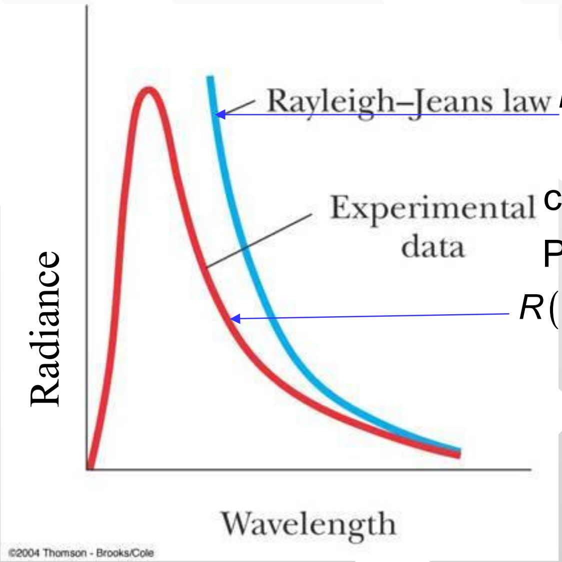
- At long wavelengths, Planck's equation reduces to the Rayleigh-Jeans expression
- This can be shown by expanding the exponential term

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda kT} + \frac{1}{2!}\left(\frac{hc}{\lambda kT}\right)^2 + \ldots \approx 1 + \frac{hc}{\lambda kT}$$

in the long wavelength limit $hc \ll \lambda kT$

- At short wavelengths, it predicts an exponential decrease in intensity with decreasing wavelength
  - This is in agreement with experimental results

Comparison between Planck's law of BB radiation and RJ's law



$$R(\lambda,T) = \frac{2\pi ckT}{\lambda^4}$$

correctely fit by Planck's derivation

$$R(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

How Planck modeled the BB

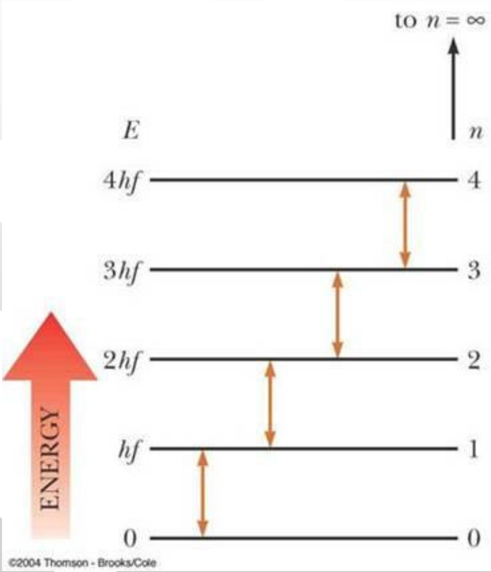
- He assumed the cavity radiation came from atomic oscillations in the cavity walls
- Planck made two assumptions about the nature of the oscillators in the cavity walls

Planck's Assumption, 1

- The energy of an oscillator can have only certain *discrete* values E_n
  - $E_n = nhf$
    - n is a positive integer called the quantum number
    - h is Planck's constant = 6.63×10^{-34} Js
    - f is the frequency of oscillation
  - This says the energy is **quantized**
  - Each discrete energy value corresponds to a different **quantum state**
  - This is in stark contrast to the case of RJ derivation according to classical theories, in which the energies of oscillators in the cavity must assume a continuous distribution

Energy-Level Diagram of the Planck Oscillator

- An **energy-level diagram** of the oscillators showing the quantized energy levels and allowed transitions
- Energy is on the vertical axis
- Horizontal lines represent the allowed energy levels of the oscillators
- The double-headed arrows indicate allowed transitions



Oscillator in Planck's theory is quantised in energies (taking only discrete values)

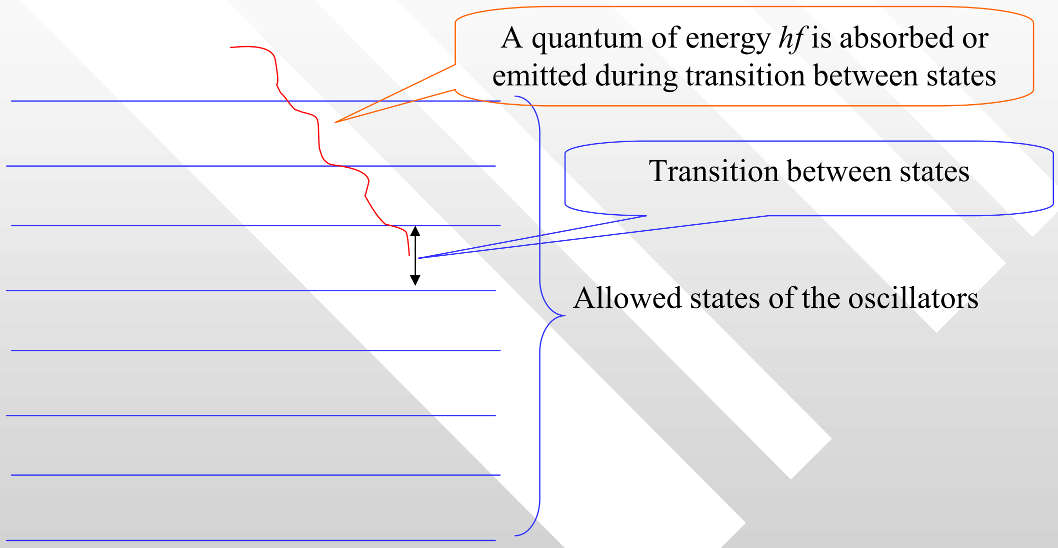
- The energy of an oscillator can have only certain *discrete* values $E_n = nhf$
- The average energy per standing wave in the Planck oscillator is

$$\langle \varepsilon \rangle = \frac{hf}{e^{hf/kT} - 1}$$ (instead of $\langle \varepsilon \rangle = kT$ in classical theories)

Planck's Assumption, 2

- The oscillators emit or absorb energy when making a transition from one quantum state to another
  - The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation
  - An oscillator emits or absorbs energy only when it changes quantum states

Pictorial representation of oscillator transition between states



Example: quantised oscillator vs classical oscillator

- A 2.0 kg block is attached to a massless spring that has a force constant k=25 N/m. The spring is stretched 0.40 m from its EB position and released.
- (A) Find the total energy of the system and the frequency of oscillation according to classical mechanics.

Solution

- In classical mechanics, $E = \frac{1}{2}kA^2 = \ldots 2.0$ J
- The frequency of oscillation is

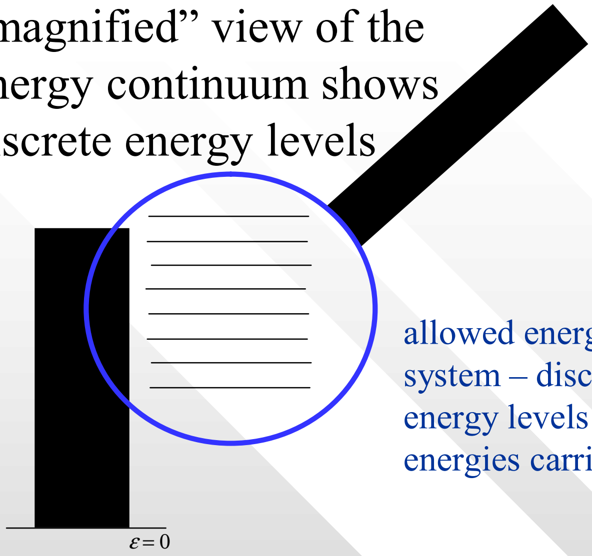
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \ldots = 0.56 \text{ Hz}$$

(B)

- (B) Assuming that the energy is quantised, find the quantum number n for the system oscillating with this amplitude
- Solution: This is a quantum analysis of the oscillator
- $E_n = nhf = n\,(6.63 \text{ x } 10^{-34} \text{ Js})(0.56 \text{ Hz}) = 2.0 \text{ J}$
- $\Rightarrow n = 5.4 \text{ x } 10^{33}$ *!!!* A very large quantum number, typical for macroscopin system

- The previous example illustrated the fact that the quantum of action, *h*, is so tiny that, from macroscopic point of view, the quantisation of the energy level is so tiny that it is almost undetectable.
- Effectively, the energy level of a macroscopic system such as the energy of a harmonic oscillator form a 'continuum' despite it is granular at the quantum scale

"magnified" view of the energy continuum shows discrete energy levels



allowed energies in quantised system – discrete (such as energy levels in an atom, energies carried by a photon)

allowed energies in classical system – continuous (such as an harmonic oscillator, energy carried by a wave; total mechanical energy of an orbiting planet, etc.)

To summarise

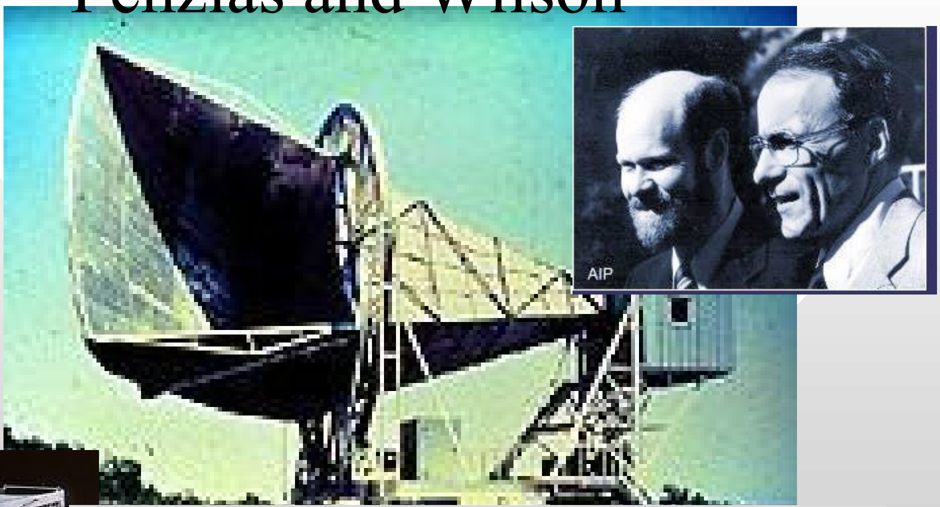
- Classical BB presents a "ultraviolet catastrophe"
- The spectral energy distribution of electromagnetic radiation in a black body CANNOT be explained in terms of classical Maxwell EM theory, in which the average energy in the cavity assumes continuous values of $<\varepsilon> = kT$ (this is the result of the wave nature of radiation)
- To solve the BB catastrophe one has to assume that the energy of individual radiation oscillator in the cavity of a BB is quantised as per $E_n = nhf$
- This picture is in conflict with classical physics because in classical physics energy is in principle a continuous variable that can take any value between 0 $\rightarrow \infty$
- One is then lead to the revolutionary concept that

ENERGY OF AN OSCILLATOR IS QUANTISED

Cosmic microwave background (CMBR) as perfect black body radiation

1965, cosmic microwave background was first detected by Penzias and Wilson

Nobel Prize 1976

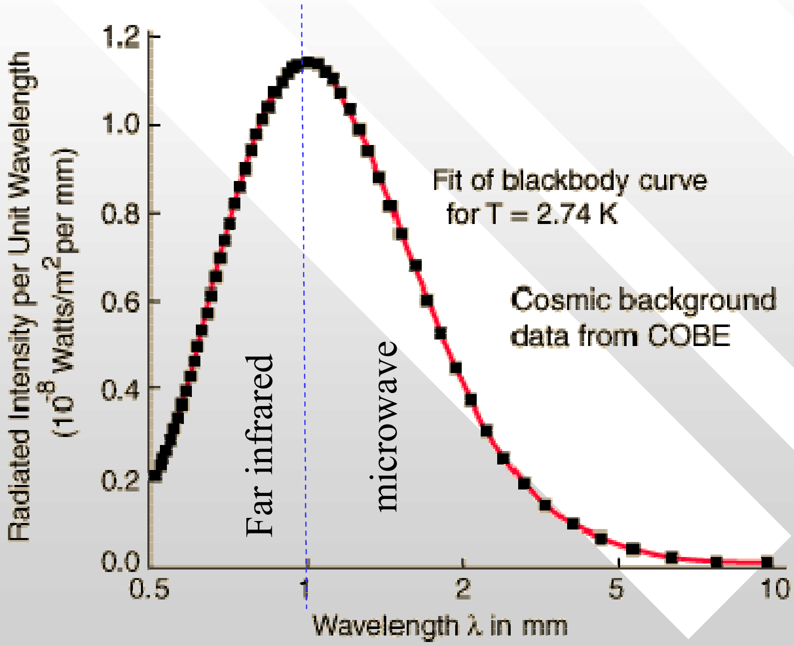


Pigeon Trap Used
Penzias and Wilson thought the static their radio antenna was picking up might be due to droppings from pigeons roosting in the antenna horn. They captured the pigeons with this trap and cleaned out the horn, but the static persisted.

CMBR – the most perfect Black Body

- Measurements of the cosmic microwave background radiation allow us to determine the temperature of the universe today.
- The brightness of the relic radiation is measured as a function of the radio frequency. To an excellent approximation it is described by a thermal of blackbody distribution with a temperature of *T*=2.735 degrees above absolute zero.
- This is a dramatic and direct confirmation of one of the predictions of the Hot Big Bang model.
- The COBE satellite measured the spectrum of the cosmic microwave background in 1990, showing remarkable agreement between theory and experiment.

The Temperature of the Universe Today, as implied from CMBR

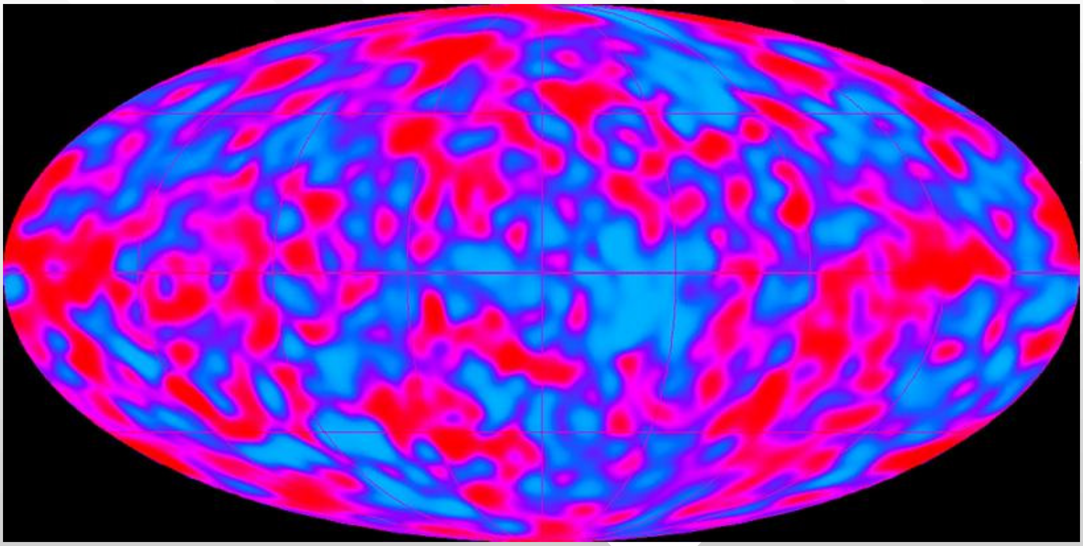


The diagram shows the results plotted in waves per centimeter versus intensity. The theoretical best fit curve (the solid line) is indistinguishable from the experimental data points (the point-size is greater than the experimental errors).

COBE

- The Cosmic Background Explorer satellite was launched twenty five years after the discovery of the microwave background radiation in 1964.
- In spectacular fashion in 1992, the COBE team announces that they had discovered `ripples at the edge of the universe', that is, the first sign of primordial fluctuations at 100,000 years after the Big Bang.
- These are the imprint of the seeds of galaxy formation.

"Faces of God"

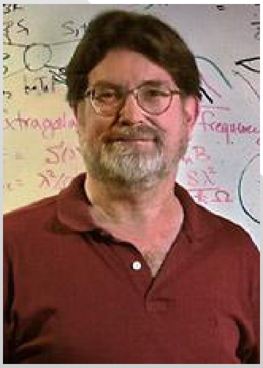


- The “faces of God”: a map of temperature variations on the full sky picture that COBE obtained.
- They are at the level of only one part in one hundred thousand.
- Viewed in reverse the Universe is highly uniform in every direction lending strong support for the cosmological principle.

The Nobel Prize in Physics 2006

"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"

John C. Mather



George F. Smoot