CHAPTER 4

The wavelike properties of particles

Schroedinger's Cat: "Am I a particle or wave?"

Wave particle duality

- "Quantum nature of light" refers to the particle attribute of light
- "Quantum nature of particle" refers to the wave attribute of a particle
- Light (classically EM waves) is said to display "wave-particle duality" – it behave like wave in one experiment but as particle in others (c.f. a person with schizophrenia)

- Not only light does have "schizophrenia", so are other microscopic "particle'' such as electron, i.e. particles also manifest wave characteristics in some experiments
- Wave-particle duality is essentially the manifestation of the quantum nature of things
- This is an very weird picture quite contradicts to our conventional assumption with is deeply rooted on classical physics or intuitive notion on things

Planck constant as a measure of quantum effect

- When investigating physical systems involving its quantum nature, the theory usually involves the appearance of the constant h
- e.g. in Compton scattering, the Compton shift is proportional to h; So is photoelectricity involves h in its formula
- In general, when h appears, it means quantum effects arise
- In contrary, in classical mechanics or classical EM theory, h never appear as both theories do not take into account of quantum effects
- Roughly quantum effects arise in microscopic system (e.g. on the scale approximately of the order 10^{-10} m or smaller)

Wavelike properties of particle

- In 1923, while still a graduate student at the University of Paris, Louis de Broglie published a brief note in the journal *Comptes rendus* containing an idea that was to revolutionize our understanding of the physical world at the most fundamental level:
- *That particle has intrinsic wave properties*
- *For more interesting details:*
- *http://www.davis-inc.com/physics/index.shtml*

Prince de Broglie, 1892-1987

de Broglie's postulate (1924)

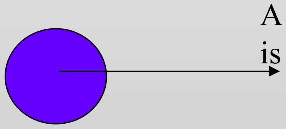
- The postulate: there should be a symmetry between matter and wave. The wave aspect of matter is related to its particle aspect in exactly the same quantitative manner that is in the case for radiation. The total (i.e. relativistic) energy E and momentum p of an entity, for both matter and wave alike, is related to the frequency f of the wave associated with its motion via Planck constant

$$p = h/\lambda,$$
$$E = hf$$

A particle has wavelength!!!

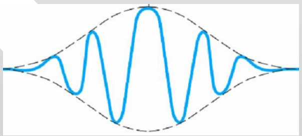
$$\lambda = h/p$$

- is the de Broglie relation predicting the wave length of the matter wave λ associated with the motion of a material particle with momentum p
- Note that classically the property of wavelength is only reserved for wave and particle was never associate with any wavelength
- But, following de Broglie's postulate, such distinction is removed



Particle with linear momentum p

A particle with momentum p is pictured as a wave (wavepulse)



Matter wave with de Broglie wavelength

$\lambda = p/h$

A physical entity possess both aspects of particle and wave in a complimentary manner

BUT why is the wave nature of material particle

not observed?

Because …

- Because…we are too large and quantum effects are too small
- Consider two extreme cases:
- (i) an electron with kinetic energy K = 54 eV, de Broglie wavelength, $\lambda = h/p = h / (2m_e K)^{1/2}$ = 1.65 Angstrom.
- Such a wavelength is comparable to the size of atomic lattice, and is experimentally detectable
- (ii) As a comparison, consider an macroscopic object, a billard ball of mass m = 100 g moving with momentum p
- $p = mv \approx 0.1$ kg $\times$ 10 m/s = 1 Ns (relativistic correction is negligible)
- It has de Broglie wavelength $\lambda = h/p \approx 10^{-34}$ m, too tiny to be observed in any experiments
- The total energy of the billard ball is
    - $E = K + m_0c^2 \approx m_0c^2 = 0.1\times(3\times10^8)^2$ J $= 9\times10^{15}$J
- (K is ignored since $K << m_0c^2$)
- The frequency of the de Broglie wave associated with the billard ball is $f = E/h = m_0c^2/h = (9\times10^{15}/6.63\times10^{34})$ Hz $=10^{78}$ Hz, impossibly high for any experiment to detect

Matter wave is a quantum phenomena

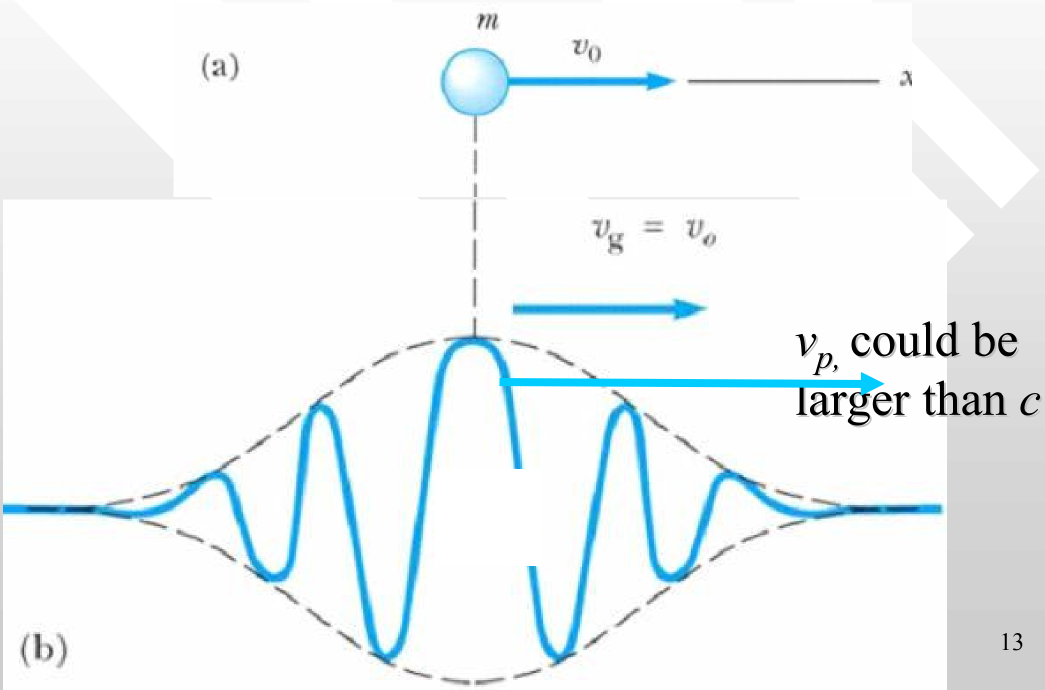
- This also means that the wave properties of matter is difficult to observe for macroscopic system (unless with the aid of some specially designed apparatus)
- The smallness of h in the relation $\lambda = h/p$ makes wave characteristic of particles hard to be observed
- The statement that when $h \rightarrow 0$, λ becomes vanishingly small means that:
- the wave nature will become effectively "shut-off" and appear to loss its wave nature whenever the relevant p of the particle is too large in comparison with the quantum scale characterised by h

How small is small?

- More quantitatively, we could not detect the quantum effect if $h/p \sim 10^{-34}$ Js/p (dimension: length, L) becomes too tiny in comparison to the length scale discernable by an experimental setup (e.g. slit spacing in a diffraction experiment)
- For a numerical example: For a slit spacing of $l \sim$ nm (inter-atomic layer in a crystal), and a momentum of p=10 Ns (100 g billard ball moving with 10 m/s),

  $h/p = 10^{-34}$ Js/p = 10^{-34} Js/10 Ns ~ 10^{-35} m << $l \sim$ nm
- LHS, i.e. h/p (~10^{-35} m), is the length scale of the de Broglie (quantum) wavelength;
- RHS, i.e. l (~ nm), is the length scale charactering the experiment
- Such an experimental set up could not detect the wave length of the moving billard ball.

The particle's velocity v_0 is identified with the de Broglie' group wave, v_g but not its phase wave v_p



Example

- An electron has a de Broglie wavelength of 2.00 pm. Find its kinetic energy and the group velocity of its de Broglie waves.
- *Hint:*
- The group velocity of the dB wave of electron v_g is equal to the velocity of the electron, v.
- Must treat the problem relativistically.
- If the electron's de Broglie wavelength λ is known, so is the momentum, p. Once p is known, so is the total energy, E and velocity v. Once E is known, so will the kinetic energy, K.

Solution

- Total energy $E^2 = c^2p^2 + m_0^2$
- $K = E - m_0c^2$

  $= (c^2p^2 + m_0^2c^4)^{½} - m_0c^2$

  $= ((hc/\lambda)^2 + m_0^2c^4)^{½} - m_0c^2 = 297$ keV
- $v_g = v$; $1/\gamma^2 = 1 - (v/c)^2$;
- $(pc)^2 = (\gamma m_0vc)^2 = (hc/\lambda)^2$ (from Relativity and de Broglie's postulate)

$\Rightarrow (\gamma v/c)^2 = (hc/\lambda)^2/(m_0c^2)^2 = (620\text{ keV}/510\text{ keV})^2 = 1.4884$;

$(\gamma v/c)^2 = (v/c)^2 / 1 - (v/c)^2$

$\Rightarrow v_g/c = \sqrt{}(1.4884/(1+1.4884)) = 0.77$

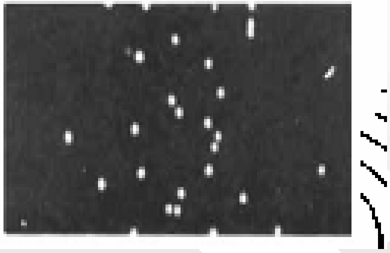
Alternatively

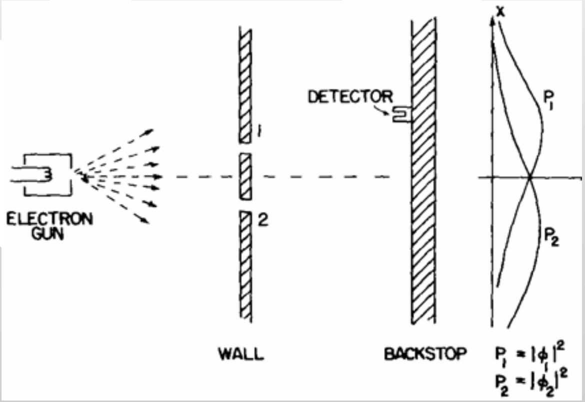
- The previous calculation can also proceed via:
- $K=(\gamma-1)m_ec^2$
- $\Rightarrow \gamma = K/(m_ec^2) + 1 = 297\text{ keV}/(510\text{ keV}) + 1$

  $=1.582$;
- $p = h/\lambda = \gamma m_e v \Rightarrow v = hc/(\lambda\gamma m_e c)$
- $\Rightarrow v/c = hc/(\lambda\, \gamma m_e c^2)$
- $= (1240\text{ nm}\cdot\text{eV}) /(2\text{pm}\cdot 1.582\cdot 0.51\text{MeV})$
- $= 0.77$

Interference experiment with a single electron, firing one in a time

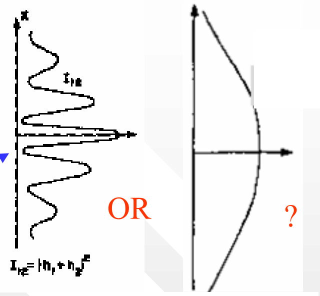
- Consider an double slit experiment using an extremely small electron source that emits only one electron a time through the double slit and then detected on a fluorescent plate
- When hole 1 (hole 2) is blocked, distribution P1 (P2) is observed.
- P1 are P2 are the distribution pattern as expected from the behaviour of particles.
- Hence, electron behaves like particle when one of the holes is blocked
- What about if both holes are not blocked? Shall we see the distribution simply be P1 + P2? (This would be our expectation for particle: Their distribution simply adds)

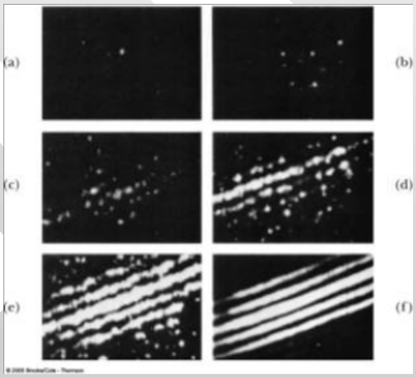




Electrons display interference pattern

- When one follows the time evolution of the pattern created by these individual electron with both hole opened, what sort of pattern do you think you will observed?
- It's the interference pattern that are in fact observed in experiments
- At the source the electron is being emitted as particle and is experimentally detected as a electron which is absorbed by an individual atom in the fluorescent plate
- In between, we must interpret the electron in the form of a wave. The double slits change the propagation of the electron wave so that it is 'processed' to forms diffraction pattern on the screen.
- Such process would be impossible if electrons are particle (because no one particle can go through both slits at the same time. Such a simultaneous penetration is only possible for wave.)
- Be reminded that the wave nature in the intermediate states is not measured. Only the particle nature are detected in this procedure.

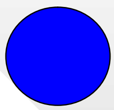




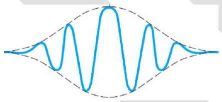
- The correct explanation of the origin and appearance of the interference pattern comes from the wave picture
- Hence to completely explain the experiment, the two pictures must somehow be taken together – this is an example for which *both pictures are complimentary to each other*
- Try to compare the last few slides with the slides from previous chapter for photon, which also displays wave-particle duality

So, is electron wave or particle?

- They are both…but not simultaneously
- In any experiment (or empirical observation) only one aspect of either wave or particle, but not both can be observed simultaneously.
- It's like a coin with two faces. But one can only see one side of the coin but not the other at any instance
- This is the so-called wave-particle duality



Electron as particle



Electron as wave

Detection of electron as particle destroy the interference pattern

- If in the electron interference experiment one tries to place a detector on each hole to determine through which an electron passes, the wave nature of electron in the intermediate states are destroyed
- i.e. the interference pattern on the screen shall be destroyed
- Why? It is the consistency of the wave-particle duality that demands such destruction must happen (think of the logics yourself or read up from the text)

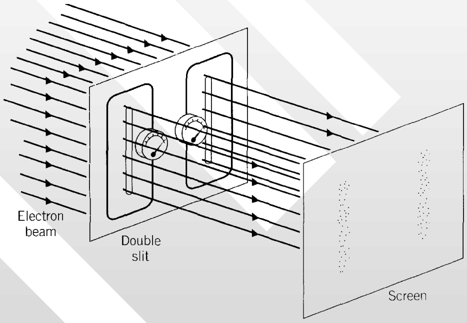


FIGURE 4.15 Apparatus to record passage of electrons through slits. Each slit is surrounded by a loop with a meter that signals the passage of an electron through the slit. No interference finges are seen on the screen.

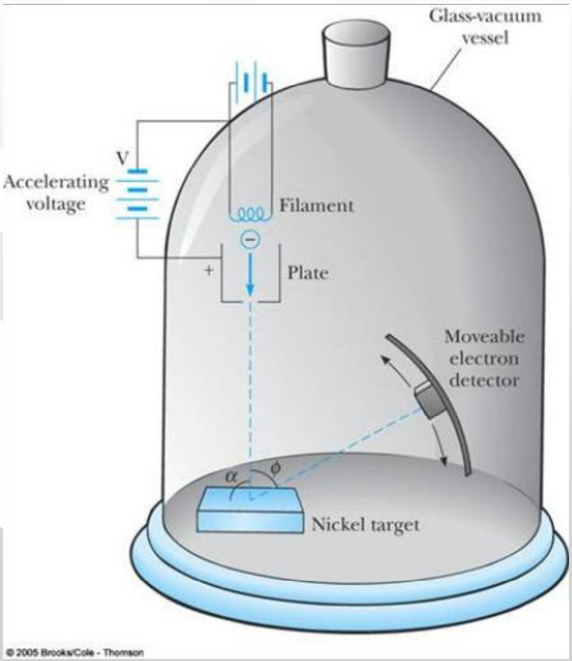
"Once and for all I want to know what I'm paying for. When the electric company tells me whether electron is a wave or a particle I'll write my check"

Extra readings

- Those quantum enthusiasts may like to read more about wave-particle duality in Section 5.7, page 179-185, Serway, Moses and Mayer.
- An even more recommended reading on wave-particle duality: the Feynman lectures on physics, vol. III, chapter 1 (Addison-Wesley Publishing)
- It's a very interesting and highly intellectual topic to investigate

Davisson and Gremer experiment

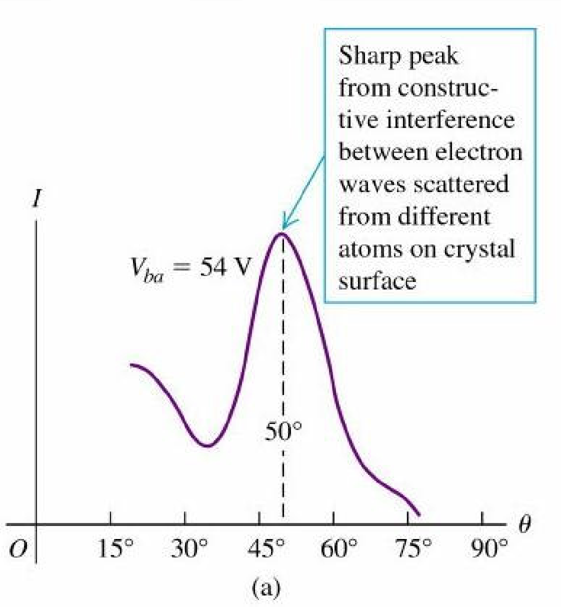
- DG confirms the wave nature of electron in which it undergoes Bragg's diffraction
- Thermionic electrons are produced by hot filament, accelerated and focused onto the target (all apparatus is in vacuum condition)
- Electrons are scattered at an angle ϕ into a movable detector



Pix of Davisson and Gremer

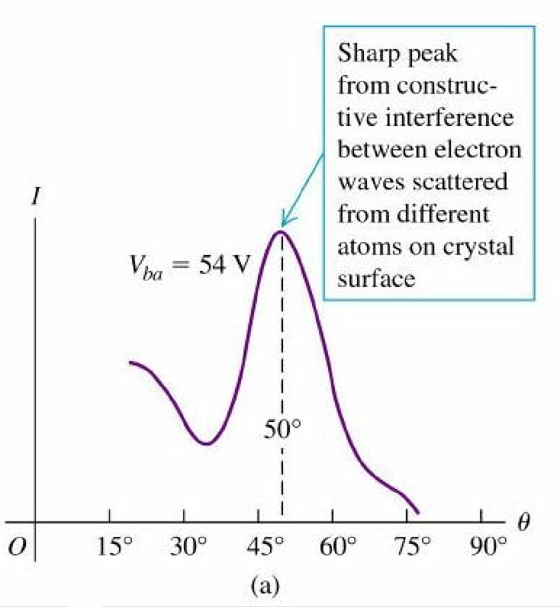
Result of the DG experiment

- Distribution of electrons is measured as a function of ϕ
- Strong scattered e-beam is detected at ϕ = 50 degree for V = 54 V



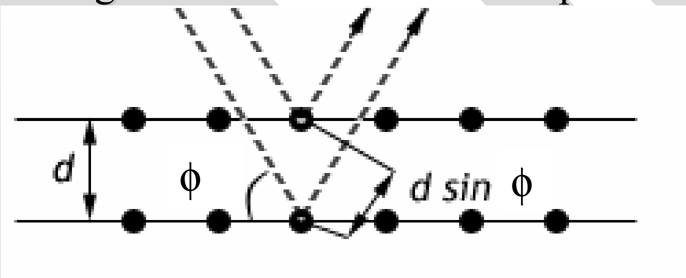
How to interpret the result of DG?

- Electrons get diffracted by the atoms on the surface (which acted as diffraction grating) of the metal as though the electron acting like they are WAVES
- Electrons do behave like waves as postulated by de Broglie



Bragg diffraction of electron by parallel lattice planes in the crystal

- Bragg law: $d \sin \phi = n\lambda$
- The peak of the diffraction pattern is the m=1st order constructive interference: $d \sin \phi = 1\lambda$
- where ϕ = 50 degree for V = 54 V
- From x-ray Bragg's diffraction experiment done independently we know d = 2.15 Amstrong
- Hence the wavelength of the electron is $\lambda = d\sin \phi$ = 1.65Angstrom
- Here, 1.65 Angstrom is the experimentally inferred value, which is to be checked against the theoretical value predicted by de Broglie



Theoretical value of λ of the electron

- An external potential V accelerates the electron via $eV=K$
- In the DG experiment the kinetic energy of the electron is accelerated to K = 54 eV (non-relativistic treatment is suffice because $K << m_e c^2$ = 0.51 MeV)
- According to de Broglie, the wavelength of an electron accelerated to kinetic energy of $K = p^2/2m_e$ = 54 eV has a equivalent matter wave wavelength

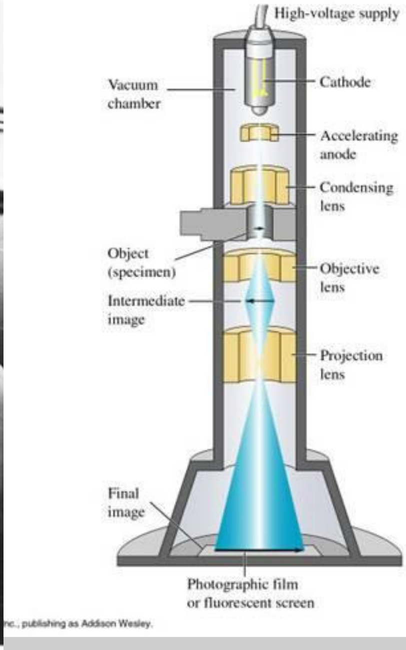
  $\lambda = h/p = h/(2Km_e)^{1/2}$ = 1.67 Amstrong
- In terms of the external potential,

  $\lambda = h/(2eVm_e)^{1/2}$

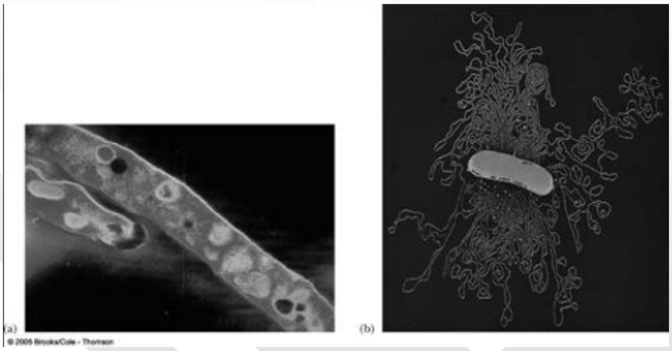
Theory's prediction matches measured value

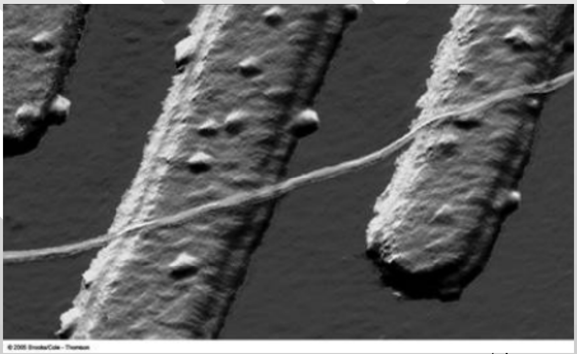
- The result of DG measurement agrees almost perfectly with the de Broglie's prediction: 1.65 Angstrom measured by DG experiment against 1.67 Angstrom according to theoretical prediction
- Wave nature of electron is hence experimentally confirmed
- In fact, wave nature of microscopic particles are observed not only in e- but also in other particles (e.g. neutron, proton, molecules etc. – most strikingly Bose-Einstein condensate)

Application of electrons wave: electron microscope, Nobel Prize 1986 (Ernst Ruska)



- Electron's de Broglie wavelength can be tuned via $\lambda = h/(2eVm_e)^{1/2}$
- Hence electron microscope can magnify specimen (x4000 times) for biological specimen or 120,000 times of wire of about 10 atoms in width

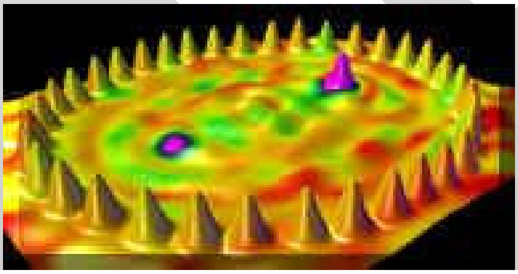




Not only electron, other microscopic particles also behave like wave at the quantum scale

- The following atomic structural images provide insight into the threshold between prime radiant flow and the interference structures called matter.
- In the right foci of the ellipse a real cobalt atom has been inserted. In the left foci of the ellipse a phantom of the real atom has appeared. The appearance of the phantom atom was not expected.
- The ellipsoid coral was constructed by placing 36 cobalt atom on a copper surface. This image is provided here to provide a visual demonstration of the attributes of material matter arising from the harmonious interference of background radiation.

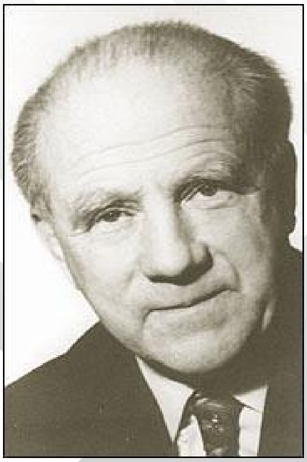
QUANTUM CORAL



http://home.netcom.com/~sbyers11/grav11E.htm

Heisenberg's uncertainty principle (Nobel Prize,1932)

- WERNER HEISENBERG (1901 - 1976)
- was one of the greatest physicists of the twentieth century. He is best known as a founder of quantum mechanics, the new physics of the atomic world, and especially for the uncertainty principle in quantum theory. He is also known for his controversial role as a leader of Germany's nuclear fission research during World War II. After the war he was active in elementary particle physics and West German science policy.
- *http://www.aip.org/history/heisenberg/p01.htm*



A particle is represented by a wave packet/pulse

- Since we experimentally confirmed that particles are wave in nature at the quantum scale *h* (matter wave) we now have to describe particles in term of waves (relevant only at the quantum scale)
- Since a real particle is localised in space (not extending over an infinite extent in space), the wave representation of a particle has to be in the form of wave packet/wave pulse

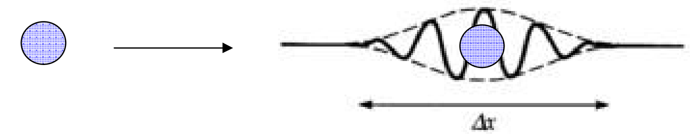
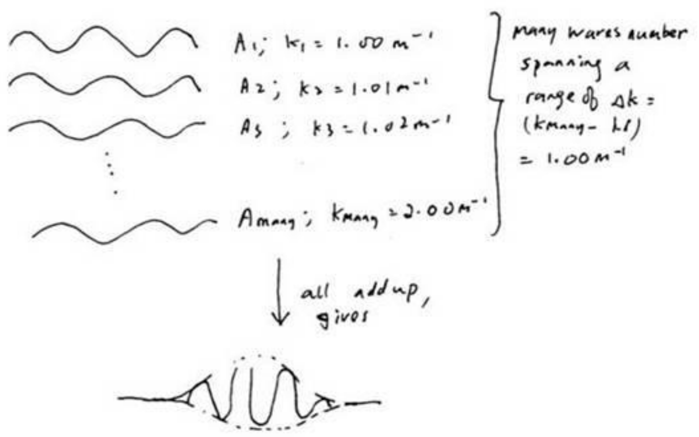


FIGURE 6.14 An idealized wave packet localized in space over a region Δx is the perposition of many waves of different amplitudes and frequencies.

- As mentioned before, wavepulse/wave packet is formed by adding many waves of different amplitudes and with the wave numbers spanning a range of Δk (or equivalently, $\Delta\lambda$)



Δx

Recall that $k = 2\pi/\lambda$, hence

$\Delta k/k = \Delta\lambda/\lambda$

Still remember the uncertainty relationships for classical waves?

- As discussed earlier, due to its nature, a wave packet must obey the uncertainty relationships for classical waves (which are derived mathematically with some approximations)

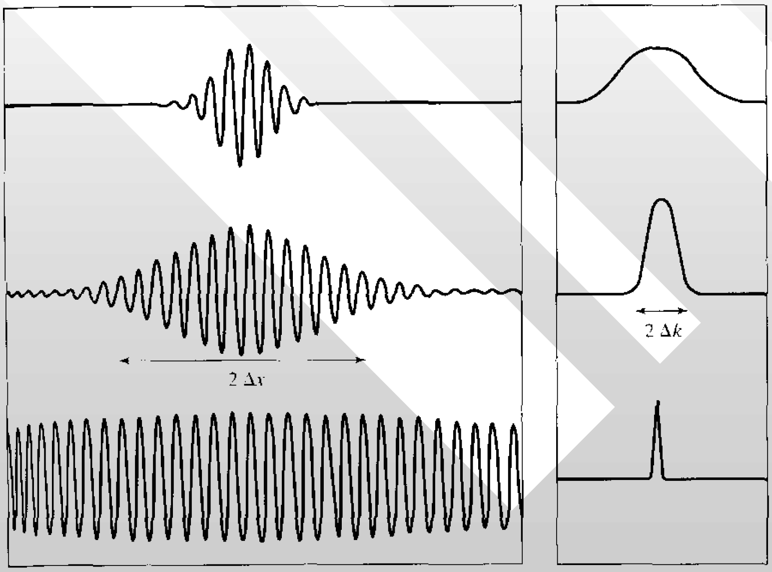
$$\Delta\lambda\Delta x \gtrsim \lambda^2 \equiv \Delta k\Delta x \gtrsim 2\pi \qquad \Delta t\Delta\nu \geq 1$$

- However a more rigorous mathematical treatment (without the approximation) gives the exact relations

$$\Delta\lambda\Delta x \geq \frac{\lambda^2}{4\pi} \equiv \Delta k\Delta x \geq 1/2 \qquad \Delta\nu\Delta t \geq \frac{1}{4\pi}$$

- To describe a particle with wave packet that is localised over a small region Δx requires a large range of wave number; that is, Δk is large. Conversely, a small range of wave number cannot produce a wave packet localised within a small distance.

- A narrow wave packet (small Δx) corresponds to a large spread of wavelengths (large Δk).
- A wide wave packet (large Δx) corresponds to a small spread of wavelengths (small Δk).



Matter wave representing a particle must also obey similar wave uncertainty relation

- For matter waves, for which their momentum and wavelength are related by $p = h/\lambda$, the uncertainty relationship of the classical wave
- $\Delta\lambda\Delta x \geq \frac{\lambda^2}{4\pi} \equiv \Delta k \Delta x \geq 1/2$ is translated into

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

- where $\hbar = h/2\pi$
- Prove this relation yourselves (hint: from $p = h/\lambda$, $\Delta p/p = \Delta\lambda/\lambda$)

Time-energy uncertainty

- Just as $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ implies position-momentum uncertainty relation, the classical wave uncertainty relation $\Delta\nu\Delta t \geq \frac{1}{4\pi}$ also implies a corresponding relation between time and energy

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- This uncertainty relation can be easily obtained:

$$h\Delta\nu\Delta t \geq \frac{h}{4\pi} = \frac{\hbar}{2};$$

$$\because E = h\nu, \Delta E = h\Delta\nu \Rightarrow \Delta E\Delta t = h\Delta\nu\Delta t = \frac{\hbar}{2}$$

Heisenberg uncertainty relations

$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

- The product of the uncertainty in momentum (energy) and in position (time) is at least as large as Planck's constant

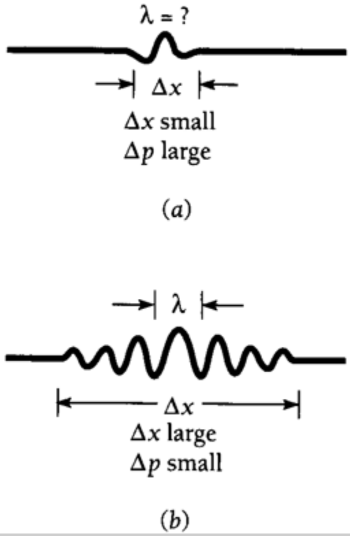
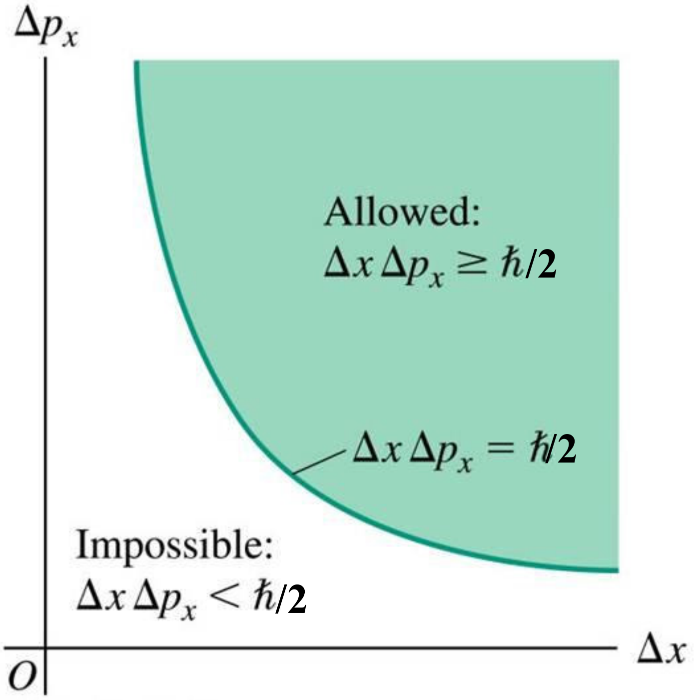


Figure 3.12 (*a*) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (*b*) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

What $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ means

- It sets the intrinsic lowest possible limits on the uncertainties in knowing the values of p_x and x, no matter how good an experiments is made
- It is impossible to specify simultaneously and with infinite precision the linear momentum and the corresponding position of a particle

It is impossible for the product $\Delta x \Delta p_x$ to be less than $h/4\pi$

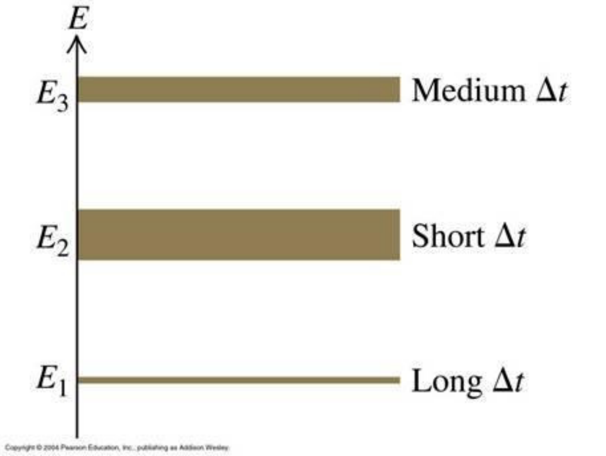


What $\Delta E \Delta t \geq \frac{\hbar}{2}$ means

- Uncertainty principle for energy.
- The energy of a system also has inherent uncertainty, ΔE
- ΔE *is* dependent on the *time interval* Δt during which the system remains in the given states.
- If a system is known to exist in a state of energy E over a limited period Δt, then this energy is uncertain by at least an amount $h/(4\pi\Delta t)$. This corresponds to the 'spread' in energy of that state (see next page)
- Therefore, the energy of an object or system can be measured with infinite precision (ΔE=0) only if the object of system exists for an infinite time ($\Delta t \rightarrow \infty$)

What $\Delta E \Delta t \geq \frac{\hbar}{2}$ means

- A system that remains in a metastable state for a very long time (large Δt) can have a very well-defined energy (small ΔE), but if remain in a state for only a short time (small Δt), the uncertainty in energy must be correspondingly greater (large ΔE).



Conjugate variables (Conjugate observables)

- $\{p_x, x\}$, $\{E, t\}$ are called *conjugate variables*
- The conjugate variables cannot in principle be measured (or known) to infinite precision simultaneously

Heisenberg's Gedanken experiment

- The U.P. can also be understood from the following gedanken experiment that tries to measure the position and momentum of an object, say, an electron at a certain moment
- In order to measure the momentum and position of an electron it is necessary to "interfere" it with some "probe" that will then carries the required information back to us – such as shining it with a photon of say a wavelength of λ

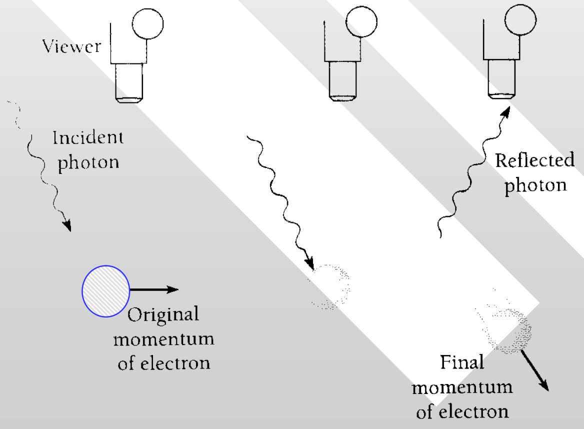
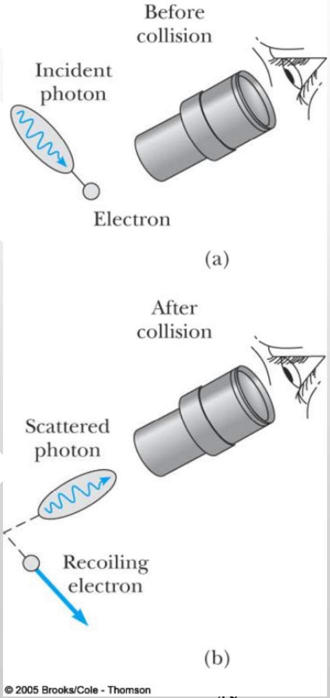


Figure 3.17 An electron cannot be observed without changing its momentum.

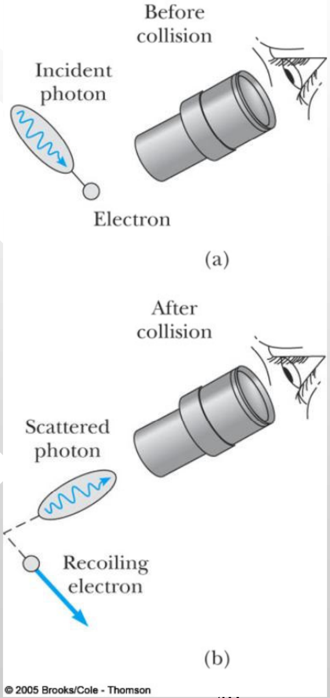
Heisenberg's Gedanken experiment

- Let's say the "unperturbed" electron was initially located at a "definite" location x and with a "definite" momentum p
- When the photon 'probes' the electron it will be bounced off, associated with a changed in its momentum by some uncertain amount, Δp.
- Δp cannot be predicted but must be of the similar order of magnitude as the photon's momentum h/λ
- Hence $\Delta p \approx h/\lambda$
- The longer λ (i.e. less energetic) the smaller the uncertainty in the measurement of the electron's momentum
- In other words, electron cannot be observed without changing its momentum



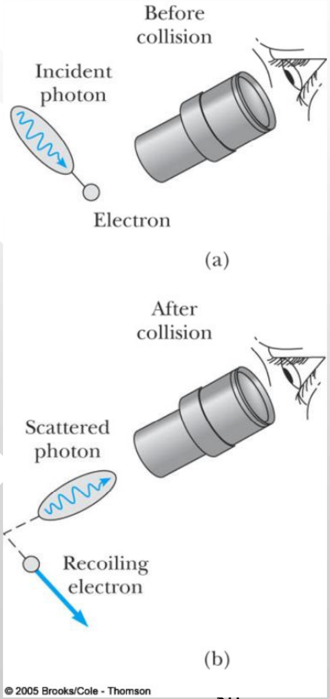
Heisenberg's Gedanken experiment

- How much is the uncertainty in the position of the electron?
- By using a photon of wavelength λ we cannot determine the location of the electron better than an accuracy of $\Delta x = \lambda$
- Hence $\Delta x \geq \lambda$
- Such is a fundamental constraint coming from optics (Rayleigh's criteria).
- The shorter the wavelength λ (i.e. more energetic) the smaller the uncertainty in the electron's position



Heisenberg's Gedanken experiment

- However, if shorter wavelength is employed (so that the accuracy in position is increased), there will be a corresponding decrease in the accuracy of the momentum, measurement (recall $\Delta p \approx h/\lambda$)
- A higher photon momentum will disturb the electron's motion to a greater extent
- Hence there is a 'zero sum game' here
- Combining the expression for Δx and Δp, we then have $\Delta p\ \Delta\lambda \geq h$, a result consistent with $\Delta p\ \Delta\lambda \geq h/2$



Heisenberg's kiosk

Example

- A typical atomic nucleus is about 5.0×10^{-15} m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus

Solution

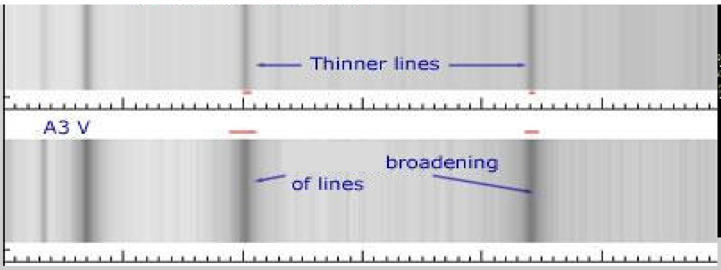
- Letting $\Delta x = 5.0\times10^{-15}$ m, we have
- $\Delta p \geq h/(4\pi\Delta x) = \ldots = 1.1\times10^{-20}$ kg·m/s

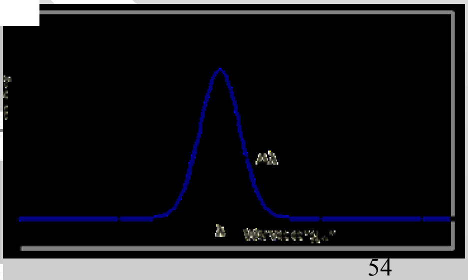
  If this is the uncertainty in a nuclear electron's momentum, the momentum p must be at lest comparable in magnitude. An electron of such a momentum has a
- KE = $pc \geq 3.3\times10^{-12}$ J

  = 20.6 MeV >> $m_e c^2$= 0.5 MeV
- i.e., if electrons were contained within the nucleus, they must have an energy of at least 20.6 MeV
- However such an high energy electron from radioactive nuclei never observed
- Hence, by virtue of the uncertainty principle, we conclude that electrons emitted from an unstable nucleus cannot comes from within the nucleus

Broadening of spectral lines due to uncertainty principle

- An excited atom gives up it excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time is radiates is 1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon.





Solution

- The photon energy is uncertain by the amount
- $\Delta E \geq hc/(4c\pi\Delta t) = 5.3\times10^{-27}$ J = 3.3×10^{-8} eV
- The corresponding uncertainty in the frequency of light is $\Delta\nu = \Delta E/h \geq 8\times10^{6}$ Hz
- This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom.
- As a result, the radiation from a group of excited atoms does not appear with the precise frequency ν.
- For a photon whose frequency is, say, 5.0×10^{14} Hz,
- $\Delta\nu/\nu = 1.6\times10^{-8}$

PYQ 2.11 Final Exam 2003/04

- Assume that the uncertainty in the position of a particle is equal to its de Broglie wavelength. What is the minimal uncertainty in its velocity, v_x?
- A. $v_x/4\pi$ B. $v_x/2\pi$ C. $v_x/8\pi$
- D. v_x E. v_x/π
- ANS: A, Schaum's 3000 solved problems, Q38.66, pg. 718

Solution

$$\Delta x \Delta p_x \geq \hbar / 2; \Delta p_x = m\Delta v_x.$$

Given $\Delta x = \lambda$,

$$\Rightarrow m\Delta x \Delta v_x = m\lambda \Delta v_x \geq \hbar / 2;$$

$$\Rightarrow \Delta v_x \geq \hbar / 2m\lambda = h / 4\pi m\lambda$$

But $p_x = h / \lambda$

$$\Rightarrow \Delta v_x \geq h / 4\pi m\lambda = p_x / 4\pi m$$

$$= mv_x / 4\pi m = v_x / 4\pi$$

Example

- A measurement established the position of a proton with an accuracy of $\pm 1.00 \times 10^{-11}$ m. Find the uncertainty in the proton's position 1.00 s later. Assume $v << c$.

Solution

- Let us call the uncertainty in the proton's position Δx_0 at the time $t = 0$.
- The uncertainty in its momentum at $t = 0$ is

$$\Delta p \geq h/(4\pi\, \Delta x_0)$$

- Since $v << c$, the momentum uncertainty is

$$\Delta p = m\Delta v$$

- The uncertainty in the proton's velocity is

$$\Delta v = \Delta p/m \geq h/(4\pi m\, \Delta x_0)$$

- The distance x of the proton covers in the time t cannot be known more accurately than

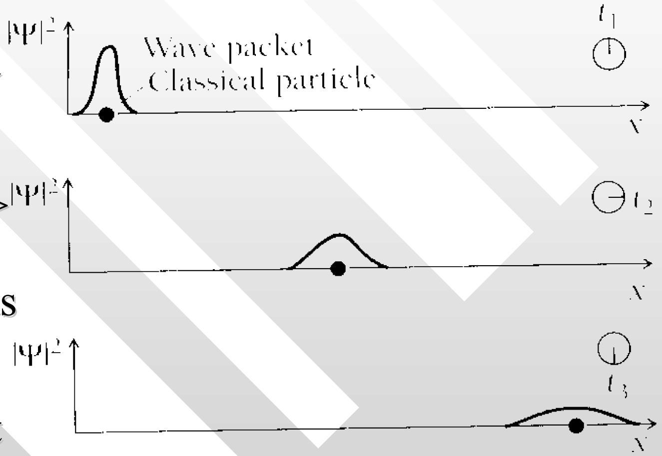
$$\Delta x = t\Delta v \geq ht/(4\pi\, m\Delta x_0)$$

- m=970 MeV/c^2
- The value of Δx at t = 1.00 s is 3.15 km.

A moving wave packet spreads out in space

$$\Delta x = t\Delta v \geq ht/(4\pi\, \Delta x_0)$$

- Note that Δx is inversely proportional to Δx_0
- It means the more we know about the proton's position at t = 0 the less we know about its later position at t > 0.
- The original wave group has spread out to a much wider one because the phase velocities of the component wave vary with wave number and a large range of wave numbers must have been present to produce the narrow original wave group



Example

Estimating quantum effect of a macroscopic particle

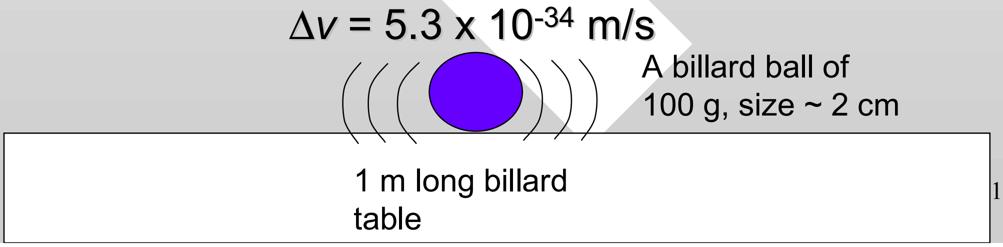
- Estimate the minimum uncertainty velocity of a billard ball (m ~ 100 g) confined to a billard table of dimension 1 m

Solution

For Δx ~ 1 m, we have

$\Delta p \geq h/4\pi\Delta x = 5.3\text{x}10^{-35}$ Ns,

- So $\Delta v = (\Delta p)/m \geq 5.3\text{x}10^{-34}$ m/s
- One can consider $\Delta v = 5.3\text{x}10^{-34}$ m/s (extremely tiny) is the speed of the billard ball at anytime caused by quantum effects
- In quantum theory, no particle is absolutely at rest due to the Uncertainty Principle



A particle contained within a finite region must has some minimal KE

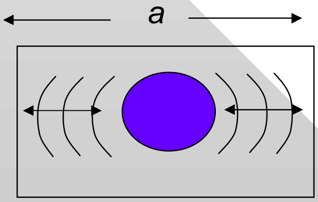
- One of the most dramatic consequence of the uncertainty principle is that a particle confined in a small region of finite width cannot be exactly at rest (as already seen in the previous example)

- Why? Because…

- ...if it were, its momentum would be precisely zero, (meaning $\Delta p = 0$) which would in turn violate the uncertainty principle

What is the K_{ave} of a particle in a box due to Uncertainty Principle?

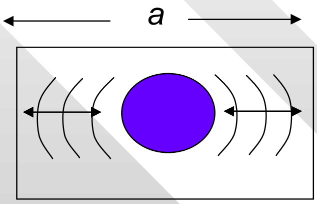
- We can estimate the minimal KE of a particle confined in a box of size *a* by making use of the U.P.
- If a particle is confined to a box, its location is uncertain by $\Delta x = a$
- Uncertainty principle requires that $\Delta p \geq (h/2\pi)a$
- (don't worry about the factor 2 in the uncertainty relation since we only perform an estimation)



Zero-point energy

$$K_{ave} = \left(\frac{p^2}{2m}\right)_{av} \gtrsim \frac{(\Delta p)^2}{2m} \gtrsim \frac{\hbar^2}{2ma^2}$$

This is the zero-point energy, the minimal possible kinetic energy for a quantum particle confined in a region of width *a*



Particle in a box of size *a* can never be at rest (e.g. has zero K.E) but has a minimal KE K_{ave} (its zero-point energy)

We will formally re-derived this result again when solving for the Schrodinger equation of this system (see later).

Recap

- Measurement necessarily involves interactions between observer and the observed system
- Matter and radiation are the entities available to us for such measurements
- The relations $p = h/\lambda$ and $E = h\nu$ are applicable to both matter and to radiation because of the intrinsic nature of wave-particle duality
- When combining these relations with the universal waves properties, we obtain the Heisenberg uncertainty relations
- In other words, the uncertainty principle is a necessary consequence of particle-wave duality