PAST YEAR TUTORIAL PROBLEM SET
(2003/04 – 2004/05)

Tutorial 1

Special Relativity

Conceptual Questions

1) What is the significance of the negative result of Michelson-Morley experiment?

ANS
The negative result of the MM experiment contradicts with the prediction of the absolute frame (the Ether frame) of reference, in which light is thought to propagate with a speed *c*. In the Ether postulate, the speed of light that is observed in other initial reference frame (such as the Earth that is moving at some constant speed relative to the Absolute frame), according to the Galilean transformation, would be different than that of the Ether frame. In other words, the MM negative result provides the first empirical evidence to the constancy of light postulate by Einstein.

2) Is it possible to have particles that travel at the speed of light?

ANS
Particle travelling at the speed of light would have an infinite mass, as per $m = \dfrac{m_0}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}$. Hence it is physically not possible to supply infinite amount of energy to boost a particle from rest to the speed of light.

postulate by Einstein.

3) A particle is moving at a speed less that $c/2$. If the speed of the particle is doubled, what happens to its momentum?

ANS

According to $\mathbf{p} = \gamma m \mathbf{u}$, doubling the speed u will make the momentum of an object increase by the factor $2\left[\dfrac{c^2-u^2}{c^2-4u^2}\right]^{1/2}$. Here's the working:

$$p = \gamma m_0 u \rightarrow p' = \gamma' m_0 u'$$

$$\left(\frac{p'}{p}\right)^2 = \left(\frac{u'}{u}\right)^2\left(\frac{\gamma'}{\gamma}\right)^2 = \left(\frac{u'}{u}\right)^2\left[\frac{1-\left(\frac{u}{c}\right)^2}{1-\left(\frac{u'}{c}\right)^2}\right] = \left(\frac{u'}{u}\right)^2\frac{c^2-u^2}{c^2-u'^2} \Rightarrow \frac{p'}{p} = \left(\frac{u'}{u}\right)\sqrt{\frac{c^2-u^2}{c^2-u'^2}}$$

$$\text{Let } u' = 2u \Rightarrow \frac{p'}{p} = \left(\frac{2u}{u}\right)\sqrt{\frac{c^2-u^2}{c^2-(2u)^2}} = 2\sqrt{\frac{c^2-u^2}{c^2-4u^2}}$$

4. The rest energy and total energy respectively, of three particles, expressed in terms of a basic amount *A* are (1) *A*, 2*A*; (2) *A*, 3*A*; (3) 3*A*, 4*A*. Without written calculation, rank the particles according to their (a) rest mass, (b) Lorentz factor, and (c) speed, greatest first.

ANS
Case 1: $\{m_0c^2, E\} = \{A, 2A\}$; Case 2: $\{m_0c^2, E\} = \{A, 3A\}$; Case 3: $\{m_0c^2, E\} = \{3A, 4A\}$

(a) Rest mass = m_0. Hence for case 1: m_0 m_0c^2=A; Case 2:m_0c^2=A; Case 3: m_0c^2 =3A. Therefore, the answer is: mass in (3) > mass in (2) = mass in (1);

(b) Lorentz factor $\gamma = E/m_0c^2$. Hence for case 1: γ= 2A/A=2; case 2: γ= 3A/A=3; case 3: γ= 4A/3A=4/3 = 1.33. Therefore, the answer is: γ in (2) > γ in (1) > γ in (3)

(c) $\gamma^{-2} = 1 - v^2/c^2 \Rightarrow v^2/c^2 = 1-\gamma^{-2}$. Hence for case 1: v^2/c^2 = 1 – 1/4 = 0.75; case 2: v^2/c^2 = 1 – 1/9 = 0.89; case 3: v^2/c^2 = 1 – 9/16 = 0.4375. Therefore, the answer is: v^2/c^2 in (2) > v^2/c^2 in (1) > v^2/c^2 in (3)

PROBLEMS

1. Space Travel (from Cutnell and Johnson, pg 861,863)
Alpha Centauri, a nearby star in our galaxy, is 4.3 light-years away. If a rocket leaves for Alpha Centauri and travels at a speed of v = 0.95c relative to the Earth, (i) by how much will the passengers have aged, according to their own clock, when they reach their destination? ii) What is the distance between Earth and Alpha Centauri as measured by the passengers in the rocket? Assume that the Earth and Alpha Centauri are stationary with respect to one another.

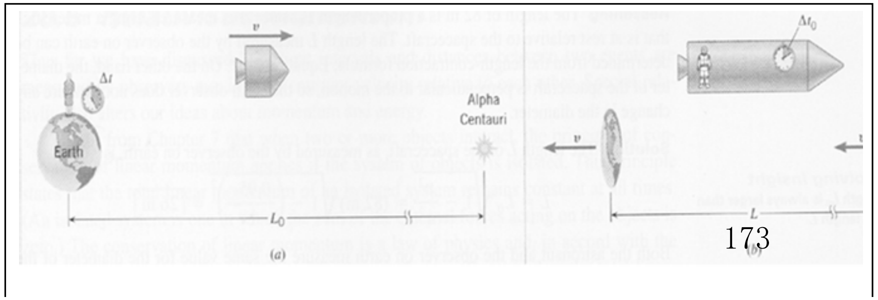

Figure: (a) As measured by an observer on the earth, the distance to Alpha Centauri is L_0, and the time required to make the trip is Δt. (b) According to the passenger on the spacecraft, the earth and Alpha Centauri move with speed v relative to the craft. The passenger measures the distance and time of the trip to be L and Δt_0 respectively, both quantities being less than those in part (a).

Reasoning

The two events in this problem are the departure from Earth and the arrival at Alpha Centauri. At departure, Earth is just outside the spaceship. Upon arrival at the destination, Alpha Centauri is just outside. Therefore, relative to the passengers, the two events occur at the same place - namely, 'just outside the spaceship. Thus, the passengers measure the proper time interval Δt_0 on their clock, and it is this interval that we must find. For a person left behind on Earth, the events occur at different places, so such a person measures the dilated time interval Δt rather than the proper time interval. To find Δt we note that the time to travel a given distance is inversely proportional to the speed. Since it takes 4.3 years to traverse the distance between earth and Alpha Centauri at the speed of light, it would take even longer at the slower speed of $v = 0.95c$. Thus, a person on earth measures the dilated time interval to be Δt = (4.3 years)/0.95 = 4.5 years. This value can be used with the time-dilation equation to find the proper time interval Δt_0.

Solution

Using the time-dilation equation, we find that the proper time interval by which the Passengers judge their own aging is $\Delta t_0 = \Delta t\sqrt{(1-v^2/c^2)}$ = 4.5 years $\sqrt{(1-0.95^2)}$ = 1.4 years.

Thus, the people aboard the rocket will have aged by only 1.4 years when they reach Alpha Centauri, and not the 4.5 years an earthbound observer has calculated.

Both the earth-based observer and the rocket passenger agree that the relative speed between the rocket and earth is $v = 0.95c$. Thus, the Earth observer determines the distance to Alpha Centauri to be $L_0 = v\Delta t$ = (0.95c)(4.5 years) = 4.3 light-years. On the other hand, a passenger aboard the rocket finds the distance is only $L = v\Delta t_0$ = (0.95c)(1.4 years) = 1.3 light-years. The passenger, measuring the shorter time, also measures the shorter distance - length contraction.

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Problem solving insight

In dealing with time dilation, decide which interval is the proper time interval as follows: (1) Identify the two events that define the interval. (2) Determine the reference frame in which the events occur at the same place; an observer at rest in this frame measures the proper time interval Δt_0.

2) The Contraction of a Spacecraft (Cutnell, pg 863)

An astronaut, using a meter stick that is at rest relative to a cylindrical spacecraft, measures the length and diameter of the spacecraft to be 82 and 21 m respectively. The spacecraft moves with a constant speed of $v = 0.95c$ relative to the Earth. What are the dimensions of the spacecraft, as measured by an observer on Earth?

Reasoning

The length of 82 m is a proper length *Lo* since it is measured using a meter stick that is at rest relative to the spacecraft. The length *L* measured by the observer on Earth can be determined from the length-contraction formula. On the other hand, the diameter of the spacecraft is perpendicular to the motion, so the Earth observer does not measure any change in the diameter.

Solution

The length *L* of the spacecraft, as measured by the observer on Earth, is

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = 82m\sqrt{1-\frac{(0.95c)^2}{c^2}} = 26 \text{ m}$$

Both the astronaut and the observer on Earth measure the same value for the diameter of the spacecraft: Diameter = 21 m

Problem solving insight The proper length L_0 is always larger than the contracted length *L*.

3) *Additional problem 36, Cutnell pg. 879.*

Two spaceship A and B are exploring a new planet. Relative to this planet, spaceship A has a speed of $0.60c$, and spaceship B has a speed of $0.80c$. What is the ratio D_A/D_B of the values for the planet's diameter that each spaceship measures in a direction that is parallel to its motion?

Solution

Length contraction occurs along the line of motion, hence both spaceship observe length contraction on the diameter of the planet. The contracted length measures by a moving observer is inversely proportional to the Lorentz factor γ. Hence,

$$\frac{L_A}{L_B}=\frac{\gamma_B}{\gamma_A}=\sqrt{\frac{1-\left(\frac{v_A}{c}\right)^2}{1-\left(\frac{v_B}{c}\right)^2}}=\sqrt{\frac{1-(0.6)^2}{1-(0.8)^2}}=4/3.$$

4) The Energy Equivalent of a Golf Ball (Cutnell, pg 866)
A 0.046-kg golf ball is lying on the green. (a) Find the rest energy of the golf ball. (b) If this rest energy were used to operate a 75-W light bulb, for how many years could the bulb stay on?

Reasoning
The rest energy E_0 that is equivalent to the mass m of the golf ball is found from the relation $E_0 = mc^2$. The power used by the bulb is 75 W, which means that it consumes 75 J of energy per second. If the entire rest energy of the ball were available for use, the bulb could stay on for a time equal to the rest energy divided by the power.

Solution
(a) The rest energy of the ball is

$$E_0 = mc^2 = (0.046\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2 = 4.1 \times 10^{15}\ \text{J}$$

(b) This rest energy can keep the bulb burning for a time t given by t = Rest energy/ Power = 4.1×10^{15} J/75 W = 5.5×10^{13} s = 1.7 million years!

5) A High-Speed electron (Cutnell pg. 867)
An electron (mass = 9.1×10^{-31} kg) is accelerated to a speed of $0.9995c$ in a particle accelerator. Determine the electron's (a) rest energy, (b) total energy, and (c) kinetic energy in MeV

(a) $E_0 = mc^2 = 9.109\times10^{-31}\,kg\times(3\times10^8)^2\,m/s = 8.19\times10^{-14}\,J = 0.51MeV$

(b) Total energy of the traveling electron,

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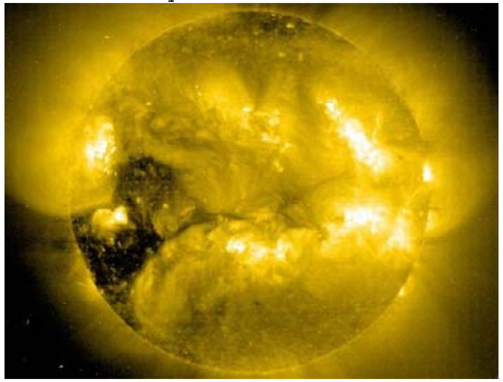
$$E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{0.51MeV}{\sqrt{1-0.995^2}}=16.2MeV$$

(c) The kinetic energy = $E - E_0$ = 15.7 MeV

6) The Sun Is Losing Mass (Cutnell, pg 868)
The sun radiates electromagnetic energy at the rate of 3.92×10^{26} W. (a) What is the change in the sun's mass during each second that it is radiating energy? (b) The mass of the sun is 1.99×10^{30} kg. What fraction of the sun's mass is lost during a human lifetime of 75 years?



Reasoning
Since a W = I J/s the amount of electromagnetic energy radiated during each second is 3.92×10^{26} J. Thus, during each second, the sun's rest energy decreases by this amount. The change ΔE_0 in the sun's rest energy is related to the change Δm in its mass by $\Delta E_0=\Delta m\ c^2$.
Solution
(a) For each second that the sun radiates energy, the change in its mass is $\Delta m = \Delta E_0/c^2 = 3.92\times10^{26}\ \text{J}/(3\times10^8\ \text{m/s})^2 = (4.36\times10^9)$ kg. Over 4 billion kilograms of mass are lost by the sun during each second.

(b) The amount of mass lost by the sun in 75 years is
$\Delta m = (4.36\times10^9)\,\text{kg} \times (3.16\times10^7\ \text{s/year}) \times (75\ \text{years}) = 1\times10^{19}$ kg
Although this is an enormous amount of mass, it represents only a tiny fraction of the sun's total mass:
$$\Delta m/m = 1.0\times10^{19}\ \text{kg}/1.99\times10^{30}\ \text{kg} = 5.0\times10^{-12}$$

Figure 28.12 An intergalactic cruiser, closing in on a hostile spacecraft, fires a beam of laser light.

7) The Speed of a Laser Beam (Cutnell, pg 871)
Figure below shows an intergalactic cruiser approaching a hostile spacecraft. The velocity of the cruiser relative to the spacecraft is $v_{CS} = +0.7c$. Both vehicles are moving at a constant velocity. The cruiser fires a beam of laser light at the enemy. The velocity of the laser beam relative to the cruiser is $v_{LC} = +c$. (a) What is the velocity of the laser beam v_{LS} relative to the renegades aboard the spacecraft? (b) At what velocity do the renegades aboard the spacecraft see the laser beam move away from the cruiser?

Reasoning and Solution

(a) Since both vehicles move at a constant velocity, each constitutes an inertial reference frame. According to the speed of light postulate, all observers in inertial reference frames measure the speed of light in a vacuum to be c. Thus, the renegades aboard the hostile spacecraft see the laser beam travel toward them at the speed of light, even though the beam is emitted from the cruiser, which itself is moving at seven-tenths the speed of light.

More formally, we can use Lorentz transformation of velocities to calculate v_{LS}. We will take the direction as +ve when a velocity is pointing from left to right. We can take view that the hostile spacecraft is at rest (as the stationary frame, O) while the cruiser is approaching it with velocity $v_{CS} = +0.7c$ (according to our choice of the sign). In this case, the cruiser is the moving frame, O'. The light beam as seen in the moving frame O' is $v_{LC} = +c$. We wish to find out what is the speed of this laser beam from O point of view, e.g. what v_{LS} is.

We may like to identify v_{LS}, v_{LC} and v_{CS} with the definitions used in the Lorentz formula: $u_x = \dfrac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}}$. In fact, a little contemplation would allow us to make the identification that, with our choice of frames (that the hostile spacecraft as the stationary frame): $v_{LC} \equiv u_{x'} = +c$; $v_{CS} \equiv u = +0.7c$ and $v_{LS} = u_x$ = the speed of laser beam as seen by the stationary frame O (the quantity we are seeking). Hence, we have

$$u_x = \frac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}} \equiv v_{LS} = \frac{v_{LC} + v_{CS}}{1 + \dfrac{v_{LC} v_{CS}}{c^2}} = \frac{(+c) + (+0.7c)}{1 + \dfrac{(+c)(+0.7c)}{c^2}} = \frac{1.7c}{1.7c} = +c,$$

i.e. the laser beam is seen, from the view point of the hostile spacecraft, to be approaching it with a velocity +c (+ve means the velocity is from left to right).

(b) The renegades aboard the spacecraft see the cruiser approach them at a relative velocity of $v_{CS} = +0.7c$, and they also see the laser beam approach them at a relative velocity Of v_{LS} +c. Both these velocities are measured relative to the same inertial reference frame-namely, that of the spacecraft. Therefore, the renegades aboard the spacecraft see the laser beam move away from the cruiser at a velocity that is the difference between these two velocities, or $+c - (+0.7c) = +0.3c$. The relativistic velocity-addition formula, is not applicable here because both velocities are measured relative to the same inertial reference frame (the spacecraft's reference frame). The relativistic velocity-addition formula can be used only when the velocities are measured relative to different inertial reference frames.

8) The Relativistic Momentum of a High-Speed Electron (Cutnell, pg 865)
The particle accelerator at Stanford University is three kilometers long and accelerates electrons to a speed of 0.999 999 999 7c, which is very nearly equal to the speed of light. Find the magnitude of the relativistic momentum of an electron that emerges from the accelerator, and compare it with the non-relativistic value.

Reasoning and Solution

The magnitude of the electron's relativistic momentum can be obtained from $p = \gamma m_0 v = 1\times10^{-17}$ Ns, where $m_0 = 9.1\times10^{-31}$ kg, $v\gamma = \dfrac{0.999999997c}{\sqrt{1-\dfrac{(0.999999997c)^2}{c^2}}} = 1.09989\times10^{13}$ m/s. The relativistic momentum is greater than the non-relativistic momentum by a factor of $\gamma = \dfrac{1}{\sqrt{1-\dfrac{(0.999999997c)^2}{c^2}}} = 4\times10^4$.

9) Resnick and Halliday, Sample problem 37-8, pg. 1047.
The most energetic proton ever detected in the cosmic rays coming to Earth from space had an astounding kinetic energy of

3.0 x 10^{20} eV. (a) What were the proton's Lorentz factor γ and speed v (both relative to the ground-based detector)?

Solution

$$\gamma = \frac{E}{m_0c^2} = \frac{m_0c^2 + K}{m_0c^2} = 1 + \frac{K}{m_0c^2} \Rightarrow \gamma = 1 + \frac{3.0\times10^{20}\,\text{eV}}{938\times10^{6}\,\text{eV}} \approx 3.2\times10^{11}$$

$$\gamma^{-1} = \sqrt{1-\left(\frac{v}{c}\right)^2} = \sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}.\ \text{But } 1+\frac{v}{c} \approx 2 \Rightarrow \gamma^{-2} \approx 2\left(1-\frac{v}{c}\right) = 9.766\times10^{-24}$$

$$\Rightarrow v \approx \left(1-5\times10^{-24}\right)c = 0.999999999999999999999995c$$

Tutorial 2

Matter and Wave; Blackbody radiation

Conceptual Questions

1. **What is ultraviolet catastrophe? What is the significance of it in the development of modern physics? (My own question)**

   ANS
   The classical theory explanation of the blackbody radiation by Rayleigh-Jeans fails in the limit $\lambda \to 0$ (or equivalently, when frequency $\to \infty$), i.e. $R(\lambda) \to \infty$ at $\lambda \to 0$. The failure prompted Planck to postulate that the energy of electromagnetic waves is quantised (via $\varepsilon = h\nu$) as opposed to the classical thermodynamics description ($\varepsilon = \kappa T$). With Planck's postulate, radiation now has particle attributes instead of wave.

2. **What assumptions did Planck make in dealing with the problem of blackbody radiation? Discuss the consequences of the assumptions.**

ANS
Planck made two new assumptions: (1) Radiation oscillator energy is quantized and (2) they emit or absorb energy in discrete irreducible packets. The "oscillator" here actually refers to the molecules or atoms that made up the walls of the blackbody cavity. These assumptions contradict the classical idea of energy as continuously divisible.

3. **The classical model of blackbody radiation given by the Rayleigh-Jeans law has two major flaws. Identify them and explain how Planck's law deals with them.**

   ANS
   The first flaw is that the Rayleigh–Jeans law predicts that the intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases. This is known as the *ultraviolet catastrophe*. The second flaw is the prediction much more power output from a black-body than is shown experimentally. The intensity of radiation from the blackbody is given by the area under the red $I(\lambda,\ T)$ *vs.* λ curve in Figure 40.5 in the text, not by the area under the blue curve.
   Planck's Law dealt with both of these issues and brought the theory into agreement with the experimental data by adding an exponential term to the denominator that

depends on $\frac{1}{\lambda}$. This both keeps the predicted intensity from approaching infinity as the wavelength decreases and keeps the area under the curve finite.

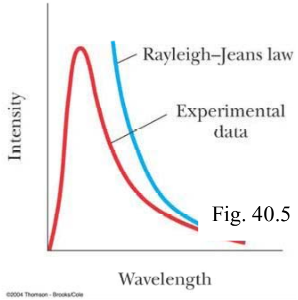


Fig. 40.5

4. What are the most few distinctive physical characteristics, according to your point of view, that exclusively differentiate a classical particle from a wave? Construct a table to compare these two.

ANS (my suggestions)

Particle	Wave
Complete localized	Cannot be confined to any particular region of space. A wave can be "simultaneously everywhere" at a given instance in time
Mass and electric charge can be identified with infinite precision	No mass is associated with a wave.
Energy carried by a particle is concentrated in it and is not spreading over the boundary that define its physical location	Energy carried by wave spreads over an infinite regions of space along the direction the wave propagates
Momentum and position can be identified with infinite precision.	Wavelength and position of a wave cannot be simultaneously measured to infinite precision, they must obey the classical wave uncertainty relation $\Delta\lambda\,\Delta x \geq \lambda^2$
There is not definition of wavelength for a particle	There is not definition of momentum for waves
Does not undergo diffraction and interference	Waves undergo diffraction and interference

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(others)	(others)

Problems

1. For a blackbody, the total intensity of energy radiated over all wavelengths, *I*, is expected to rise with temperature. In fact one find that the total intensity increases as the fourth power of the temperature. We call this the *Stefan's law*: $I = \sigma T^4$, where σ is the Stefan's constant $\sigma = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}$. How does the total intensity of thermal radiation vary when the temperature of an object is doubled?

ANS

Intensity of thermal radiation $I \propto T^4$. Hence, when T is double, ie. $T\rightarrow 2T$, $I \rightarrow I'(2)^4 = 16I$, i.e. the total intensity of thermal radiation increase by 16 times.

2. (Krane, pg. 62)

In the spectral distribution of blackbody radiation, the wavelength λ_{max} at which the intensity reaches its maximum value decreases as the temperature is increased, in inverse proportional to the temperature: $\lambda_{\max} \propto 1/T$. This is called the *Wein's displacement law*. The proportional constant is experimentally determined to be $\lambda_{\max}T = 2.898\times10^{-3}\,\mathrm{m\cdot K}$

(a) At what wavelength does a room-temperature (T = 20°C) object emit the maximum thermal radiation?

(b) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum?

(c) How many times as much thermal radiation does it emit at the higher temperature?

ANS

(a) Converting to absolute temperature, T = 293 K, and from Wien's displacement law, $\lambda_{\max}T = 2.898\times10^{-3}\,\mathrm{m\cdot K}$

λ_{max} = 2.898×10^{-3} m·K/293K = 9.89 μm

(b) Taking the wavelength of red light to be =650 nm, we again use Wien's displacement law to find T:

T = 2.898×10^{-3} m·K/650×10^{-9} m = 4460 K

(c) Since the total intensity of radiation is proportional to T^4, the ratio of the total thermal emissions will be

$$\frac{T_2^4}{T_1^4} = \frac{4460^4}{293^4} = 5.37\times 10^4$$

Be sure to notice the use of absolute (Kelvin) temperatures.

3. **Show that the spectral distribution derived by Planck,** $I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)}$ **reduces to the Rayleigh-Jeans law,** $I(\lambda,T) = \frac{2\pi c k_B T}{\lambda^4}$ **in the long wavelength limit.**

ANS

In long wavelength limit, $hc \ll \lambda k_B T$, the exponential term is approximated to

$e^{hc/\lambda k_B T} = 1 + \frac{hc}{\lambda k_B T} + \frac{1}{2!}\left(\frac{hc}{\lambda k_B T}\right)^2 + \ldots \approx 1 + \frac{hc}{\lambda k_B T}$. Hence, substituting $e^{hc/\lambda k_B T} \approx 1 + \frac{hc}{\lambda k_B T}$ into the Planck's distribution, we have

$$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)} \approx \frac{2\pi hc^2}{\lambda^5\left[\left(1+\frac{hc}{\lambda k_B T}\right)-1\right]} = \frac{2\pi hc^2}{\lambda^5\left[\frac{hc}{\lambda k_B T}\right]} = \frac{2\pi hc^2 \lambda k_B T}{\lambda^5 hc} = \frac{2\pi c k_B T}{\lambda^4},$$

which is nothing but just the RJ's law.

Tutorial 3

Photoelectricity, Compton scatterings, pair-production/annihilation, X-rays

Conceptual Questions

1. **What is the significance of the Compton wavelength of a given particle (say an electron) to a light that is interacting with the particle? (Own question)**

ANS

The Compton wavelength (a characteristic constant depend solely on the mass of a given particle) characterises the length scale at which the quantum property (or wave) of a given particle starts to show up. In an interaction that is characterised by a length scale larger than the Compton wavelength, particle behaves classically. For interactions that occur at a length scale comparable than the Compton wavelength, the quantum (or, wave) nature of the particle begins to take over from classical physics.

In a light-particle interaction, if the wavelength of the light is comparable to the Compton wavelength of the interacting particle, light displays quantum (granular/particle) behaviour rather than like a wave.

2. **Why doesn't the photoelectric effect work for free electron? (Krane, Question 7, pg 79)**

ANS (verify whether the answer make sense)

Essentially, Compton scattering is a two-body process. The free electron within the target sample (e.g. graphite) is a unbounded elementary particle having no internal structure that allows the photons to be `absorbed'. Only elastic scattering is allowed here.

Whereas PE effect is a inelastic scattering, in which the absorption of a whole photon by the atom is allowed due to the composite structure (the structure here refers the system of the orbiting electrons and nuclei hold together via electrostatic potential) of the atom. A whole photon is allowed to get absorbed by the atom in which the potential energy acts like a medium to transfer the energy absorbed from the photon, which is then `delivered' to the bounded electrons (bounded to the atoms) that are then `ejected' out as photoelectrons.

3. **How is the wave nature of light unable to account for the observed properties of the photoelectric effect? (Krane, Question 5, pg 79)**

ANS

See lecture notes

4. In the photoelectric effect, why do some electrons have kinetic energies smaller than K_{max}? (Krane, Question 6, pg 79)

ANS

By referring to $K_{max} = h\nu - \phi$, K_{max} corresponds to those electrons knocked loose from the surface by the incident photon whenever $h\nu > \phi$. Those below the surface required an energy greater than ϕ and so come off with less kinetic energy.

5. Must Compton scattering take place only between x-rays and free electrons? Can radiation in the visible (say, a green light) Compton scatter a free electron? (My own question)

ANS

In order to Compton scatter the electron, the wavelength of the radiation has to be comparable to the Compton wavelength of the electron. If such criterion is satisfied the cross section (the probability for which a scattering process can happen) of Compton scattering between the radiation and the electron would be highly enhanced. It so happen that the Compton wavelength of the electron, $\lambda_e = \frac{h}{m_e c} \sim 10^{-12}\text{m}$ is ~ the order the X-rays', $\lambda_{X-ray} \sim 10^{-12}\text{m}$, hence X-rays' Compton scattering with electrons is most prominent compared to radiation at other wavelengths. This means that at other wavelength (such as in the green light region, where $\lambda_{green} << \lambda_e$) the cross section of Compton scattering would be suppressed.

Problems

1. The diameter of an atomic nucleus is about 10×10^{-15} m. Suppose you wanted to study the diffraction of photons by nuclei. What energy of photons would you choose? Why? (Krane, Question 1, pg 79)

Solution

Diffraction of light by the nucleus occurs only when the wavelength of the photon is smaller or of the order of the size of the nucleus, $\lambda \sim D$ (D = diameter of the nucleus). Hence, the minimum energy of the photon would be $E = hc/\lambda \sim hc/D \sim 120$ MeV.

2. Photons from a Light Bulb (Cutnell, pg884)
In converting electrical energy into light energy, a sixty-watt incandescent light bulb operates at about 2.1% efficiency. Assuming that all the light is green light (vacuum wavelength 555 nm), determine the number of photons per second given off by the bulb.

Reasoning

The number of photons emitted per second can be found by dividing the amount of light energy emitted per second by the energy E of one photon. The energy of a single photon is $E = hf$. The frequency of the photon is related to its wavelength λ by $\nu = c/\lambda$.

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Solution

At an efficiency of 2.1%, the light energy emitted per second by a sixty-watt bulb is (0.021)(60.0 J/s)=1.3 J/s. The energy of a single photon is

$E = hc/\lambda = (6.63\times10^{-34}\text{Js})(3\times10^{8}\text{ m/s})/555\times10^{-9}\text{ nm} = 3.58\times10^{-19}$ J

Therefore, number of photons emitted per second =
1.3 J/s/ (3.58×10^{-19} J/photon) = 3.6×10^{18} photon per second

3. Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (a) Find the maximum KE of the photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2? (Beiser, pg. 63)

Solution

(a) The energy of the photons is, $E_p = hc/\lambda$ = 3.5eV. The work function of potassium is 2.2 eV. So,
KE = $h\nu - \phi$ = 3.5 eV - 2.2 eV = 5.68×10^{-19} J

(b) The photon energy in joules is 5.68×10^{-19} J. Hence the number of photons that reach the surface per second is
$n_p = (E/t)/E_p = (E/A)(A)/E_p$
$= (1.00\text{ W/m}^2)(1.00\times10^{-4}\text{ m}^2)/5.68\times10^{-19}$ J
$= 1.76\times10^{14}$ photons/s
The rate at which photoelectrons are emitted is therefore $n_e = (0.0050)n_p = 8.8\times10^{11}$ photoelectrons/s

4. The work function for tungsten metal is 4.53 eV. (a) What is the cut-off wavelength for tungsten? (b) What is the maximum kinetic energy of the electrons when radiation of wavelength 200.0 nm is used? (c) What is the stopping potential in this case? (Krane, pg. 69)

Solution

(a) The cut-off frequency is given by $\lambda_c = \frac{hc}{\phi} = \frac{1240\text{eV}\cdot\text{nm}}{200\text{nm}} = 274\text{nm}$, in the uv region

(b) At the shorter wavelength,

$$K_{\max} = h\frac{c}{\lambda} - \phi = \frac{1240eV\cdot nm}{200nm} - 4.52eV = 1.68eV$$

(c) The stopping potential is just the voltage corresponding to $K_{\max}$:

$$V_s = K_{\max}/e = \frac{1.68eV}{e} = 1.68\text{V}$$

5. X-rays of wavelength 10.0 pm (1 pm = 10^{-12} m) are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons. (Beiser, pg. 75)

Solution

(a) The Compton shift is given by $\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\varphi)$, and so $\lambda' = \lambda + \lambda_c(1-\cos 45^o) = 10.0\,pm + 0.293\lambda_c = 10.7$ pm

(b) $\Delta\lambda$ is a maximum when $1-\cos\varphi$ = 2, in which case, $\Delta\lambda = \lambda + 2\lambda_c$ = 10.0 pm + 4.9 pm = 14.9 pm

(c) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$KE_{max} = h(\nu - \nu') = hc(\frac{1}{\lambda} - \frac{1}{\lambda'}) = 40.8$ eV

6. Gautreau and Savin, page 70, Q 9.28

A photon of wavelength 0.0030 Å in the vicinity of a heavy nucleus produces an electron-positron pair. Determine the kinetic energy of each of the particles if the kinetic energy of the positron is twice that of the electron.

Solution:
From (total relativistic energy before) = (total relativistic energy after),

$$\frac{hc}{\lambda} = 2m_ec^2 + K_+ + K_- = 2m_ec^2 + 3K_-$$

$$\frac{12.4\times10^{-3}\,\text{MeV}\cdot\text{Å}}{0.0030\,\text{Å}} = 2(0.511\text{MeV})\cdot 3K_-$$

$$K_- = 1.04\text{MeV};\ K_+ = 2K_- = 2.08\text{MeV}$$

7. Gautreau and Savin, page 71, Q 9.32

Annihilation occurs between an electron and positron at rest, producing three photons. Find the energy of the third photon of the energies of the two of the photons are 0.20 MeV and 0.30 MeV.

Solution:
From conservation of energy, 2(0.511 MeV) = 0.20 MeB + 0.30 MeV = E_3 or E_3 = 0.522 MeV

8. Gautreau and Savin, page 71, Q 9.33
How Many positrons can a 200 MeV photon produce?

Solution:
The energy needed to produce an electron-positron pair at rest is twice the rest energy of an electron, or 1.022 MeV. Therefore, Maximum number of positrons =

$$(200\text{ MeV})\left(\frac{1\text{ pair}}{1.022\text{ MeV}}\right)\left(1\frac{\text{positron}}{\text{pair}}\right) = 195\text{ positrons}$$

Tutorial 4
Wave particle duality, de Brolie postulate, Heisenberg Uncertainty principle

Conceptual Questions

1. **What difficulties does the uncertainty principle cause in trying to pick up an electron with a pair of forceps? (Krane, Question 4, pg. 110)**

ANS
When the electron is picked up by the forceps, the position of the electron is ``localised' (or fixed), i.e. $\Delta x$ = 0. Uncertainty principle will then render the momentum to be highly uncertainty. In effect, a large $\Delta p$ means the electron is ``shaking'' furiously against the forceps' tips that tries to hold the electron ``tightly''.

2. **An electron and a proton both moving at nonrelativistic speeds have the same de Broglie wavelength. Which of the following are also the same for the two particles?**
(a) speed (b) kinetic energy (c) momentum
(d) frequency

ANS
(c). According to de Broglie's postulate, $\lambda = \frac{h}{p} = \frac{h}{mv}$, two particles with the same de Broglie wavelength will have the same momentum $p = mv$. If the electron and proton have the same momentum, they cannot have the same speed (a) because of the difference in their masses. For the same reason, because $K = p^2/2m$, they cannot have the same kinetic energy (b). Because the particles have different kinetic energies, Equation $\lambda = \frac{h}{p} = \frac{h}{mv}$ tells us that the particles do not have the same frequency (d).

3. **The location of a particle is measured and specified as being exactly at *x* = 0, with *zero* uncertainty in the *x* direction. How does this affect the uncertainty of its velocity component in the *y* direction?**
(a) It does not affect it.
(b) It makes it infinite.
(c) It makes it zero.

ANS
(a). The uncertainty principle relates uncertainty in position and velocity along the same axis. The zero uncertainty in

position along the x axis results in infinite uncertainty in its velocity component in the x direction, but it is unrelated to the y direction.

4. **You use a large potential difference to accelerate particles from rest to a certain kinetic energy. For a certain potential difference, the particle that will give you the highest resolution when used for the application as a microscope will be a) an electron, b) a proton, c) a neutron, or d) each particle will give you the same resolution under these circumstances. (Serway QQ)**

   ANS

   (b). The equation $\lambda = h/(2mq\Delta V)^{1/2}$ determines the wavelength of a particle. For a given potential difference and a given charge, the particle with the highest mass will have the smallest wavelength, and can be used for a microscope with the highest resolution. Although neutrons have the highest mass, their neutral charge would not allow them to be accelerated due to a potential difference. Therefore, protons would be the best choice. Protons, because of their large mass, do not scatter significantly off the electrons in an atom but can be used to probe the structure of the nucleus.

5. **Why was the demonstration of electron diffraction by Davisson and Germer and important experiment? (Serway, Q19, pg. 1313)**

   ANS

   The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

6. **If matter has wave nature why is this wave-like character not observed in our daily experiences? (Serway, Q21, pg. 1313)**

   ANS

   Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength and does not exhibit quantum behavior.

Problems

1. **Beiser, pg. 100, example 3.3**

   An electron has a de Broglie wavelength of 2.00 pm. Find its kinetic energy and the phase and the group velocity of its de Broglie waves.

   Solution

   (a) First calculate the pc of the electron

   $pc = hc/\lambda$ = 1.24 keV.nm / 2.00 pm = 620 keV

   The rest energy of the electron is E_0=511 keV, so the KE of the electron is

   $KE = E - E_0 = [E_0^2 - (pc)^2]^{1/2} - E_0 = \ldots 292$ keV

   (b) The electron's velocity is to be found from

   $$\gamma = E/E_0 = (KE + E_0)/E_0 = 803/511 = 1.57$$

   $$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} = 0.405 \Rightarrow \frac{v^2}{c^2} = 1 - 0.405 = 0.595$$

   $$\Rightarrow v = 0.771c$$

   Hence, the phase velocity is $v_p = \dfrac{c^2}{v} = \dfrac{c^2}{0.771c} = 1.29c$

   The group velocity is $v_g = v = 0.771c$

2. **Find the de Broglie wave lengths of (a) a 46-g ball with a velocity of 30 m/s, and (b) an electron with a velocity of 10^7 m/s (Beiser, pg. 92)**

   Solution

   (a) Since $v \ll c$, we can let $m = m_o$. Hence

   $\lambda = h/mv = 6.63\times10^{-34}$ Js/(0.046 kg)(30 m/s)
   $= 4.8\times10^{-34}$ m

   The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behaviour.

   (b) Again $v \ll c$, so with $m = m_o = 9.1\times10^{-31}$ kg, we have

   $\lambda = h/mv = 6.63\times10^{-34}$ Js/(9.1×10^{-31} kg)(10^7 m/s)
   $= 7.3\times10^{-11}$ m

   The dimensions of atoms are comparable with this figure - the radius of the hydrogen atom, for instance, is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behaviour.

3. **The de Broglie Wavelength (Cutnell, pg. 897)**
An electron and a proton have the same kinetic energy and are moving at non-relativistic speeds. Determine the ratio of the de Broglie wavelength of the electron to that of the proton.

ANS
Using the de Broglie wavelength relation $p = h/\lambda$ and the fact that the magnitude of the momentum is related to the kinetic energy by $p = (2mK)^{1/2}$, we have

$$\lambda = h/p = h/(2mK)^{1/2}$$

Applying this result to the electron and the proton gives

$$\lambda_e/\lambda_p = (2m_pK)^{1/2}/(2m_eK)^{1/2}$$
$$= (m_p/m_e)^{1/2} = (1.67\times10^{-27}\,\text{kg}/9.11\times10^{-31}\,\text{kg})^{1/2} = 42.8$$

As expected, the wavelength for the electron is greater than that for the proton.

4. **Find the kinetic energy of a proton whose de Broglie wavelength is 1.000 fm = 1.000×10^{-15} m, which is roughly the proton diameter (Beiser, pg. 92)**

ANS
A relativistic calculation is needed unless pc for the proton is much smaller than the proton rest mass of E_o = 0.938 GeV.

So we have to first compare the energy of the de Broglie wave to E_o:

$E = pc = \dfrac{hc}{\lambda} = \dfrac{1242 eV\cdot nm}{10^{-6} nm} = 1.24$ GeV, c.f. E_o = 0.938 GeV. Since the energy of the de Broglie wave is larger than the rest mass of the proton, we have to use the relativistic kinetic energy instead of the classical $K = p^2/2m$ expression.

The total energy of the proton is
$E = \sqrt{E_0^2 + (pc)^2} = \sqrt{(0.938\,GeV)^2 + (1.24\,GeV)^2} = 1.555$ GeV.
The corresponding kinetic energy is
KE = $E - E_o$ = (1.555 - 0.938) GeV = 0.617 GeV = 617 MeV

5. **A hydrogen atom is 5.3x10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom. (Beiser, pg 114)**

ANS
Here we find that with Δx = 5.3×10^{-11} m.
$\Delta p \geq \dfrac{\hbar}{2\pi}$ = 9.9×10^{-25} Ns.
An electron whose momentum is of this order of magnitude behaves like a classical particle, an its kinetic energy is $K = p^2/2m \geq (9.9\times10^{-25}\text{ Ns})^2/2\times9.110^{-31}$ kg = 5.4×10^{-19} J, which is 3.4 eV. The kinetic energy of an electron in the lowest energy level of a hydrogen atom is actually 13.6 eV.

6. **A measurement established the position of a proton with an accuracy of $\pm1.00\times10^{-11}$m. Find the uncertainty in the proton's position 1.00 s later. Assume *v* << *c*. (Beiser, pg. 111)**

ANS
Let us call the uncertainty in the proton's position Δx_0 at the time t = 0. The uncertainty in its momentum at this time is therefore $\Delta p \geq \dfrac{\hbar}{2\Delta x_0}$. Since $v << c$, the momentum uncertainty is $\Delta p \geq \Delta(mv) = m_0\Delta v$ and the uncertainty in the proton's velocity is $\Delta v \geq \dfrac{\Delta p}{m_0} \geq \dfrac{\hbar}{2m_0\Delta x_0}$. The distance x of the proton covers in the time t cannot be known more accurately than $\Delta x \geq t\Delta v \geq \dfrac{\hbar t}{2m_0\Delta x_0}$. Hence Δx is inversely proportional to Δx_0: the more we know about the proton's position at t = 0 the les we know about its later position at t. The value of Δx at t = 1.00 s is $\Delta x \geq \dfrac{(1.054\times10^{-34}Js)(1.00s)}{2(1.672\times10^{-27}kg)(1.00\times10^{-11}m)} = 3.15\times10^3$ m. This is 3.15 km! What has happened is that the original wave group has spread out to a much wider one because the phase velocities of the component wave vary with wave number and a large range of wave numbers must have been present to produce the narrow original wave

7. **Broadening of spectral lines due to uncertainty principle: An excited atom gives up it excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time is radiates is**

1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon. (Beiser, pg. 115)

ANS

The photon energy is uncertain by the amount $\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054\times10^{-34}\,Js}{2(1.0\times10^{-8}\,s)} = 5.3\times10^{-27}$ J. The corresponding uncertainty in the frequency of light is $\Delta\nu = \frac{\Delta E}{h} \geq 8\times10^{6}$ Hz. This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom. As a result, the radiation from a group of excited atoms does not appear with the precise frequency ν. For a photon whose frequency is, say, 5.0×10^{14} Hz, $\frac{\Delta\nu}{\nu} = 1.6\times10^{-8}$.

Tutorial 5
Atomic models

Conceptual Questions

1. **What is the ONE essential difference between the Rutherford model and the Bohr's model? (My own question)**

   ANS
   Rutherford's model is a classical model, in which EM wave will be radiated rendering the atom to collapse. Whereas the Bohr's model is a semi-classical model in which quantisation of the atomic orbit happens.

2. **Conventional spectrometers with glass components do not transmit ultraviolet light ($\lambda \lesssim$ 380 nm). Explain why non of the lines in the Lyman series could be observed with a conventional spectrometer. (Taylor and Zafiratos, pg. 128)**

   ANS

   For Lyman series, n_f = 1. According to $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, the wavelength corresponding to n_i = 2 in the Lyman series is predicted to be $\lambda = \frac{4}{3R_\infty} = \frac{4}{3(109{,}737\,\text{cm}^{-1})}$=121.5 nm. Similarly, for n_i = 3, one finds that λ=102 nm, and inspection of $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ shows that the larger we take n, the smaller the corresponding wavelength. Therefore, all lines in the Lyman series lie well into the ultraviolet and are unobservable with a conventional spectrometer.

3. **Does the Thompson model fail at large scattering angles or at the small scattering angle? Why? (Krane, Questions 1, pg. 173)**

   ANS
   Thompson model fails at large angle (but is consistent with scattering experiments at small angle). Thompson model predicts that the average scattered angle is given by a small value of θ_{ave} ~ 1°. However, in the experiment, alpha particles are observed to be scattered at angle in excess of 90°. This falsifies Thompson model at large angle.

4. **In which Bohr orbit does the electron have the largest velocity? Are we justified in treating the electron non-relativistically? (Krane, Questions 6. pg. 174)**

ANS

The velocity in an orbit n is given by $v = h/2\pi mnr_0$, which means that the velocity is inversely proportional to the n number. Hence the largest velocity corresponds to the $n = 1$ state,

$v(n=1)/c = h/2c\pi mr_0$

$= 6.63\times10^{-34}/2\pi(9.1\times10^{-31})(0.53\times10^{-10})/c$

$= 0.007.$

Hence, nonrelativistic treatment is justified.

5. **How does a Bohr atom violate the $\Delta x\Delta p \geq \frac{\hbar}{2}$ uncertainty relation? (Krane, Question 11, pg. 174)**

ANS

The uncertainty relation in the radial direction of an electron in a Bohr orbit is $\Delta r\Delta p_r \geq \frac{\hbar}{2}$. However, in the Bohr model, the Bohr orbits are assumed to be precisely known $(=r_n = n^2 r_0)$ for a given n. This tantamount to $\Delta r = 0$, which must render the momentum in the radial direction to become infinite. But in the Bohr atom the electron does not have such radial motion caused by this uncertainty effect. So in this sense, the discrete Bohr orbit violates the uncertainty relation $\Delta x\Delta p \geq \frac{\hbar}{2}$.

Problem

1. **If we assume that in the ground of the hydrogen the position of the electron along the Bohr orbit is not known and not knowable, then the uncertainty in the position is about $\Delta x \approx 2r_0 = 10^{-10}$ m, (a) what is the magnitude of the momentum of the electron at the ground state? (b) What is the corresponding quantum uncertainty in the momentum? (Ohanian, pg. 152)**

ANS

(a) Angular momentum, $|L| \equiv |p| r = n\hbar$. Hence, in the ground state, $|p| = \hbar/r_0 = 2.1\times10^{-24}$ Ns

(b) $\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2(2r_0)} = 5.3\times10^{-25}$ Ns.

2. **Serway and Mosses, Problem 13(a), page 148**
What value of n is associated with the Lyman series line in hydrogen whose wavelength is 102.6 nm?

Solution:

$\lambda = 102.6\text{ nm}$; $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right) \Rightarrow n = \frac{R}{\left(R - \frac{1}{\lambda}\right)^{1/2}} = \frac{R}{\left(R - \frac{1}{102.6\times10^{-9}\text{ m}}\right)^{1/2}} = 2.99 \approx 3$

3. **Serway and Moses, Problem 22**
Find the potential energy and kinetic energy of an electron in the ground state of the hydrogen atom.

Solution:

$E = K + U = \frac{mv^2}{2} - \frac{ke^2}{r}$. But $\frac{mv^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r}$. Thus $E = \left(\frac{1}{2}\right)\left(\frac{-ke^2}{r}\right) = \frac{U}{2}$, so $U = 2E = 2(-13.6\text{ eV}) = -27.2\text{ eV}$ and $K = E - U = -13.6\text{ eV} - (-27.2\text{ eV}) = 13.6\text{ eV}$.

4. **Serway and Moses, Problem 21**
Calculate the longest and shortest wavelengths for the Paschen series. (b) Determine the photon energies corresponding to these wavelengths.

Solution

(a) For the Paschen series; $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$; the maximum wavelength corresponds to $n_i = 4$, $\frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$; $\lambda_{max} = 1\,874.606\text{ nm}$. For minimum wavelength, $n_i \to \infty$, $\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$; $\lambda_{min} = \frac{9}{R} = 820.140\text{ nm}$.

(b) $\frac{hc}{\lambda_{min}} = \frac{\left(\frac{hc}{1\,874.606\text{ nm}}\right)}{1.6\times10^{-19}\text{ J/eV}} = 0.662\,7\text{ nm}$, $\frac{hc}{\lambda_{min}} = \frac{\left(\frac{hc}{820.140\text{ nm}}\right)}{1.6\times10^{-19}\text{ J/eV}} = 1.515\text{ nm}$

5. **Hydrogen atoms in states of high quantum number have been created in the laboratory and observed in space. (a) Find the quantum number of the Bohr orbit in a hydrogen atom whose radius is 0.0199 mm. (b) What is the energy of a hydrogen atom in this case? (Beiser, pg. 133)**

Solution

(a) From $r_n = n^2 r_0$, we have $n = \sqrt{\frac{r_n}{r_0}} = \sqrt{\frac{0.0100\times10^{-3}}{5.3\times10^{-11}}} = 434$

(b) From $E_n = -\frac{13.6}{n^2}$eV, we have $E_n = -\frac{13.6}{434^2}$eV = -0.000072 eV. Such an atom would obviously be extremely fragile and be easily ionised (compared to the kinetic energy of the atom at temperature T, kT ~ $(1.38\times10^{-23}$ J/K$)\times(300$ K$)$ =0.03 eV)

6. **(a) Find the frequencies of revolution of electrons in $n = 1$ and $n = 3$ Bohr orbits. (b) What is the frequency of the photon emitted when an electron in the $n = 2$ orbit drops to an $n = 1$ orbit? (c) An electron typically spends about 10^{-8} s in an excited state before it drops to a lower state by emitting a photon. How many revolutions does an electron in an $n = 2$ Bohr orbit make in 10^{-8} s? (Beiser, pg. 137)**

Solution

(a) Derive the frequency of revolution from scratch: Forom Bohr's postulate of quantisation of angular momentum, $L = (mv)r = nh/2\pi$, the velocity is related to the radius as $v = nh/2mr\pi$. Furthermore, the quantised radius is given in terms of Bohr's radius as $r_n = n^2 r_0$. Hence, $v = h/2\pi mnr_0$.

The frequency of revolution $f = 1/T$ (where T is the period of revolution) can be obtained from $v = 2\pi r/T = 2\pi n^2 r_0 f$. Hence, $f = v/2\pi r = (h/2\pi mnr_0)/2\pi r = h/4\pi^2 mn^3 (r_0)^2$.

For $n = 1$, $f_1 = h/4\pi^2 m(r_0)^2 = 6.56\times10^{15}$ Hz.
For $n = 2$, $f_2 = h/4\pi^2 m(2)^3(r_0)^2 = 6.56\times10^{15}/8$ Hz $= 8.2\times10^{14}$.

(b) $$\nu = \frac{\Delta E}{h} = \frac{13.6eV}{h}\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3c}{4}\frac{13.6eV}{1242eV\cdot nm} = 0.00821\times(3\times10^8 m/s)/10^{-9} m$$
$= 2.463\times10^{15}$ Hz. The frequency is intermediate between f_1 and f_2.

(c) The number of revolutions the electron makes is $N = f_2\Delta t = (8.2\times10^{14})\times10^8 = 8.2\times10^{22}$ rev.

Tutorial 1

Special Relativity

Conceptual Questions

1) **The speed of light in water is c/n, where $n = 1.33$ is the index of refraction of water. Thus the speed of light in water is less than c. Why doesn't this violate the speed of light postulate?**

ANS
The constancy of light postulate only applies to light propagating in vacuum. So, a light propagating in a medium which is otherwise could still has a travelling speed other than c.

2) **What is the significance of the negative result of Michelson-Morley experiment?**

ANS
The negative result of the MM experiment contradicts with the prediction of the absolute frame (the Ether frame) of reference, in which light is thought to propagate with a speed c. In the Ether postulate, the speed of light that is observed in other initial reference frame (such as the Earth that is moving at some constant speed relative to the Absolute frame), according to the Galilean transformation, would be different than that of the Ether frame. In other words, the MM negative result provides the first empirical evidence to the constancy of light postulate by Einstein.

3) **Is it possible to have particles that travel at the speed of light?**

ANS
Particle travelling at the speed of light would have an infinite mass, as per $m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$. Hence it is physically not possible to supply infinite amount of energy to boost a particle from rest to the speed of light.

4) **What is the twin-paradox? What is the solution to the paradox?**

ANS
Refer to page 43-44, Krane.

PROBLEMS

1) **Space Travel (from Cutnell and Johnson, pg 861,863)**
Alpha Centauri, a nearby star in our galaxy, is 4.3 light-years away. If a rocket leaves for Alpha Centauri and travels at a speed of v = 0.95c relative to the Earth, (i) by how much will the passengers have aged, according to their own clock, when they reach their destination? ii) What is the distance between Earth and Alpha Centauri as measured by the passengers in the rocket? Assume that the Earth and Alpha Centauri are stationary with respect to one another.

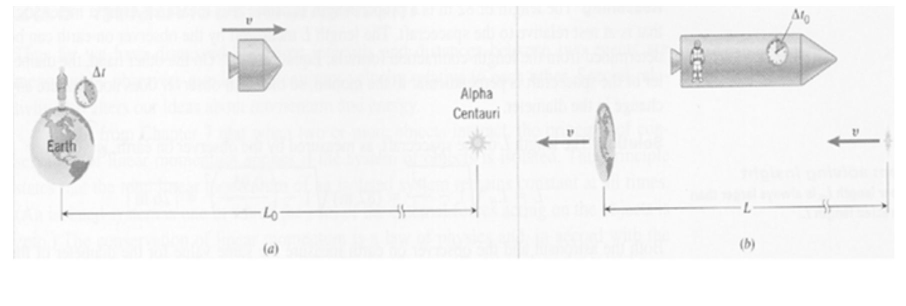


Figure: (a) As measured by an observer on the earth, the distance to Alpha Centauri is L_0, and the time required to make the trip is Δt. (b) According to the passenger on the spacecraft, the earth and Alpha Centauri move with speed v relative to the craft. The passenger measures the distance and time of the trip to be L and Δt_0 respectively, both quantities being less than those in part (a).

Reasoning

The two events in this problem are the departure from Earth and the arrival at Alpha Centauri. At departure, Earth is just outside the spaceship. Upon arrival at the destination, Alpha Centauri is just outside. Therefore, relative to the passengers, the two events occur at the same place - namely, 'just outside the spaceship. Thus, the passengers measure the proper time interval Δt_0 on their clock, and it is this interval that we must find. For a person left behind on Earth, the events occur at different places, so such a person measures the dilated time interval Δt rather than the proper time interval. To find Δt we note that the time to travel a given distance is inversely proportional to the speed. Since it takes 4.3 years to traverse the distance between earth and Alpha Centauri at the speed of light, it would take even longer at the slower speed of v = 0.95c. Thus, a person on earth measures the dilated time interval to be Δt = (4.3 years)/0.95 = 4.5 years. This value can be used with the time-dilation equation to find the proper time interval Δt_0.

Solution

Using the time-dilation equation, we find that the proper time interval by which the Passengers judge their own aging is
$\Delta t_0 = \Delta t\sqrt{(1-v^2/c^2)}$ = 4.5 years $\sqrt{(1-0.95^2)}$ = 1.4 years.

Thus, the people aboard the rocket will have aged by only 1.4 years when they reach Alpha Centauri, and not the 4.5 years an earthbound observer has calculated.

Both the earth-based observer and the rocket passenger agree that the relative speed between the rocket and earth is v = 0.95c. Thus, the Earth observer determines the distance to Alpha Centauri to be $L_0 = v\Delta t$ = (0.95c)(4.5 years) = 4.3 light-years. On the other hand, a passenger aboard the rocket finds the distance is only L = $v\Delta t_0$ = (0.95c)(1.4 years) = 1.3 light-years. The passenger, measuring the shorter time, also measures the shorter distance - length contraction.

Problem solving insight

In dealing with time dilation, decide which interval is the proper time interval as follows: (1) Identify the two events that define the interval. (2) Determine the reference frame in which the events occur at the same place; an observer at rest in this frame measures the proper time interval Δt_0.

2) **The Contraction of a Spacecraft (Cutnell, pg 863)**

An astronaut, using a meter stick that is at rest relative to a cylindrical spacecraft, measures the length and diameter of the spacecraft to be 82 and 21 m respectively. The spacecraft moves with a constant speed of v = 0.95c relative to the Earth. What are the dimensions of the spacecraft, as measured by an observer on Earth?

Reasoning

The length of 82 m is a proper length Lo since it is measured using a meter stick that is at rest relative to the spacecraft. The length L measured by the observer on Earth can be determined from the length-contraction formula. On the other hand, the diameter of the spacecraft is perpendicular to the motion, so the Earth observer does not measure any change in the diameter.

Solution

The length L of the spacecraft, as measured by the observer on Earth, is

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = 82m\sqrt{1-\frac{(0.95c)^2}{c^2}} = 26 \text{ m}$$

Both the astronaut and the observer on Earth measure the same value for the diameter of the spacecraft: Diameter = 21 m

Problem solving insight The proper length L_0 is always larger than the contracted length L.

3) ***Additional problem 36, Cutnell pg. 879.***
Two spaceship A and B are exploring a new planet. Relative to this planet, spaceship A has a speed of 0.60*c*, and spaceship B has a speed of 0.80*c*. What is the ratio D_A/D_B of the values for the planet's diameter that each spaceship measures in a direction that is parallel to its motion?

Solution

Length contraction occurs along the line of motion, hence both spaceship observe length contraction on the diameter of the planet. The contracted length measures by a moving observer is inversely proportional to the Lorentz factor γ. Hence,

$$\frac{L_A}{L_B} = \frac{\gamma_B}{\gamma_A} = \sqrt{\frac{1-\left(\frac{v_A}{c}\right)^2}{1-\left(\frac{v_B}{c}\right)^2}} = \sqrt{\frac{1-(0.6)^2}{1-(0.8)^2}} = 4/3.$$

4) **The Relativistic Momentum of a High-Speed Electron (Cutnell, pg 865)**
The particle accelerator at Stanford University is three kilometers long and accelerates electrons to a speed of 0.999 999 999 7*c*, which is very nearly equal to the speed of light. Find the magnitude of the relativistic momentum of an electron that emerges from the accelerator, and compare it with the non-relativistic value.

Reasoning and Solution

The magnitude of the electron's relativistic momentum can be obtained from $p = \gamma m_0 v = 1\times10^{-17}$ Ns, where $m_0 = 9.1\times10^{-31}$ kg, $v\gamma = \dfrac{0.999999997c}{\sqrt{1-\dfrac{(0.999999997c)^2}{c^2}}} = 1.09989\times10^{13}$ m/s. The relativistic momentum is greater than the non-relativistic momentum by a factor of $\gamma = \dfrac{1}{\sqrt{1-\dfrac{(0.999999997c)^2}{c^2}}} = 4\times10^4$.

5) **The Energy Equivalent of a Golf Ball (Cutnell, pg 866)**
A 0.046-kg golf ball is lying on the green. (a) Find the rest energy of the golf ball. (b) If this rest energy were used to operate a 75-W light bulb, for how many years could the bulb stay on?

Reasoning

The rest energy E_0 that is equivalent to the mass m of the golf ball is found from the relation $E_0 = mc^2$. The power used by the bulb is 75 W, which means that it consumes 75 J of energy per second. If the entire rest energy of the ball were available for use, the bulb could stay on for a time equal to the rest energy divided by the power.

Solution

(a) The rest energy of the ball is

$E_0 = mc^2 = (0.046 \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 = 4.1 \times 10^{15}$ J

(b) This rest energy can keep the bulb burning for a time t given by

t = Rest energy/ Power = 4.1×10^{15} J/75 W = 5.5×10^{13} s = 1.7 million years!

6) **A High-Speed electron (Cutnell pg. 867)**
An electron (mass = 9.1×10^{-31} kg) is accelerated to a speed of 0.9995*c* in a particle accelerator. Determine the electron's (a) rest energy, (b) total energy, and (c) kinetic energy in MeV

(a) $E_0 = mc^2 = 9.109\times10^{-31} kg \times (3\times10^8)^2 m/s = 8.19\times10^{-14} J = 0.51 MeV$

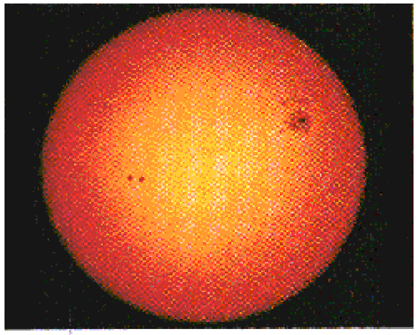
(b) Total energy of the traveling electron,

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0.51MeV}{\sqrt{1-0.995^2}} = 16.2MeV$$

(c) The kinetic energy = $E - E_0$ = 15.7 MeV

7) **The Sun Is Losing Mass (Cutnell, pg 868)**

The sun radiates electromagnetic energy at the rate of 3.92×10^{26} W. (a) What is the change in the sun's mass during each second that it is radiating energy? (b) The mass of the sun is 1.99×10^{30} kg. What fraction of the sun's mass is lost during a human lifetime of 75 years?



Visible light image.

Reasoning

Since a W = I J/s the amount of electromagnetic energy radiated during each second is 3.92×10^{26} J. Thus, during each second, the sun's rest energy decreases by this amount. The change ΔE_0 in the sun's rest energy is related to the change Δm in its mass by $\Delta E_0 = \Delta m\ c^2$.

Solution

(a) For each second that the sun radiates energy, the change in its mass is

$\Delta m = \Delta E_0/c^2 = 3.92\times10^{26}\ \text{J}/(3\times10^{8}\ \text{m/s})^2 = (4.36\times10^{9})$ kg.

Over 4 billion kilograms of mass are lost by the sun during each second.

(b) The amount of mass lost by the sun in 75 years is

$\Delta m = (4.36\times10^{9})\,\text{kg} \times (3.16\times10^{7}\ \text{s/year}) \times (75\ \text{years}) = 1\times10^{19}$ kg

Although this is an enormous amount of mass, it represents only a tiny fraction of the sun's total mass:

$$\Delta m/m = 1.0\times10^{19}\ \text{kg}/1.99\times10^{30}\ \text{kg} = 5.0\times10^{-12}$$

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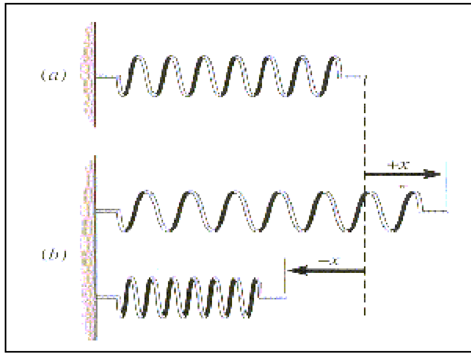
8) **Figure below shows the top view of a spring lying on a horizontal table. The spring is initially unstrained. Suppose that the spring is either stretched or compressed by an amount *x* from its unstrained length, as part (b) of the drawing shows. Has the mass of the spring changed? If so, is the change greater, smaller, or the same when the spring is stretched rather than compressed? (Cutnell, pg 868)**



(a) This spring is unstrained. (b) When the spring is either stretched or compressed by an amount *x*, it gains elastic potential energy and hence, mass.

Reasoning and Solution

Whenever a spring is stretched or compressed, its elastic potential energy changes. The elastic potential energy of an ideal spring is equal to $1/2kx^2$ where k is the spring constant and x is the amount of stretch or compression. Consistent with the theory of special relativity, any change in the total energy of a system, including a change in the elastic potential energy, is equivalent to a change in the mass of the system. Thus, the mass of a strained spring is greater than that of an unstrained spring. Furthermore, since the elastic potential energy depends on x^2, the increase in mass of the spring is the same whether it is compressed or stretched, provided the magnitude of x is the same in both cases. The increase is exceedingly small because the factor c^2 is so large.

9) The Speed of a Laser Beam (Cutnell, pg 871)
Figure below shows an intergalactic cruiser approaching a hostile spacecraft. The velocity of the cruiser relative to the spacecraft is v_{CS} = +0.7*c*. Both vehicles are moving at a constant velocity. The cruiser fires a beam of laser light at the enemy. The velocity of the laser beam relative to the cruiser is v_{LC} = +*c*. (a) What is the velocity of the laser beam v_{LS} relative to the renegades aboard the spacecraft? (b) At what velocity do the renegades aboard the spacecraft see the laser beam move away from the cruiser?

Figure 28.12 An intergalactic cruiser, closing in on a hostile spacecraft, fires a beam of laser light.

Reasoning and Solution

(a) Since both vehicles move at a constant velocity, each constitutes an inertial reference frame. According to the speed of light postulate, all observers in inertial reference frames measure the speed of light in a vacuum to be *c*. Thus, the renegades aboard the hostile spacecraft see the laser beam travel toward them at the speed of light, even though the beam is emitted from the cruiser, which itself is moving at seven-tenths the speed of light.

More formally, we can use Lorentz transformation of velocities to calculate v_{LS}. We will take the direction as +ve when a velocity is pointing from left to right. We can take view that the hostile spacecraft is at rest (as the stationary frame, O) while the cruiser is approaching it with velocity v_{CS} = + 0.7c (according to our choice of the sign). In this case, the cruiser is the moving frame, O'. The light beam as seen in the moving frame O' is v_{LC} = +c. We wish to find out what is the speed of this laser beam from O point of view, e.g. what v_{LS} is.

We may like to identify v_{LS}, v_{LC} and v_{CS} with the definitions used in the Lorentz formula: $u_x = \dfrac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}}$. In fact, a little contemplation would allow us to make the identification that, with our choice of frames (that the hostile spacecraft as the stationary frame): $v_{LC} \equiv u_{x'}$ = +c; $v_{CS} \equiv u$ = + 0.7c and $v_{LS} = u_x$ = the speed of laser beam as seen by the stationary frame O (the quantity we are seeking). Hence, we have

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$$u_x = \frac{u'_x + u}{1 + \dfrac{u'_x u}{c^2}} \equiv v_{LS} = \frac{v_{LC} + v_{CS}}{1 + \dfrac{v_{LC} v_{CS}}{c^2}} = \frac{(+c) + (+0.7c)}{1 + \dfrac{(+c)(+0.7c)}{c^2}} = \frac{1.7c}{1.7c} = +c$$

, i.e. the laser beam is seen, from the view point of the hostile spacecraft, to be approaching it with a velocity +c (+ve means the velocity is from left to right).

(b) The renegades aboard the spacecraft see the cruiser approach them at a relative velocity of v_{CS} = +0.7c, and they also see the laser beam approach them at a relative velocity Of v_{LS} +c. Both these velocities are measured relative to the same inertial reference frame-namely, that of the spacecraft. Therefore, the renegades aboard the spacecraft see the laser beam move away from the cruiser at a velocity that is the difference between these two velocities, or +*c* - (+0.7*c*) = +0.3*c*. The relativistic velocity-addition formula, is not applicable here because both velocities are measured relative to the same inertial reference frame (the spacecraft's reference frame). The relativistic velocity-addition formula can be used only when the velocities are measured relative to different inertial reference frames.

10) **Mass and Energy (Cutnell, pg 873)**
The rest energy E_0 and the total energy *E* of three particles, expressed in terms of a basic amount of energy *E'* = 5.98 × 10^{-10} J, are listed in the table below. The speeds of these particles are large, in some cases approaching the speed of light. For each particle, determine its mass and kinetic energy.

Particle	Rest Energy	Total Energy
a	E'	2E'
b	E'	4E'
c	5E'	6E'

Concept Questions and Answers

Given the rest energies specified in the table, what is the ranking (largest first) of the masses of the particles?

Answer

The rest energy is the energy that an object has when its speed is zero. According to special relativity, the rest energy E_0 and the mass *m* are equivalent. Thus, the rest energy is directly proportional to the mass. From the table it can be seen that particles a and b have identical rest energies, so they have identical masses. Particle c has the greatest rest energy, so it has the greatest mass. The ranking of the masses, largest first, is c, then a and b.

What is the ranking (largest first) of the kinetic energies of the particles?

According to special relativity, the kinetic energy is the difference between the total energy E and the rest energy E_0, so KE = $E - E_0$. Therefore, we can examine the table and determine the kinetic energy of each particle in terms of E'. The kinetic energies of particles a, b, and c are, respectively, $2E' - E' = E'$, $4E' - E' = 3E'$, and $6E' - 5E' = E'$. The ranking of the kinetic energies, largest first, is b, then a and c.

Solution

(a) The mass of particle a can be found from its rest energy $E_0 = mc^2$. Since $E_0 = E'$ (see the table), its mass is $m_a = E'/c^2 = 5.98\text{x}10^{-10}$ J/(3x10^8 m/s)^2 = $6.64\text{x}10^{-27}$ kg

In a similar manner, we find that the masses of particles b and c are

$$m_b = 6.64\text{x}10^{-27}\text{ kg},\ m_c = 33.2\text{x}10^{-27}\text{ kg},$$

As expected, the ranking is $m_c > m_a = m_b$

(b) The kinetic energy KE of a particle is KE = $E - E_0$. For particle a, its total energy is $E = 2E'$ and its rest energy is $E_0 = E'$, so its kinetic energy is

$$KE_a = 2E' - E' = E' = 5.98\text{x}10^{-10}\text{ J}.$$

The kinetic energies of particles b and c can be determined in a similar fashion:

$$KE_b = 17.9\text{x}10^{-10}\text{ J},\ KE_c = 5.98\text{x}10^{-10}\text{ J}$$

As anticipated, the ranking is $KE_b > KE_a = KE_c$.

Tutorial 2
Preliminaries, Blackbody radiation, particle nature of waves

Conceptual Questions

1. **Explain in your own words the essential differences between the concept of wave from that of particle (Own question)**

ANS
Particle is finite in size and is localised both in space and time, whereas wave is not.

2. **What is ultraviolet catastrophe? What is the significance of it in the development of modern physics? (Own question)**

ANS
The classical theory explanation of the blackbody radiation by Rayleigh-Jeans fails in the limit $\lambda \to 0$ (or equivalently, when frequency $\to \infty$), i.e. $R(\lambda) \to \infty$ at $\lambda \to 0$. The failure prompted Planck to postulate that the energy of electromagnetic waves is quantised (via $\varepsilon = h\nu$) as opposed to the classical thermodynamics description ($\varepsilon = kT$). With Planck's postulate, radiation now has particle attributes instead of wave.

3. **What is the significance of the Compton wavelength of a given particle? What does the Compton wavelength of a particle mean to light that interacts with it? (Own question)**

ANS
The Compton wavelength (a characteristic constant depend solely on the mass of a given particle) characterises the length scale at which the quantum property (or wave) of a given particle starts to show up. In an interaction that is characterised by a length scale larger than the Compton wavelength, particle behaves classically. For interaction that occurs at a length scale comparable or smaller than the Compton wavelength, the quantum (or, wave) nature starts of the particle begins to take over from classical physics.

In a light-particle interaction, if the wavelength of the light is comparable to the Compton wavelength of the interacting particle, light displays quantum (granular/particle) behaviour rather than as a wave.

4. **How does the Rayleigh scattering could be explained by the Compton scattering relation, $\Delta\lambda = \lambda_c(1-\cos\theta)$? In the γ-ray region, which effect, Compton scattering or Rayleigh scattering is dominant? Explain. (Own question)**

ANS
Rayleigh scattering refers to unresolved peaks of the scattered x-ray, ie. $\Delta\lambda = 0$, which is due to the extremely small Compton wavelength of the whole ATOM, as seen by the x-ray $\lambda_c = h/Mc \to 0$, where M = mass of the atom (instead of $m_e << M$).

5. **Why doesn't the photoelectric effect work for free electron? (Krane, Question 7, pg 79)**

ANS (to be verified)
Essentially, Compton scattering is a two-body process. The free electron within the target sample (e.g. graphite) is a unbounded elementary particle having no internal structure that allows the photons to be `absorbed'. Only elastic scattering is allowed here.

Whereas PE effect is a inelastic scattering, in which the absorption of a whole photon by the atom is allowed due to the composite structure (the structure here refers the system of the orbiting electrons and nuclei hold together via electrostatic potential) of the atom. A whole photon is allowed to get absorbed by the atom in which the potential energy acts like a medium to transfer the energy absorbed from the photon, which is then `delivered' to the bounded electrons (bounded to the atoms) that are then `ejected' out as photoelectrons.

6. **How is the wave nature of light unable to account for the observed properties of the photoelectric effect? (Krane, Question 5, pg 79)**

ANS
See lecture notes

7. **In the photoelectric effect, why do some electrons have kinetic energies smaller than K_{max}? (Krane, Question 6, pg 79)**

ANS
By referring to $K_{max} = h\nu - \phi$, K_{max} corresponds to those electrons knocked loose from the surface by the incident photon whenever $h\nu > \phi$. Those below the surface required an energy greater than ϕ and so come off with less kinetic energy.

Problems

1. **The diameter of an atomic nucleus is about 10×10^{-15} m. Suppose you wanted to study the diffraction of photons by nuclei. What energy of photons would you choose? Why? (Krane, Question 1, pg 79)**

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ANS
Diffraction of light by the nucleus occurs only when the wavelength of the photon is smaller or of the order of the size of the nucleus, $\lambda \sim D$ (D = diameter of the nucleus). Hence, the minimum energy of the photon would be $E = hc/\lambda \sim hc/D \sim 120$ MeV.

2. **How does the total intensity of thermal radiation vary when the temperature of an object is doubled? (Krane, Question 4, pg 79)**

ANS
Intensity of thermal radiation $I \propto T^4$. Hence, when T is double, ie. $T \to 2T$, $I \to I'(2)^4 = 16I$, i.e. the total intensity of thermal radiation increase by 16 times.

3. **Photons from a Light Bulb (Cutnell, pg884) In converting electrical energy into light energy, a sixty-watt incandescent light bulb operates at about 2.1% efficiency. Assuming that all the light is green light (vacuum wavelength 555 nm), determine the number of photons per second given off by the bulb.**

Reasoning
The number of photons emitted per second can be found by dividing the amount of light energy emitted per second by the energy E of one photon. The energy of a single photon is $E = hf$. The frequency of the photon is related to its wavelength λ by $\nu = c/\lambda$.

Solution
At an efficiency of 2. 1%, the light energy emitted per second by a sixty-watt bulb is (0.021)(60.0 J/s)=1.3 J/s. The energy of a single photon is

$$E = hc/\lambda$$
$$= (6.63\times10^{-34}\text{Js})(3\times10^{8}\text{ m/s})/555\times10^{-9}\text{ nm} = 3.58\times10^{-19}\text{ J}$$

Therefore,
Number of photons emitted per second =
1.3 J/s/ 3.58×10^{-19} J/photon = 3.6×10^{18} photon per second

4. **Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (a) Find the maximum KE of the photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2? (Beiser, pg. 63)**

(a) The energy of the photons is, $E_P = hc/\lambda$ = 3.5eV. The work function of potassium is 2.2 eV. So, KE = $h\nu - \phi$ = 3.5 eV - 2.2 eV = 5.68×10^{-19} J
(b) The photon energy in joules is 5.68×10^{-19} J. Hence

the number of photons that reach the surface per second is

$n_p = (E/t)/E_p = (E/A)(A)/E_p$
$= (1.00\ \mathrm{W/m^2})(1.00\times10^{-4}\ \mathrm{m^2})/5.68\times10^{-19}\ \mathrm{J}$
$= 1.76\times10^{14}$ photons/s

The rate at which photoelectrons are emitted is therefore

$n_e = (0.0050)n_p = 8.8\times10^{11}$ photoelectrons/s

5. **(Krane, pg. 62)**
(a) At what wavelength does a room-temperature (T = 20°C) object emit the maximum thermal radiation?
(b) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum?
(c) How many times as much thermal radiation does it emit at the higher temperature?

ANS

(a) Converting to absolute temperature, T = 293 K, and from Wien's displacement law,

$\lambda_{max}T = 2.898\times10^{-3}\ \mathrm{m\cdot K}$

$\lambda_{max} = 2.898\times10^{-3}\ \mathrm{m\cdot K}/\ 293\mathrm{K} = 9.89\ \mu\mathrm{m}$

(b) Taking the wavelength of red light to be =650 nm, we again use Wien's displacement law to find T:

$T = 2.898\times10^{-3}\ \mathrm{m\cdot K}/650\times10^{-9}\ \mathrm{m} = 4460\ \mathrm{K}$

(c) Since the total intensity of radiation is proportional to T^4, the ratio of the total thermal emissions will be

$$\frac{T_2^4}{T_1^4} = \frac{4460^4}{293^4} = 5.37\times10^4$$

Be sure to notice the use of absolute (Kelvin) temperatures.

6. **The work function for tungsten metal is 4.53 eV. (a) What is the cut-off wavelength for tungsten? (b) What is the maximum kinetic energy of the electrons when radiation of wavelength 200.0 nm is used? (c) What is the stopping potential in this case? (Krane, pg. 69)**

ANS

(a) The cut-off frequency is given by

$\lambda_c = \frac{hc}{\phi} = \frac{1240\mathrm{eV\cdot nm}}{200\mathrm{nm}} = 274\mathrm{nm}$, in the uv region

(b) At the shorter wavelength,

$$K_{max} = h\frac{c}{\lambda} - \phi = \frac{1240eV\cdot nm}{200nm} - 4.52eV = 1.68eV$$

(c) The stopping potential is just the voltage corresponding to K_{max}:

$$V_s = K_{max}/e = \frac{1.68eV}{e} = 1.68\,\mathrm{V}$$

7. **X-rays of wavelength 10.0 pm (1 pm = 10^{-12} m) are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons. (Beiser, pg. 75)**

Solution

(a) The Compton shift is given by

$\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\varphi)$, and so

$\lambda' = \lambda + \lambda_c(1-\cos 45^o) = 10.0\,pm + 0.293\lambda_c = 10.7$ pm

(b) $\Delta\lambda$ is a maximum when $1-\cos\varphi$ = 2, in which case,

$\Delta\lambda = \lambda + 2\lambda_c$ = 10.0 pm + 4.9 pm = 14.9 pm

(C) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$KE_{max} = h(\nu - \nu') = hc(\frac{1}{\lambda} - \frac{1}{\lambda'}) = 40.8$ eV

Tutorial 3
Matter waves, The Uncertainty Principle and Schrodinger Equation

Conceptual Questions

1. **What difficulties does the uncertainty principle cause in trying to pick up an electron with a pair of forceps? (Krane, Question 4, pg. 110)**

ANS
When the electron is picked up by the forceps, the position of the electron is ``localised' (or fixed), i.e. $\Delta x = 0$. Uncertainty principle will then render the momentum to be highly uncertainty. In effect, a large $\Delta p$ means the electron is ``shaking'' furiously against the forceps' tips that tries to hold the electron ``tightly''.

2. **Is it possible for v_{phase} to be greater than c? Can v_{group} be greater than c? (Krane, Question 12, pg. 111)**

ANS
Is it possible for v_{phase} to be greater than c but not so for v_{group}. This is because the group velocity is postulated to be associated with the physical particle. Since a physical particle (with mass) can never move greater than the speed of light (according to SR), so is v_{group}.

3. **Why is it important for a wave function to be normalised? Is an unrenomalised wave function a solution to the Schrodinger equation? (Krane, Question 2, pg. 143)**

ANS
Due to the probabilistic interpretation of the wave function, the particle must be found within the region in which it exists. Statistically speaking, this means that the probability to find the particle in the region where it exists must be 1. Hence, the square of the wave function, which is interpreted as the probably density to find the particle at an intervals in space, integrated over all space must be one in accordance with this interpretation. Should the wave function is not normalised, that would lead to the consequence that the probability to find the particle associated with the wave function in the integrated region where the particle is suppose to be in is not one, which violates the probabilistic interpretation of the wave function.

A wave function that is not normalised is also a solution to the Schrodinger equation. However, in order for the wave function to be interpreted in accordance to the probabilistic interpretation (so that the wave function could has a physical meaning) it must be normalised.

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4. How would the solution to the infinite potential well be different if the width of the well is extended from L to $L + x_0$, where x_0 is a nonzero value of x? How would the energies be different? **(Krane, Question 7, pg. 143)**

ANS
The form of the solutions to the wave functions inside the well remains the same. They still exist as stationary states described by the same sinusoidal functions, except that in the expressions of the observables, such as the quantised energies and the expectation values, the parameter L be replaced by $L + x_0$. For the quantised energies, they will be modified as per

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \rightarrow \frac{n^2\pi^2\hbar^2}{2m(L+x_0)^2}.$$

5. The infinite quantum well, with width L, as defined in the lecture notes is located between $x = 0$ and $x = L$. If we define the infinite quantum well to be located between $x = -L/2$ to $x = +L/2$ instead (the width remains the same, L), find the solution to the time-independent Schrodinger equation. Would you expect the normalised constant to the wave function and the energies be different than that discussed in the notes? Explain. **(Brehm and Mullin, pg. 234 - 237)**

ANS
By applying the boundary conditions that the solution must vanish at both ends, i.e. $\psi(x=-L/2)=\psi(x=L/2)=0$, the solution takes the form

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\cos\frac{n\pi x}{L} \ (odd\ \ n) \\ \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \ (even\ n) \end{cases} \text{ for } -\frac{L}{2} \le x \frac{L}{2}$$

This question is tantamount to re-analyse the same physical system in a shifted coordinates, $x \rightarrow x - L/2$. The normalisation and energies shall remain unchanged under the shift of coordinate system $x \rightarrow x - L/2$. Both of these quantities depends only on the width of the well but not on the coordinate system used.

Problems

1. **Find the de Broglie wave lengths of (a) a 46-g ball with a velocity of 30 m/s, and (b) an electron with a velocity of 10^7 m/s (Beiser, pg. 92)**

ANS

(a) Since $v << c$, we can let $m = m_o$. Hence

$\lambda = h/mv = 6.63\times10^{-34}$ Js/(0.046 kg)(30 m/s)
$= 4.8\times10^{-34}$ m

The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behaviour.

(b) Again $v << c$, so with $m = m_o = 9.1\times10^{-31}$ kg, we have

$\lambda = h/mv = 6.63\times10^{-34}$ Js/(9.1×10^{-31} kg)(10^7 m/s)
$= 7.3\times10^{-11}$ m

The dimensions of atoms are comparable with this figure - the radius of the hydrogen atom, for instance, is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behaviour.

2. **The de Broglie Wavelength (Cutnell, pg. 897)**
An electron and a proton have the same kinetic energy and are moving at non-relativistic speeds. Determine the ratio of the de Broglie wavelength of the electron to that of the proton.

ANS

Using the de Broglie wavelength relation $p = h/\lambda$ and the fact that the magnitude of the momentum is related to the kinetic energy by $p = (2mK)^{1/2}$, we have

$$\lambda = h/p = h/(2mK)^{1/2}$$

Applying this result to the electron and the proton gives

$$\lambda_e/\lambda_p = (2m_pK)^{1/2}/(2m_eK)^{1/2}$$
$$= (m_p/m_e)^{1/2} = (1.67\times10^{-27}\text{ kg}/9.11\times10^{-31}\text{ kg})^{1/2} = 42.8$$

As expected, the wavelength for the electron is greater than that for the proton.

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3. **Find the kinetic energy of a proton whose de Broglie wavelength is 1.000 fm = 1.000×10^{-15} m, which is roughly the proton diameter (Beiser, pg. 92)**

ANS

A relativistic calculation is needed unless pc for the proton is much smaller than the proton rest mass of E_o = 0.938 GeV.

So we have to first compare the energy of the de Broglie wave to E_o:

$E = pc = \dfrac{hc}{\lambda} = \dfrac{1242 eV\cdot nm}{10^{-6} nm} = 1.24$ GeV, c.f. E_o = 0.938 GeV. Since the energy of the de Broglie wave is larger than the rest mass of the proton, we have to use the relativistic kinetic energy instead of the classical $K = p^2/2m$ expression.

The total energy of the proton is

$E = \sqrt{E_0^2 + (pc)^2} = \sqrt{(0.938\,GeV)^2 + (1.24\,GeV)^2} = 1.555$ GeV.

The corresponding kinetic energy is

KE = $E - E_o$ = (1.555 - 0.938) GeV = 0.617 GeV = 617 MeV

4. **An electron is in a box 0.10 nm across, which is the order of atomic dimensions. Find its permitted energies. (Beiser, pg. 106)**

ANS

Here $m = 9.1\times10^{-31}$ kg and $L = 1\times10^{-10}$ m, so that the permitted electron energies are

$E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2} = 6.0\times10^{-18} n^2$ J $= 38n^2$ eV.

The minimal energy the electron can have is 38 eV, corresponding to $n = 1$. The sequence of energy levels continues with E_2 = 152 eV, E_3 = 342 eV, E_4 = 608 eV and so on. If such a box existed, the quantisation of a trapped electron's energy would be a prominent feature of the system. (And indeed energy quantisation is prominent in the case of an atomic electron.)

5. **A 10-g marble is in a box 10 cm across. Find its permitted energies.**

ANS

With m = 1.0×10^{-2} kg and L = 1.0×10^{-1} m,

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = 5.5\times10^{-64}n^2 \text{ J}$$

The minimum energy the marble can have is 5.5×10^{-64} J, corresponding to n = 1. A marble with this kinetic energy has a speed of only 3.3×10^{-31} m/s and therefore cannot be experimentally distinguished from a stationary marble. A reasonable speed a marble might have is, say, 1/3 m/s - which corresponds to the energy level of quantum number n = 10^{30}! The permissible energy levels are so very close together, then, that there is no way to determine whether the marble can take on only those energies predicted by $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ or any energy whatever. Hence in the domain of everyday experience, quantum effects are imperceptible, which accounts for the success of Newtonian mechanics in this domain.

6. **A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom. (Beiser, pg 114)**

ANS

Here we find that with Δx = 5.3×10^{-11} m.

$\Delta p \geq \frac{\hbar}{2\pi}$ = 9.9×10^{-25} Ns.

An electron whose momentum is of this order of magnitude behaves like a classical particle, an its kinetic energy is $K = p^2/2m \geq (9.9\times10^{-25}\text{ Ns})^2/2\times9.110^{-31}$ kg = 5.4×10^{-19} J which is 3.4 eV. The kinetic energy of an electron in the lowest energy level of a hydrogen atom actually 13.6 eV.

7. **A measurement established the position of a proton with an accruracy of $\pm1.00\times10^{-11}$m. Find the uncertainty in the proton's position 1.00 s later. Assume $v \ll c$. (Beiser, pg. 111)**

ANS

Let us call the uncertainty in the proton's position Δx_0 at the time t = 0. The uncertainty in its momentum at this time is therefore

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$\Delta p \geq \frac{\hbar}{2\Delta x_0}$. Since $v \ll c$, the momentum uncertainty is $\Delta p \geq \Delta(mv) = m_0\Delta v$ and the uncertainty in the proton's velocity is $\Delta v \geq \frac{\Delta p}{m_0} \geq \frac{\hbar}{2m_0\Delta x_0}$. The distance x of the proton covers in the time t cannot be known more accurately than

$\Delta x \geq t\Delta v \geq \frac{\hbar t}{2m_0\Delta x_0}$. Hence Δx is inversely proportional to Δx_0: the more we know about the proton's position at t = 0 the les we know about its later position at t. The value of Δx at t = 1.00 s is

$$\Delta x \geq \frac{(1.054\times10^{-34}Js)(1.00s)}{2(1.672\times10^{-27}kg)(1.00\times10^{-11}m)} = 3.15\times10^3\text{ m}$$

This is 3.15 km! What has happened is that the original wave group has spread out to a much wider one because the phase velocities of the component wave vary with wave number and a large range of wave numbers must have been present to produce the narrow original wave

8. **Broadening of spectral lines due to uncertainty principle: An excited atom gives up it excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time is radiates is 1.0×10^{-8} s. Find the inherent uncertainty in the frequency of the photon. (Beiser, pg. 115)**

ANS

The photon energy is uncertain by the amount

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054\times10^{-34}Js}{2(1.0\times10^{-8}s)} = 5.3\times10^{-27}\text{ J}$$

The corresponding uncertainty in the frequency of light is $\Delta\nu = \frac{\Delta E}{h} \geq 8\times10^6$ Hz.

This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom. As a result, the radiation from a group of excited atoms does not appear with the precise frequency ν. For a photon whose frequency is, say, 5.0×10^{14} Hz, $\frac{\Delta\nu}{\nu} = 1.6\times10^{-8}$. In practice, other phenomena such as the doppler effect contribute more ian this to the broadening of spectral lines.

9. **If we assume that in the ground of the hydrogen the position of the electron along the Bohr orbit is not known and not knowable, then the uncertainty in the position is about $\Delta x \approx 2r_0 = 10^{-10}$ m, (a) What is the magnitude of the momentum of the electron at the ground state? (b) What is the corresponding quantum uncertainty in the momentum? (Ohanian, pg. 152)**

ANS
(a) Angular momentum, $|L| \equiv |p|r = n\hbar$. Hence, in the ground state, $|p| = \hbar / r_0 = 2.1\times10^{-24}$ Ns

(b) $\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2(2r_0)} = 5.3\times10^{-25}$ Ns.

10. **Show that $\psi = A\exp(kx - \omega t)$ is solution to the time-independent Schrodinger equation.**

ANS
Taking the partial derivative of ψ wrp to x,

$$\frac{\partial^2}{\partial x^2}\psi = (ik)^2 A\exp(kx-\omega t) = -k^2\psi . \qquad (1)$$

The total energy of the particle is

$E = K + U = p^2/2m + U = \frac{\hbar^2 k^2}{2m} + U$

$\Rightarrow k^2 = \frac{2m(E-U)}{\hbar^2}$.

Hence, Eq. (1) becomes $\frac{\partial^2}{\partial x^2}\psi = -\frac{2m(E-U)}{\hbar^2}\psi$. This shows that $\psi = A\exp(kx-\omega t)$ is the solution to the Schrodinger equation.

11. **Consider a quantum particle trapped in an infinite well with width *a*. Assuming that the particle is in the ground state, calculate the expectation values of its position <x> and <x²>. Obtain the uncertainty in its position, Δx, given**

by standard statistical definition, Δx = <x²> - <x>². (Brehm and Mullin, pg.265)

ANS
The solution of the ground state wave function for a particle in an infinite box is $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$.

$$\langle x\rangle = \int_{-\infty}^{\infty}\psi x\psi dx = \frac{2}{a}\int_0^a x\sin^2\frac{\pi x}{a}dx = \frac{2a}{\pi^2}\int_0^\pi y\sin^2 y dy$$

$$\int_0^\pi y\sin^2 y dy = \frac{y^2}{4} - \frac{y\sin 2y}{4} - \frac{\cos 2y}{8}\bigg|_0^\pi = \frac{\pi^2}{4}$$

$\therefore \langle x\rangle = \frac{a}{2}$. Likewise,

$$\langle x^2\rangle = \int_{-\infty}^{\infty}\psi x^2\psi dx = \frac{2}{a}\int_0^a x^2\sin^2\frac{\pi x}{a}dx = \frac{2a^2}{\pi^3}\int_0^\pi y^2\sin^2 y dy$$

$$\int_0^\pi y^2\sin^2 y dy = \frac{x^3}{6} - \frac{x\cos 2x}{4} + \frac{(1-2x^2)\sin 2x}{8}\bigg|_0^\pi = \frac{\pi^3}{6} - \frac{\pi}{4}$$

$$\therefore \langle x^2\rangle = \int_{-\infty}^{\infty}\psi x^2\psi dx = a^2\left(\frac{1}{3} - \frac{1}{2\pi^2}\right)$$

Δx = <x²> - <x>² = $a^2\left(\frac{1}{3} - \frac{1}{2\pi^2}\right) - \frac{a^2}{4} = a^2\left(\frac{1}{12} - \frac{1}{2\pi^2}\right)$

Tutorial 4
Atomic model

Conceptual Questions

1. **What is the ONE essential difference between the Rutherford model and the Bohr's model? (Own question)**

ANS
Rutherford's model is a classical model, in which EM wave will be radiated rendering the atom to collapse. Whereas the Bohr's model is a semi-classical model in which quantisation of the atomic orbit happens.

2. **Conventional spectrometers with glass components do not transmit ultraviolet light ($\lambda \lesssim$ 380 nm). Explain why non of the lines in the Lyman series could be observed with a conventional spectrometer. (Taylor and Zafiratos, pg. 128)**

ANS
For Lyman series, n_f = 1. According to $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, the wavelength corresponding to n_i = 2 in the Lyman series is predicted to be $\lambda = \frac{4}{3R_\infty} = \frac{4}{3(109{,}737\text{cm}^{-1})}$=121.5 nm. Similarly, for n_i = 3, one finds that λ=102 nm, and inspection of $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ shows that the larger we take n, the smaller the corresponding wavelength. Therefore, all lines in the Lyman series lie well into the ultraviolet and are unobservable with a conventional spectrometer.

3. **Does the Thompson model fail at large scattering angles or at the small scattering angle? Why? (Krane, Questions 1, pg. 173)**

ANS
Thompson model fails at large angle (but is consistent with scattering experiments at small angle). Thompson model predicts that the average scattered angle is given by $\theta_{ave} = \sqrt{N} \cdot \frac{\pi}{4} \cdot z\left(\frac{Ze^2}{4\pi\varepsilon_0 R^3}\right) R^2 \cdot \left(\frac{1}{mv^2}\right)$. One can estimate the order of θ_{ave} in an atomic scattering experiment: R ~ 0.1 nm (a typical atomic radius), N ~ 10^4 (no. of collisions in the target metal foil), kinetic energy of the alpha particle, mv^2 ~ 10 MeV, z = 2 (charge of alpha particle); Z ~ 79 for gold. Putting in all figures, one expects that alpha particle is scattered only for a small scattering angle of θ_{ave} ~ 1°. However, in the experiment, alpha particles are observed to be scattered at angle in excess of 90°. This falsifies Thompson model at large angle.

4. **In which Bohr orbit does the electron have the largest velocity? Are we justified in treating the electron non-relativistically? (Krane, Questions 6. pg. 174)**

ANS
The velocity in an orbit n is given by $v = h/2\pi mnr_0$, which means that the velocity is inversely proportional to the n number. Hence the largest velocity corresponds to the n = 1 state,

$v(n=1)/c = h/2c\pi mr_0$
$= 6.63\times10^{-34}/2\pi(9.1\times10^{-31})(0.53\times10^{-10})/c$
$= 0.007.$

Hence, nonrelativistic treatment is justified.

5. **How does a Bohr atom violate the $\Delta x \Delta p \geq \frac{\hbar}{2}$ uncertainty relation? (Krane, Question 11, pg. 174)**

ANS
The uncertainty relation in the radial direction of an electron in a Bohr orbit is $\Delta r \Delta p_r \geq \frac{\hbar}{2}$. However, in the Bohr model, the Bohr orbits are assumed to be precisely known $(= r_n = n^2 r_0)$ for a given n. This tantamount to $\Delta r = 0$, which must render the momentum in the radial direction to become infinite. But in the Bohr atom the electron does not have such radial motion caused by this uncertainty effect. So in this

sense, the discrete Bohr orbit violates the uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2}$.

Problem

1. **Hydrogen atoms in states of high quantum number have been created in the laboratory and observed in space. (a) Find the quantum number of the Bohr orbit in a hydrogen atom whose radius is 0.0199 mm. (b) What is the energy of a hydrogen atom in this case? (Beiser, pg. 133)0**

ANS

(a) From $r_n = n^2 r_0$, we have $n = \sqrt{\frac{r_n}{r_0}} = \sqrt{\frac{0.0100\times10^{-3}}{5.3\times10^{-11}}} = 434$

(b) From $E_n = -\frac{13.6}{n^2}$eV, we have $E_n = -\frac{13.6}{434^2}$eV = -0.000072 eV. Such an atom would obviously be extremely fragile and be easily ionised (compared to the kinetic energy of the atom at temperature T, kT ~ $(1.38\times10^{-23}$ J/K$)\times(300$ K$)$ =0.03 eV)

2. **(a) Find the frequencies of revolution of electrons in n = 1 and n = 3 Bohr orbits. (b) What is the frequency of the photon emitted when an electron in the n = 2 orbit drops to an n = 1 orbit? (c) An electron typically spends about 10^{-8} s in an excited state before it drops to a lower state by emitting a photon. How many revolutions does an electron in an n = 2 Bohr orbit make in 10^{-8} s? (Beiser, pg. 137)**

ANS

(a) Derive the frequency of revolution from scratch: Forom Bohr's postulate of quantisation of angular momentum,

$L = (mv)r = nh/2\pi$, the velocity is related to the radius as $v = nh/2mr\pi$. Furthermore, the quantised radius is given in terms of Bohr's radius as $r_n = n^2 r_0$. Hence, $v = h/2\pi mnr_0$.

The frequency of revolutionm $f = 1/T$ (where T is the period of revolution) can be obtained from $v = 2\pi r/T = 2\pi n^2 r_0 f$. Hence, $f = v/2\pi r = (h/2\pi mnr_0)/2\pi r = h/4\pi^2 mn^3(r_0)^2$.

For n = 1, $f_1 = h/4\pi^2 m(r_0)^2 = 6.56\times10^{15}$ Hz.
For n = 2, $f_2 = h/4\pi^2 m(2)^3(r_0)^2 = 6.56\times10^{15}/8$ Hz = 8.2×10^{14}.

(b)
$$\nu = \frac{\Delta E}{h} = \frac{13.6eV}{h}\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3c}{4}\frac{13.6eV}{1242eV\cdot nm} = 0.00821\times(3\times10^8\,m/s)/10^{-9}m =$$
2.463×10^{15}s. The frequency is intermediate between f_1 and f_2.

(c) The number of revolutions the electron makes is $N = f_2\Delta t = (8.2\times10^{14})\times10^8 = 8.2\times10^{22}$ rev.

3. **Consider a positronium atom consisting of a positron and electron revolving about their common centre of mass, which lies halfway between them. (a) If such a system were a normal atom, how would its emission spectrum compared to that of hydrogen atom? (b) What would be the electron-positron separation, *r*, in the ground state orbit of positronium? (Eisberg, pg. 106)**

ANS
(a) The emission spectrum is described by the general form of $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$, where $R \equiv \frac{\mu e^4}{4c\pi\hbar^3(4\pi\varepsilon_0)^2}$, the reduced mass of the positronium is $\mu = \frac{mM}{M+m} = \frac{m_e\cdot m_e}{m_e+m_e} = \frac{m_e}{2}$. Compared to the emission spectrum of hydrogen, which is given by $\frac{1}{\lambda_H} = Z^2 R_\infty\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$. Hence we have

$\frac{\lambda_{positronium}}{\lambda_H} = \frac{R_\infty}{R_{positronium}} = \frac{m_e}{\mu_{positronium}} = 2$. That is, the spacing between the spectral lines in the positronium is doubled as compared to the corresponding spacing in that of the hydrogen.

(b) The ground state radius is

$$r_0(positronium) = \frac{4\pi\hbar^2\varepsilon_0}{Ze^2\mu} = 2\left(\frac{4\pi\hbar^2\varepsilon_0}{e^2 m_e}\right) = 2r_0$$

4. **Ordinary hydrogen atom contains about one part in 6000 of deuterium, or heavy hydrogen. This is a hydrogen atom whose nucleus contains a proton and a neutron. How does the doubled nuclear mass affect the atomic spectrum? (Eisberg, pg 102)**

ANS

The reduced mass is $\mu = \frac{mM}{M+m} = \frac{m_e \cdot 2M}{2M+m}$. The numerical ratio $\frac{\lambda_d}{\lambda_H} = \frac{R_\infty}{R_d} = \frac{m_e}{\mu_d} = m_e \frac{2M+m_e}{m_e \cdot 2M} = \frac{2M+m_e}{2M} \approx m_e$ is the same for both limits $2M >> m$ (for deuterium) or $M >> m$ (for hydrogen). Hence the double nuclear mass does not affect the atomic spectrum in a significant sense. To be more quantitative, the ratio $\frac{m_e}{\mu_d} = \frac{2M+m_e}{2M} = \frac{2(934\text{MeV})+(0.51\text{MeV})}{2(934\text{MeV})} = 1.0003$. The nuclear mass to the atomic spectrum only cases a 0.03% shift to the wavelengths of the spectral lines.

5. A muonic atom contains a nucleus of charge *e* and a negative muon, μ^-, moving about it. The μ^- is an elementary particle with charge $-e$ and a mass 207 times as large as an electron. (a) Calculate the biding energy of the muonic atom. (b) What is the wavelength of the first line in the Lyman series for such an atom? (Eisberg, pg. 106)

ANS

(a) $\mu = \frac{mM}{M+m} = \frac{m_\mu \cdot m_\mu}{m_\mu + m_\mu} = \frac{m_\mu}{2} = \frac{207}{2} m_e = 103.5 m_e$. The energy levels are given by

$E_n^{muon} = \frac{\mu e^4}{(4\pi\varepsilon_0)^2 2\hbar^2}\frac{1}{n^2} = 103.5 E_n = 103.5 \times \frac{-13.6eV}{n^2}$. Hence the biding energy is $\Delta E = E_\infty - E_{n=1} = 0 - (-1407.6)eV$ = 1407.6 eV.

(b) $\frac{1}{\lambda} = \frac{E_i^{muon} - E_f^{muon}}{hc} = 103.5 \frac{m_e e^4}{(4\pi\varepsilon_0)^2 2\hbar^2 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = 103.5 R_\infty \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$,

where $R_\infty \equiv \frac{m_e e^4}{4c\pi\hbar^3(4\pi\varepsilon_0)^2} = 109{,}737\text{cm}^{-1}$. The first line in Lyman series correspond to n_i = 2, n_f = 1. Hence this wavelength is given by

$\frac{1}{\lambda} = 103.5 R_\infty \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3\times 103.5}{4} R_\infty = 8518334.625\,\text{cm}^{-1}$, or

$\lambda = 117.4\ \text{nm}$