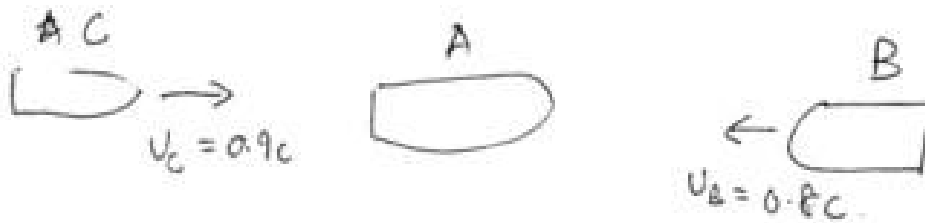


①



→ Assume that the direction to the right-hand side is positif

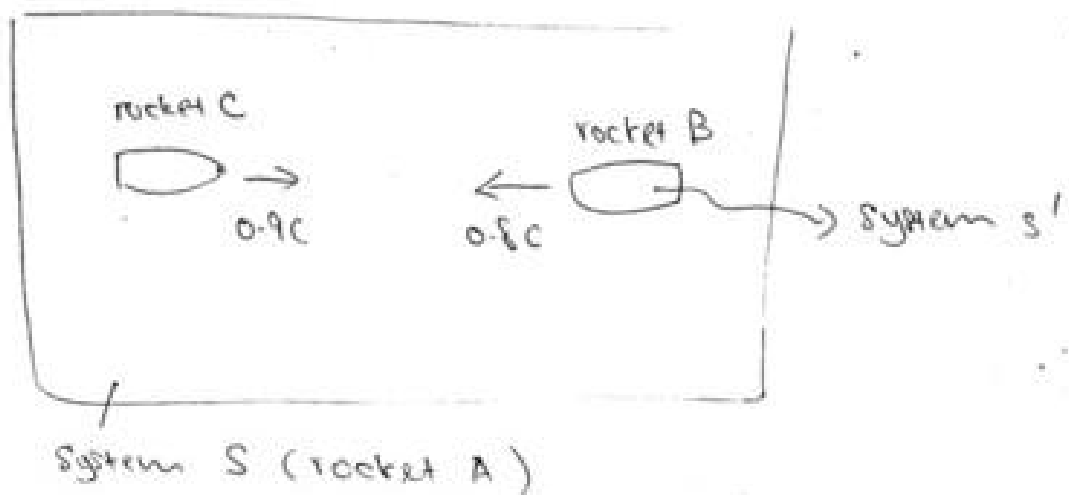
→ With respect to the rocket A,

→ the speed of rocket B, $v_B = -0.8c$

→ the speed of rocket C, $v_C = 0.9c$.

→ the speed of rocket C as measured by B :

→ Now, we may assume that the rocket B is a moving system (system S').



→ Here, $V = -0.8c$, $v_x (v_C) = 0.9c$ and

we need to determine v'_x (speed of rocket C as measured by B).

①

① By Galileo approach

$$v_x' = v_x - v$$

$$= 0.9c - (-0.8c) = 1.7c \quad \#$$

By Special relativity approach:

$$v_x' = \frac{v_x - v}{1 - \left(\frac{v}{c}\right)v_x}$$

$$= \frac{0.9c - (-0.8c)}{1 - \left(\frac{-0.8c}{c}\right)(0.9c)}$$

$$= \frac{1.7c}{1 + 0.72}$$

$$= \frac{1.7c}{1.72}$$

$$= 0.9884c \quad \#$$

② Given that the ^(or cut-off wavelength) threshold wavelength of the metal is
 $\lambda_{\text{cutoff}} = 600 \text{ nm}$.

→ The work function of this metal is given by

$$W_0 = \frac{hc}{\lambda_{\text{cutoff}}} \quad (1)$$

or

$$W_0 = hf_{\text{cutoff}} \quad (2)$$

where $h = \text{Planck constant}$

$$f_{\text{cutoff}} = \text{cut-off or threshold frequency} = \frac{c}{\lambda_{\text{cutoff}}}$$

→ From (1), hence.

$$W_0 = \frac{1240 \text{ eV} \cdot \text{nm}}{600 \text{ nm}} = 2.067 \text{ eV}$$

→ The photoelectric (PE) formula is given by

$$eV_s = hf - W_0 \quad (3)$$

or

$$eV_s = \frac{hc}{\lambda} - W_0 \quad (4)$$

where

$e = \text{charge of an electron}$

$V_s = \text{stopping potential}$

$$\left. \begin{array}{l} \text{or} \\ k_{\text{max}} = eV_s \\ \therefore k_{\text{max}} = hf - W_0 \end{array} \right\}$$

2 (a) Given that $\lambda = 400 \text{ nm}$.

→ From Eq. (1)

$$\begin{aligned} \therefore eV_s &= \frac{1240 \text{ eV} \cdot \text{nm}}{400} - 2.067 \text{ eV} \\ &= 3.100 \text{ eV} - 2.067 \text{ eV} \\ &= 1.033 \text{ eV} \end{aligned}$$

$$\therefore V_s = 1.033 \text{ V}$$

2b) Assume that the ^{light} frequency ~~at~~ in 2(a) is $f_1 = f = \frac{c}{\lambda}$

Now, the light frequency is $f_2 = 2f = \frac{2c}{\lambda}$

→ By replacing $f_2 = \frac{2c}{\lambda}$ into Eq. (3), hence.

$$\begin{aligned} eV_s &= \frac{2hc}{\lambda} - W_0 \\ &= 2 \left(\frac{1240 \text{ eV} \cdot \text{nm}}{400} \right) - 2.067 \text{ eV} \\ &= 2(3.100 \text{ eV}) - 2.067 \text{ eV} \\ &= 6.200 \text{ eV} - 2.067 \text{ eV} \\ &= 4.133 \text{ eV} \end{aligned}$$

$$\therefore V_s = 4.133 \text{ V}$$

∴ (c) → the stopping potential V_s is independent of the light intensity of light.

→ Hence, the V_s remain the same as in ∴(a)

$$\therefore V_s = 1.033 \text{ V}$$

3.

→ The Compton shift relationship is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos\theta) \quad (1)$$

or $\lambda' - \lambda = \lambda_c (1 - \cos\theta)$

where λ = wavelength of the incident photon

λ' = wavelength of the scattered photon.

m_0 = rest mass of electron.

θ = angle of the scattered photon.

λ_c = Compton wavelength = 2.43×10^{-3} nm.

3 (a) Here, $\lambda = 0.0248$ nm, $\theta = 90^\circ$, $\lambda' = ?$

→ From Eq. (1),

$$\lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos\theta)$$

$$\lambda' = 2.43 \times 10^{-3} [1 - \cos(90)] + \lambda$$

$$= 2.43 \times 10^{-3} \text{ nm} + 0.0248 \text{ nm}$$

$$= 0.02723 \text{ nm}$$

$$= 2.723 \times 10^{-2} \text{ nm}$$

3b)

$$P_x = \frac{h}{\lambda}$$

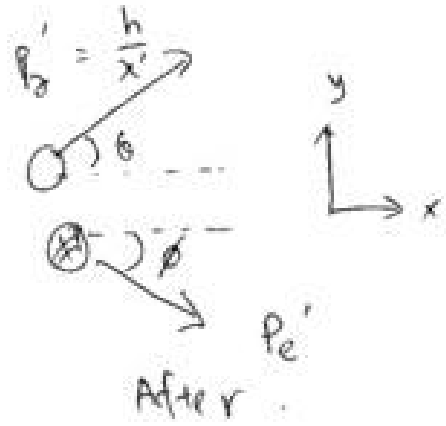


before

$$P_e = 0$$



m_e



After

- Momentum of the incident photon

$$\begin{aligned}
 P_x &= \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js}}{0.0248 \text{ nm}} \\
 &= \frac{6.63 \times 10^{-34} \text{ Js}}{2.48 \times 10^{-11} \text{ m}} \\
 &= 2.673 \times 10^{-23} \text{ kg ms}^{-1} \quad \#
 \end{aligned}$$

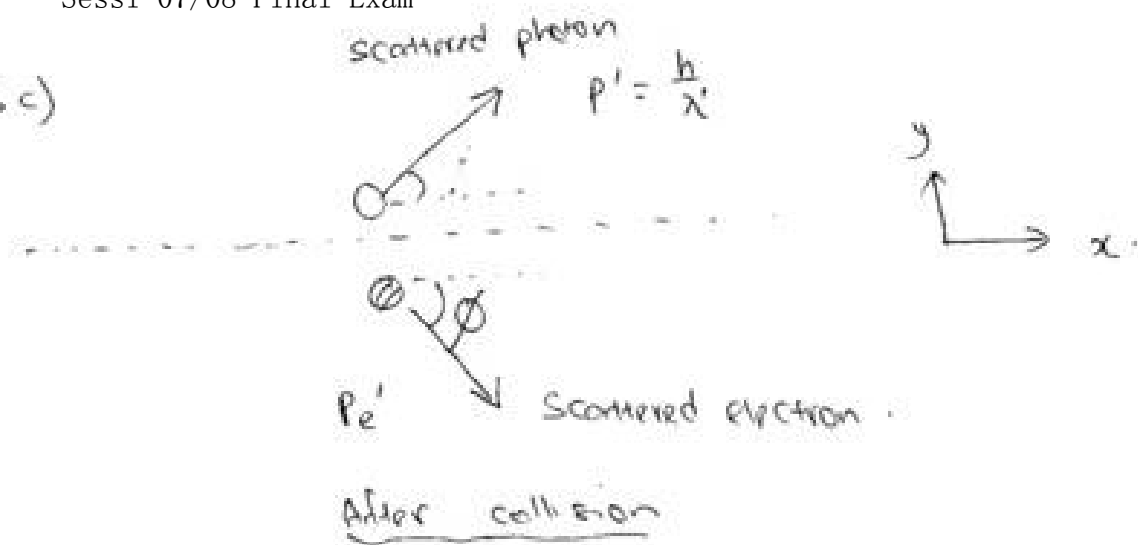
- Momentum of the scattered photons.

$$\begin{aligned}
 P'_x &= \frac{h}{\lambda'} \\
 &= \frac{6.63 \times 10^{-34} \text{ Js}}{2.723 \times 10^{-11} \text{ nm}} \\
 &= \frac{6.63 \times 10^{-34} \text{ Js}}{2.723 \times 10^{-11} \text{ m}} \\
 &= 2.435 \times 10^{-23} \text{ kg ms}^{-1} \quad \#
 \end{aligned}$$

or

$$\begin{aligned}
 |P'_x| &= \sqrt{(P'_x \sin \theta)^2 + (P'_x \cos \theta)^2} \\
 &= \sqrt{P'^2_x} \\
 &= P'_x \\
 &= \frac{h}{\lambda'} \\
 &= 2.435 \times 10^{-23} \text{ kg ms}^{-1}
 \end{aligned}$$

3c)



- Use the conservation of linear momentum in x- and y- directions.

Total momentum before = Total momentum after.

$$\therefore p + p_e = p' + p_e' \quad (1)$$

→ In x- direction

$$p + 0 = p' \cos \theta + p_e' \cos \phi$$

$$\therefore p_e' \cos \phi = p - p' \cos \theta \quad (2)$$

→ In y- direction

$$0 = p' \sin \theta - p_e' \sin \phi$$

$$\therefore p_e' \sin \phi = p' \sin \theta \quad (3)$$

$$3 \text{ c) } \frac{\textcircled{3}}{\textcircled{3}} \quad \frac{p_e' \sin \phi}{p_e' \cos \phi} = \frac{p' \sin \theta}{p - p' \cos \theta}$$

$$\therefore \tan \phi = \frac{\sin \theta}{p/p' - \cos \theta} \quad \textcircled{4}$$

- magnitude of the ^{momentum of the} scattered electron can be calculated using Eq. (1).

$$p = p' + p_e'$$

$$\therefore p_e' = p - p'$$

$$= 2.673 \times 10^{-23} \text{ kg m s}^{-1} - 2.435 \times 10^{-23} \text{ kg m s}^{-1}$$

$$= 0.238 \times 10^{-23} \text{ kg m s}^{-1}$$

$$= 2.38 \times 10^{-24} \text{ kg m s}^{-1} \quad \checkmark$$

→ the direction of the momentum of the scattered electron can be determined by using Eq. (4)

$$\therefore \tan \phi = \frac{\sin \theta}{p/p' - \cos \theta}$$

$$= \frac{\sin(90^\circ)}{\left(\frac{2.673 \times 10^{-23}}{2.435 \times 10^{-23}}\right) - \cos(90^\circ)} = \frac{1}{1.0977} = 0.911$$

$$\phi = 42.33^\circ$$

(h) d)

→ Heisenberg's uncertainty principle is given by

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \text{where } \hbar = \frac{h}{2\pi}$$

$$\therefore \Delta x \geq \frac{\hbar}{2} \cdot \frac{1}{\Delta p}$$

→ The minimum uncertainty in the position is then

$$\Delta x = \frac{\hbar}{2} \cdot \frac{1}{\Delta p} \quad (1)$$

→ Given that $m = 46 \text{ gm} = 46 \times 10^{-3} \text{ kg}$
 $v = 2 \text{ ms}^{-1}$.

→ then, the momentum of the gold ball is

$$p_g = mv = (46 \times 10^{-3} \text{ kg}) \times (2 \text{ ms}^{-1}) \\ = 9.2 \times 10^{-2} \text{ kg ms}^{-1}$$

→ Given that the accuracy of the momentum = 0.1%

$$\therefore \Delta p_g = 0.1\% \times p_g = \frac{0.1}{100} \times 9.2 \times 10^{-2} \text{ kgms}^{-1} \\ = 9.2 \times 10^{-5} \text{ kg ms}^{-1}$$

→ From Eq. (1), then

$$\Delta x = \frac{\hbar}{2} \cdot \frac{1}{\Delta p} \\ = \frac{6.63 \times 10^{-34}}{4\pi (9.2 \times 10^{-5})} = 5.735 \times 10^{-31} \text{ m.}$$

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A b) Here, $m_e = 9.1 \times 10^{-31} \text{ kg}$, $v = 2.4 \times 10^8 \text{ ms}^{-1}$.

- In this case, we must treat the problem relativistically, namely,

$$p_e = \gamma m_0 v$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v}{c}}} = \frac{1}{\sqrt{1 - \frac{2.4 \times 10^8}{3.0 \times 10^8}}} = 2.236$

$$\therefore p_e = 2.236 (9.1 \times 10^{-31}) (2.4 \times 10^8) = 4.883 \times 10^{-23} \text{ kgms}^{-1}$$

→ Given that the uncertainty of momentum = 0.1%

$$\begin{aligned} \therefore \Delta p_e &= 0.1\% \times p_e \\ &= \frac{0.1}{100} \times 4.883 \times 10^{-23} = 4.883 \times 10^{-25} \text{ kgms}^{-1} \end{aligned}$$

→ From Eq (1), then

$$\begin{aligned} \Delta \lambda &= \frac{h}{\lambda} \cdot \frac{1}{\Delta p_e} \\ &= \frac{6.63 \times 10^{-34}}{4\pi (4.883 \times 10^{-25})} \\ &= 1.080 \times 10^{-10} \text{ m} \end{aligned}$$

3 a)

→ The Paschen series for hydrogen is given by

$$\frac{1}{\lambda} = R_H \left[\frac{1}{3^2} - \frac{1}{n_i^2} \right] \quad (1)$$

where $R_H = \text{Rydberg constant} = 1.097 \times 10^7 \text{ m}^{-1}$

$$n_i = 4, 5, 6, \dots$$

→ the longest wavelength of the Paschen series for hydrogen corresponds to

$$n_i = \infty$$

$$\begin{aligned} \frac{1}{\lambda} &= R_H \left[\frac{1}{3^2} - \frac{1}{\infty^2} \right] \\ &= 1.097 \times 10^7 \left(\frac{1}{9} - 0 \right) \\ &= 1.2189 \times 10^6 \text{ m}^{-1} \end{aligned}$$

$$\therefore \lambda = 8.204 \times 10^{-7} \text{ m}$$

*

5 b)

→ The shortest wavelength of the Paschen series for hydrogen is corresponds to

$$n_i = n_f + 1$$

$$= 3 + 1$$

$$= 4$$

$$\frac{1}{\lambda} = R_H \left[\frac{1}{3^2} - \frac{1}{4^2} \right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{9} - \frac{1}{16} \right]$$

$$= 5.333 \times 10^5 \text{ m}^{-1}$$

$$\therefore \lambda = 1.875 \times 10^6 \text{ m}$$

5 b)

→ The general formula of the spectral line series for hydrogen is given by

$$\frac{1}{\lambda} = R_H \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$$

→ Given that $\lambda = 4861 \text{ \AA} = 4861 \times 10^{-10} \text{ m}$
and $n_f = 2$.

→ from 5(a), $R_H = 1.097 \times 10^7 \text{ m}^{-1}$.

$$\frac{1}{4861 \times 10^{-10}} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{n_i^2} \right]$$

$$\frac{1}{(4861 \times 10^{-10})(1.097 \times 10^7)} = \left(\frac{1}{4} - \frac{1}{n_i^2} \right)$$

$$0.1875 = 0.25 - \frac{1}{n_i^2}$$

$$\frac{1}{n_i^2} = 0.25 - 0.1875$$

$$\frac{1}{n_i^2} = 0.0625$$

$$n_i^2 = \frac{1}{0.0625}$$

$$n_i^2 = 16$$

$$n_i = 4 \quad \checkmark$$