make-up test 0708

FIZIK IV (MODERN PHYSICS)
ZCT 104
TEST
24 March 2008

Instruction: Two questions are prepared. Please answer both of them in this question sheet. Use extra blank paper if you need more space. Explain your steps as clearly as possible.

1. In Compton scattering experiment, X-ray photons with wavelength λ are incident on a target. After collision with electrons which initially at rest, the scattered photons are observed at angle θ while the recoiled electrons are observed at angle ϕ.

   (a) Show that the angle between the directions of the recoil electron and the incident photon is given by

   $$\tan\phi = \frac{\sin\theta}{(1+\cos\theta)\left[\frac{h}{m_e c\lambda}+1\right]}.$$

   Here h = Planck constant
   λ = wavelength of incident photons
   m_e = rest mass of electron
   c = speed of light

   (b) If the wavelength of the X-rays is 1.5406Å and the scattered photons are observed at 120°, find
   (i) the wavelength of the scattered photons,
   (ii) the angle of the recoil electron, and
   (iii) the kinetic energy of the recoil electron (in keV).

[25 marks]

make-up test 0708

2. (a) Derive a non-relativistic formula that gives the de Broglie wavelength of a particle, of charge q and mass m, in terms of the potential difference V through which it has been accelerated.

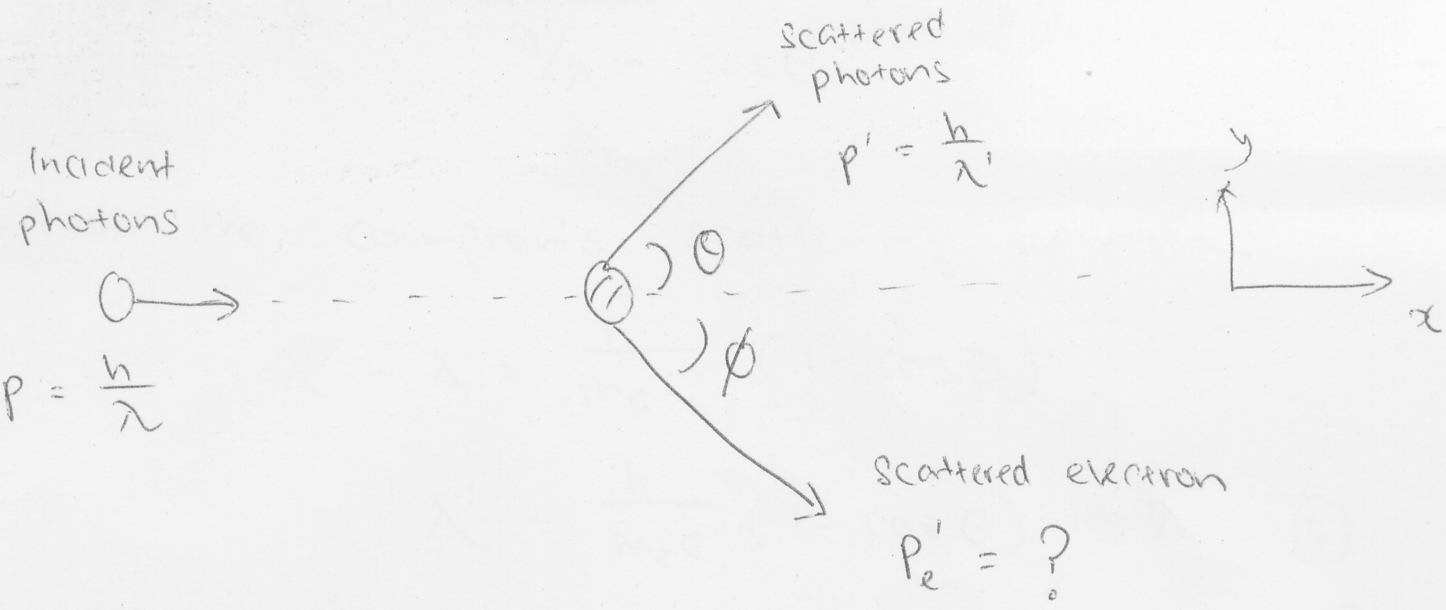
(b) Derive the relativistic version for the de Broglie wavelength in (a).

(c) Show that your formula in (b) reduces to that of (a) in the limit of $qV << mc^2$.

[25 marks]

Solution:
make-up test 0708

a) Let consider the conservation of momentum in 2-D



$\Rightarrow$ Based on the Principle of conservation of Momentum

Initial momentum = final momentum.

$\Rightarrow$ In x- direction:

$$P + 0 = P'\cos\theta + P_e'\cos\phi$$

$$P_e'\cos\phi = P - P'\cos\theta \qquad (1)$$

$\Rightarrow$ In y- direction

$$0 + 0 = P'\sin\theta - P_e'\sin\phi$$

$$\therefore \quad P_e'\sin\phi = P'\sin\theta \qquad (2)$$

$$\Rightarrow \frac{(2)}{(1)} \qquad \tan\phi = \frac{P'\sin\theta}{P - P'\cos\theta}$$

$$= \frac{\sin\theta}{P/P' - \cos\theta} \qquad (3)$$

Since $p = \frac{h}{\lambda}$ & $p' = \frac{h}{\lambda'}$, we have

$$\tan\phi = \frac{\sin\theta}{\lambda'/\lambda - \cos\theta} \quad (4)$$

⇒ From the compton's scattering formula:

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

$$\lambda' = \frac{h}{m_e c}(1-\cos\theta) + \lambda \quad (5)$$

⇒ ~~Replace (5)~~ By substituting (5) into (4)

$$\therefore \tan\phi = \frac{\sin\theta}{\frac{\left[\frac{h}{m_e c}(1-\cos\theta)+\lambda\right]}{\lambda} - \cos\theta}$$

$$= \frac{\sin\theta}{\left(\frac{h}{m_e c\lambda}(1-\cos\theta)+1\right) - \cos\theta}$$

$$= \frac{\sin\theta}{(1-\cos\theta)\left[\frac{h}{m_e c\lambda}+1\right]} \quad (6)$$

(shown)

b) Here $\lambda = 1.5406\times10^{-10}$ m , $\theta = 120^\circ$

make-up test 0708

i) $$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

$$\lambda = \frac{h}{m_e c}(1-\cos\theta) + \lambda$$

$$= 2.43\times10^{-12}(1-\cos 120) + 1.5406\times10^{-10}\ \text{m}$$

$$= 1.5771\times10^{-10}\ \text{m}$$

$$= 1.5771\ \text{Å}$$

ii) From Equation (6)

$$\tan\phi = \frac{\sin 120}{(1-\cos 120)\left[\frac{2.43\times10^{-12}}{1.5406\times10^{-10}}+1\right]}$$

$$= \frac{0.866}{(1.5)(1.0158)}$$

$$= 0.5684$$

$$\phi = 29.61^\circ$$

iii) The kinetic energy of the recoil electron:

$$KE\ (\text{recoil electron}) = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

$$= \frac{1240\ \text{eV.nm}}{0.15406\text{nm}} - \frac{1240\ \text{eV.nm}}{0.15771\ \text{nm}}$$

$$= 8.049\ \text{keV} - 7.863\ \text{keV}$$

$\therefore$ KE (recoil electron) $= 0.186$ keV

make-up test 0708

Solution:

Q2. Beiser, Ex. 12, pg 117.

(a) Non-relativistic scenario:

If the electron is non-relativistic, $K = p^2/2m$;

According to de Broglie's postulate, $p = h/\lambda$;

$K = qV$;

Hence,

$K = qV = p^2/2m = (h/\lambda)^2/2m$

$\Rightarrow qV = h^2/2m\lambda^2$

$\Rightarrow \lambda = h/\sqrt{(2mqV)}$.

(b) $K = qV$;

K is related to momentum p via $E^2 = (K + mc^2)^2 = p^2c^2 + m^2c^4 \Rightarrow K^2 + 2Kmc^2 = p^2c^2$

$$\Rightarrow K = \frac{-2mc^2 \pm \sqrt{4m^2c^4 + 4p^2c^2}}{2} = -mc^2 \pm c^2\sqrt{m^2 + p^2/c^2}$$

$$\Rightarrow K = c^2\left(\sqrt{m^2 + p^2/c^2} - m\right) = qV$$

$$p^2 = \left(\frac{qV}{c^2} + m\right)^2 c^2 - m^2c^2 = \frac{q^2V^2}{c^2} + 2mqV = \frac{h^2}{\lambda^2}$$

$$\Rightarrow \lambda = \frac{hc}{\sqrt{q^2V^2 + 2mc^2qV}}$$

(c) λ In non-relativistic limit:

$$\lambda = \frac{hc}{\sqrt{2qVmc^2}\sqrt{\frac{qV}{2mc^2} + 1}} = \frac{hc}{\sqrt{2qVmc^2}}\left(1 + \frac{qV}{2mc^2}\right)^{-1/2} = \frac{hc}{\sqrt{2qVmc^2}}\left(1 - \frac{qV}{4mc^2} + \ldots\right)$$

$$\Rightarrow \lim_{qV \ll mc^2} \lambda = \frac{hc}{\sqrt{2qVmc^2}} = \frac{h}{\sqrt{2qVm}}$$