ZCT 104/3E Modern Physics
Semester I, Sessi 2007/08
1. Test I (26 Jan 2008)

Name:____

Matrix Number:______________________________________

Class A / Class B

Instruction: Do all questions in Section A and Section B.

Duration : 1 Hour

Section A (Objective questions)

1. In the following, which statement(s) is (are) correct regarding two events, A and B, at the space-time coordinates (x_A,t_A) and (x_B,t_B)? Assume $x_A > x_B$, $t_B > t_A$.

   I. If $x_B - x_A > c(t_B - t_A)$, events A and B can never be causally related.
   II. If $x_B - x_A < c(t_B - t_A)$, events A and B could be causally related.
   III. If $x_B - x_A = c(t_B - t_A)$, events A and B must be causally related.
   IV. If $x_B - x_A = c(t_B - t_A)$, events A and B could be causally related.

A. I, II   B. II, IV   C. I, II, III   D. I, II, IV   E. None of A, B, C, D
ANS: D (I, II, IV true).

2. Referring to Question 1 above, the space-time interval of the events A and B, $c^2(t_B - t_A)^2 - (x_B - x_A)^2$, is

   A. always the same in all inertial frames of reference.
   B. unpredictable when measured in other frame of reference.
   C. always positive
   D. always negative
   E. None of A, B, C, D.

ANS: A

3. O' is running at a velocity of v towards O. O' throws out a ball towards O. The velocity of the ball as measured by O is u. What is the velocity of the ball as measured by O'? Assume the velocities are non-relativisitic.

   A. $u + v$ B. $u - v$ C. $v - u$ D. $\sqrt{|u^2 - v^2|}$

   E. (None of **A**, **B**, **C**)
   ANS: B

4. Reconsider Question 3 above. Which of the following statement(s) is (are) true?

   I. Galilean transformation of velocities can be used to calculate the correct relative velocities in Question 3.
   II. Lorentz transformation of velocities can be used to calculate the correct relative velocities in Question 3.
   III. Either Galilean or Lorentz transformation of velocities can be used to calculate the correct relative velocities in Question 3.
   IV. Only one of the Galillean or Lorentz transformations of velocities, BUT not both, can be used to calculate the correct relative velocities in Question 3.

A. I,II B. IV only C. I,II,IV D. I, IV E. None of A,B,C,D

ANS: E (I,II, III are true.)

5. Reconsider the similar scenario as in Question 3 above but with both *u*, *v relativistic.* Which of the following statements is true?

A. the speed of the ball as measured by O' would be smaller than that in Question 3.
B. the speed of the ball as measured by O' would be larger than that in Question 3.
C. the speed of the ball as measured by O' would be equal to that in Question 3.
D. the speed of the ball as measured by O' could be larger or smaller than that in Question 3.
E. None of A, B,C,D.
ANS: B

ANS: Let *u* be the velocity of the ball observed by O. O' is moving with a velocity *v* with respect to O. The Lorentz transformation for *u*,*v* and *u*', the velocity of the ball as measured by O', is simply

$u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$. Since u,v has the same direction, $1-\dfrac{uv}{c^2}<1, \therefore |u'| = \left|\dfrac{u-v}{1-\dfrac{uv}{c^2}}\right| > |u-v|$

6. Given a species of fly has an average lifespan of t. Let say you put many of them in a box and send them to a destination at some remote destination in deep space using a rocket that travel at speed *v*. The destination is located at a distance of *L* from Earth. Considering only special relativistic effect and assuming that none of the flies die of any cause other than aging, which of the following statements is (are) correct? (Lorentz factor is defined as $\gamma = [1-(v/c)^2]^{-1/2}$).

I. To the Earth observer, the time taken by the rocket to arrive at its destination is L/v.
II. To the flies, the time taken by the rocket to arrive at its destination is $(1/\gamma)(L/v)$.
III. Most of the flies would survive if $(1/\gamma)(L/v) < \tau$
IV. Most of the flies would survive if $(L/v) < \tau$

A. I,II B. I,II,III C. I,II,IV D. I, II,III,IV
E. None of A,B,C,D

ANS: D (ALL are true.)

7. Consider an object moving in a straight line with constant speed. Say in frame O, the momentum of the object is measured to be P. In a frame O$'$ moving with a non-zero relative constant velocity with respect to O, the momentum of the same object is measured to be P'. Which of the following statements are (is) true regarding P and P'?

I. In non-relativistic regime, P and P' have a same numerical value.
II. In relativistic regime, P and P' have a same numerical value.
III. P and P' have a same numerical value in the relativistic regime but not in the non-relativistic one.
IV. P and P' have a same numerical value in the non-relativistic regime but not in the relativisitic one.

A. I,II B. II,III C. I, IV D. III,IV
E. None of A, B, C,D

ANS: E (*P*,*P*' in general are different in different frames of reference, since momentum is not an invariant.)

8. Consider an object moving in a straight line with constant speed. Say in frame O, the kinetic energy of the object is measured to be K. In a frame O$'$ moving with a non-zero relative constant velocity with respect to O,

the kinetic energy of the same object is measured to be K'. Which of the following statements are (is) true regarding K and K'?

I. In non-relativistic regime, K and K' have a same numerical value.
II. In relativistic regime, K and K' have a same numerical value.
III. K and K' have a same numerical value in the relativistic regime but not in the non-relativistic one.
IV. K and K' have a same numerical value in the non-relativistic regime but not in the relativisitic one.

A. I,II B. II,III C. I, IV D. III,IV
E. None of A, B, C,D

ANS: E (K,K' in general are different in different frames of reference, since kinetic energy is not an invariant.)

9. A subatomic particle of rest mass of M, initially at rest, decays into two daughter subatomic particles with rest masses m_1 and m_2 respectively. Which statements in the following is (are) true?

I. $M = m_1 + m_2$
II. $(M - m_1 - m_2)c^2$ equals the sum of kinetic energy of the daughter subatomic particles.
III. The kinetic energy of the daughter subatomic particle with rest mass m_1 equals the kinetic energy of the daughter subatomic particle with rest mass m_2.
IV. The momentum of the daughter subatomic particle with rest mass m_1 equals the momentum of the daughter subatomic particle with rest mass m_2.

A. I,IV B. II,III, IV C. III, IV D. II,IV
E. None of A,B,C,D

ANS: D

10. A clock moving with a finite speed v is observed to run slow. If the speed of light were to be halved, you would observe the clock to be

A. Even slower.
B. Still slow but not as much
C. As slow as it was
D. To start to actually run fast.
E. None of A, B, C,D

ANS: A. The gamma-factor would be modified. The time dilation effect will be greater if c becomes $c/2$.

$$g(c) \to g(c/2) \equiv g'(c)$$

$$\Rightarrow (g')^2 = \frac{1}{1-\left(\frac{v}{c/2}\right)^2} = \frac{1}{1-4\left(\frac{v}{c}\right)^2} > g^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2}$$

Alternatively, in an ordinary world with speed of light c, the asymptote of the Lorentz factor g is at v=c, whereas in a world with speed of light c'=c/2, the asymptote for g' occurs at v = c/2. This intuitive argument leads to the conclusion that $g'$is generally larger than g at the same v.

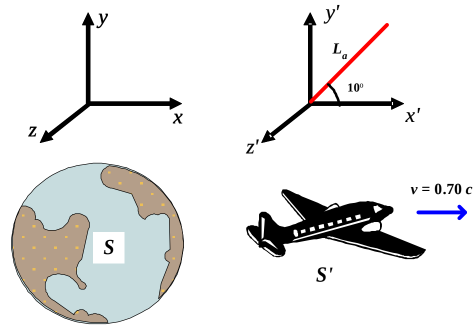
Section B (Structured questions)

1. A spacecraft antenna is at an angle of 10° relative to the axis of the spacecraft. If the spacecraft moves away from the earth at a speed of 0.70*c*, what is the angle of the antenna as seen from the earth? (biser, p.50) [Ans. 14°]

2. At what velocity does the KE of a particle equal its rest energy? [Ans. 2.6×10^{8} m s^{-1}] (Chand, p.25)

Solutions:

1. Let say the length of the antenna as measured by an observer on the spacecraft (system S') is L_a.



According to system S' :

The projection of the antenna onto the spacecraft,

$$L_{a,x'} = L_a \cos(10°).$$

The projection of the antenna onto an axis perpendicular to the spacecraft's axis,

$$L_{a,y'} = L_a \sin(10°).$$

To an observer on the earth (system S):

The length in the direction of the spacecraft's axis will be contracted:

$$L_{a,x} = \frac{1}{\gamma} L'_{a,x'} = L'_{a,x'}\sqrt{1-\frac{v^2}{c^2}}$$

$$\therefore \qquad L_{a,x} = L_a \cos(10°)\sqrt{1-\left(\frac{0.70c}{c}\right)^2}\ . \qquad (1)$$

The length perpendicular to the spacecraft's motion will appear unchanged:

$$L_{a,y} = L_{a,y'} = L_a \sin(10°). \qquad (2)$$

The angle as seen from the earth will then be [Eq. (2) / Eq. (1)]:

$$\tan q = \frac{L_{a,y}}{L_{a,x}}$$

$$= \frac{L_a \sin(10°)}{L_a \cos(10°)\sqrt{1-\left(\frac{0.70c}{c}\right)^2}}$$

$$= \frac{\tan(10°)}{\sqrt{1-\left(\frac{0.70c}{c}\right)^2}}$$

$$\therefore \qquad q = \arctan\left[\frac{\tan(10°)}{\sqrt{1-\left(\frac{0.70c}{c}\right)^2}}\right]$$

$$= 14°$$

2. If the kinetic energy KE = $E_0 = m_0c^2$, then the total energy will become

$$E = E_0 + KE = m_0c^2 + m_0c^2 = 2m_0c^2 \qquad (1)$$

Since,

$$E = mc^2 = gm_0c^2 = \frac{1}{\sqrt{1-v^2/c^2}}m_0c^2 \qquad (2)$$

Therefore, Eq. (1) = Eq. (2)

$$\therefore \qquad \frac{1}{\sqrt{1-v^2/c^2}}m_0c^2 = 2m_0c^2$$

$$\frac{1}{\sqrt{1-v^2/c^2}} = 2$$

Solving for *v*,

$$v = \frac{\sqrt{3}}{2}c = 2.60\times10^8 \text{ m s}^{-1}$$